Uncertainty Quantification Based Receptivity Modelling of Crossflow Instabilities Induced by Distributed Surface Roughness in Swept Wing Boundary Layers

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A high fidelity methodology based on the rapid solution of a linearised Navier-Stokes equation set is used to model the role of distributed surface roughness in generating stationary crossflow disturbances on swept wing flows. The technique is based on a stochastic description of surface roughness linked to high precision measurement data for both anodised aluminium and painted surfaces. A Monte Carlo analysis of laminar-turbulent transition is demonstrated for data from the AERAST wind tunnel test which was designed to show control of stationary cross-flow disturbances through the forced excitation of a sub-dominant disturbance mode generated by the artificial placement of periodically distributed roughness elements. The method provides both the roughness induced naturally occurring and forced control mode disturbance amplitudes for initialisation of a simulation based on solution of the non-linear parabolised stability equations. Monte-Carlo based uncertainty quantification analysis enables propagation of uncertainties from the roughness field through the receptivity phase, to ultimately providing bounds on the predicted transition location.

I. Introduction

Receptivity modelling and its inclusion in advanced predictive methods for transition, is now recognised as the key requirement, for the accurate simulation of the breakdown process in laminar to turbulent flow. Transition modelling in shear flows is of both fundamental and practical interest to many technologies currently being pursued in the aerospace sector. To date, most effort in transition modelling has been motivated by the concept of developing a laminar flow wing in order to reduce drag. However, turbulent wing design and in general CFD tools have evolved to the stage where uncertainties associated with the unknown transition location has a significant effect on the predicted flow development and hence aerodynamic performance. The work of the group led by Saric and colleagues at Arizona State University, on the experimental detection of stationary crossflow (CF) disturbances on swept leading edges,1 and their stabilisation by the forcing of sub-dominant crossflow vortices, has further highlighted the need to incorporate the receptivity process. Agreement with experiments only arises once an adequate description of the key underlying mechanisms of disturbance generation are included in the initialisation of nonlinear simulations via Parabolised Stability Equations (PSE).2 The relative efficiency, and almost Direct Numerical Simulation (DNS) like properties3, 4 of the PSE are now well documented, thus the coupling of nonlinear PSE models with receptivity models appears to be the most obvious means to devise a high-fidelity physics based transition analysis capability. The PSE method brings to the fore the importance of building into the prediction approach, the role of freestream environment (sound, turbulence) and wing surface roughness, vibration etc.

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The empirical N factors based approach omits this important and crucial so-called receptivity mechanism in the transition process. Non-linearity requires the setting of initial amplitudes, and the full benefits of non-linear modelling can only be achieved by inclusion of the receptivity stages. However, modelling of the receptivity phenomenon generally requires a non-PSE based model due to the inherent inability of PSE models to capture the short-scale conversion processes, arising in the disturbance generation.

Theoretical and computational methods have to be developed to resolve the problem and the successful coupling of receptivity methods to conventional linear and nonlinear instability tools in an integrated manner will provide a major enhancement to predictive capability. This is the key driver of the work reported here.

The process of receptivity modelling is a complex phenomenon with numerous actors governing the disturbance generation process. The most common scenarios involve the natural surface roughness acting on its own or in combination with external excitations of mainly acoustical and/or vortical origin. Three-dimensionality or sweep and compressibility are additional complicating factors. It is now accepted that in atmospheric flight environments, low levels of freestream turbulence mean stationary crossflow induced transition is the primary instability mechanism to control. This is generally induced by the natural surface roughness characteristics of the wing. There is still some debate in the wider research community about the role, if any, in atmospheric flight of acoustic/vortical disturbances coupling to surface-roughness to induce travelling crossflow disturbances and the relative magnitude of this compared to pure stationary crossflow induced disturbances. In noisier environments such as wind-tunnels, travelling crossflow mechanisms, involving a coupling of freestream vortical/acoustic fluctuations with the natural surface roughness, will usually be the dominant actor in the breakdown of laminar flow to turbulence.

Another aspect causing much confusion is the need or otherwise to model geometrical surface curvature in the context of receptivity. It has been known for some considerable time that curvature included PSE models predict (as is verified by experiment) a significant stabilising effect of curvature on stationary crossflow waves, particularly around the curved leading edge of a swept wing. In the recent work of Malik et al., on the NASA Environmentally Responsible Aviation Project, computational assessment of the Gulfstream-III (G-III) aircraft wing-glove design revealed strong curvature dependent effects on instability amplitudes. Inclusion of curvature and non-parallel modelling within the receptivity modelling arena is contentious. However, Ng & Crouch report good agreement using simple finite Reynolds number residue based local analysis, while Collis & Lele claim curvature and non-parallelism have an effect.

Considerable work in the general area of modelling receptivity by various theoretical and numerical approaches has been published in the literature. Approximate, finite Reynolds number theory (FRNT)\(^9\) approaches based on the asymptotic works of Ruban\(^9\) and Goldstein,\(^10\) are simple to devise though they have many weaknesses. For detailed quantitative comparison with experiment adjoint PSE, direct and adjoint linearised Navier-Stokes (LNS) (for latter, see Carpenter et al.) and even more demanding DNS equations are now the preferred way forward. As a result of both continued speed-ups in computing hardware, parallel processing and advances in numerical algorithms, solutions to LNS and DNS of transitional flows has become increasingly practical during recent years. Early works, among others are those of Liu et al., and Lin & Reed in the low speed context, and more recently a concerted effort led by various groups such as Pruet et al., Zhong\(^14\) and Balakumar,\(^15\) has extended the work to high speed flows with and without shocks, into the hypersonic regime. The work of Piot et al. is also worthy of mention; a full unsteady DNS method is used to examine periodic crossflow instabilities generated by a roughness array. Detailed flow field structure around a single micron-sized square-element in a periodic spanwise domain is computed.

Recent receptivity modelling work in a similar vein to Piot et al.,\(^16\) using brute force numerics, includes that of Tempelmann et al.\(^4\) and Rizzetta et al.\(^17\) The latter solve the fully unsteady compressible Navier Stokes equations with a time-accurate, approximate implicit factored finite differenced algorithm, with Newton-like sub-iterations to treat the nonlinear terms. Their results are interesting in that they show, for small enough roughness elements, no evidence of any horseshoe vortex produced downstream of the elements, as would occur if the height was considerably larger. This is important, since in the linearised perturbation type framework used in this paper, it justifies the use of steady boundary-layer parabolic marching method to compute the background steady field.

All LNS and DNS methods discussed have been developed over a long and sustained period of time and rely heavily on the use of parallel processing for the computations. In order to limit computational overhead Rizzetta et al. only solve the flow field in the immediate vicinity of the roughness elements using complex gridding techniques around the elements and making use of an assumption of a zero-lift surface pressure distribution to truncate their computational domain. For rapid design and parametric investigations such
techniques are not appropriate due to considerable computational resource utilisation, requiring long compute times on a dedicated HPC cluster.

In the work described here the background basic field is computed using a standard compressible boundary-layer solver based on the infinite swept approximation. The pressure distribution which drives the solution of the solver is, in one instance (AERAST test data) based on experimentally measured values, while in the other (EADS-ATC3 data set), surface pressure from Reynolds Averaged Navier Stokes (RANS) based simulation is used as the input to the boundary-layer code.

In this paper, we use the technique adopted by Streett,\cite{Streett1995} namely solving a discretised version of the harmonic form of the incompressible and compressible LNS equations via a direct matrix inversion approach. Significant improvements in computer hardware over the past decade, have made the harmonic approach very appealing today. Typically, a highly resolved solution for the fully compressible LNS equation set is computable in under three minutes on a high-end standalone multi-core workstation. The drawback in this approach is the excessive memory requirements (~16 gigabytes RAM). However, present day workstations can easily hold in excess of 128 gigabytes of RAM thus, in our view, this is not a major limiting issue. The research being undertaken in the general area of receptivity modelling is wide ranging in scope but specifically in this paper the focus is on the incorporation of a receptivity model for coupling stationary crossflow vortices to nonlinear PSE based models and the use of uncertainty quantification tools of analysis to incorporate variance in the transition location. This must naturally arise due to the stochastic nature of the transition process as it is governed primarily by very small scale random and machined in, surface roughness fields, forcing and generating instability waves in the boundary-layer.

The precise manner of providing a quantitative link between height, shape and width of micron-sized surface roughness elements and amplitudes of the generated linear modes, provoked to control natural flow disturbances is one aspect modelled. The issue of developing a stochastic based transition prediction method incorporating the effects of surface finish on natural modes is the primary focus. The work and results reported below, goes some way towards addressing the current shortfall in predictive capability in this area though considerable further work remains.

### I.A. DRE Control of Crossflow Vortices

Saric and co-workers,\cite{Saric2008} in their flow control and stabilisation work, utilised distributed roughness elements (DRE) as the seeding to force a sub-dominant stationary CF vortex to suppress the naturally most amplified disturbance. Roughness elements were placed at periodic intervals in a single row along the spanwise extent of the wing, just ahead of the neutral locus. It was shown that provided the generated sub-dominant vortex was strong enough, it suppressed the development of the naturally occurring mode through nonlinear interaction and no further intervention or further unnatural forcing was required to achieve the desired effect.

The precise mechanism to invoke for the control to be effective was well elucidated in the AERAST project,\cite{AERAST2010} and was partially demonstrated in wind-tunnel experiments (ONERA F2 tunnel, Toulouse). Theoretical objectives of the project were to improve upon modelling capability in this area. The 40° swept model with 0.8 m chord and span of 1.2 m, was designed using the linear and nonlinear PSE model, with the nonlinear numerics initiated by the assumption that the forcing mode and control modes could simply be prescribed small amplitudes and, by performing a number of runs with differing levels of forcing, a feel for the 'correct' seeding amplitudes to attain sufficient enough control effectiveness, could be gauged. Transition was observed by the use of infra red camera imaging. Detailed boundary-layer measurements of the disturbance structure were not made, thus precise comparison of experimental data with the theoretical tool was not possible in this case. The experimentally measured pressure distribution for the designed geometry, having the key requirements for effective DRE control, is shown in figure 1.

With such a control technique, the key requirement is the ability to generate the sub-dominant stationary vortex by whatever means, whether this be by applying roughness, bumps, dips, localised suction/blowing actuators on their own or coupled to a flexible membrane\cite{Karniadakis2002} or more exotic devices such as localised hot/cold spots. Steady suction has been known for decades to delay the onset of the drag-increasing turbulent state of the boundary layer by significantly enhancing its laminar stability and thus pushing back laminar-turbulent transition. Discrete suction, through groups of micro-holes or slots can also be used to excite crossflow vortices.\cite{Streett1995}

We use the DRE control work of Saric and colleagues\cite{Saric2008} and the experimental data from the AERAST test, as a basis to guide the development of the analysis tool. There are two requirements for the modelling of the DRE concept, namely receptivity due to spanwise periodic roughness elements of various types and...
shapes,\textsuperscript{17} which generate the forced sub-dominant (0, 3) CF modes, and a model for a randomised distributed surface roughness field which gives rise to the naturally occurring (0, 2) CF mode to be stabilised. The latter is modelled using height data from real measured surface roughness fields, comprising a painted and an unpainted aluminium surface (figure 5), typically used in aircraft wing structures, as prototypes to investigate the possibility of including real roughness fields in receptivity models.

The work reported below, focuses on the ability to predict precisely the control effectiveness achievable, prior to the design of the experimental programme. Ideally one would like to be able to perform a computational prediction and expect the experiment to reproduce the numerical result. Such practicality, in addition to including effects of roughness shape, width, different forcing actuators, surface curvature, non-parallelism etc. can be most readily and directly modelled by solutions to LNS equations. Saric’s group found excellent agreement between experiment and nonlinear PSE theory, however their seeding for the initialisation of the nonlinear PSE numerics, was from the experimentally measured disturbance amplitudes, rather than using detailed receptivity modelling.

In \textsection II we describe the numerical model and \textsection III describes the manner of modelling natural distributed surface roughness. Usage of a Monte-Carlo (MC) based uncertainty quantification (UQ) technique is then described and used in the analysis of two specific swept wing data sets. \textsection V presents the analysis for the AERAST data set, where pressure data from experiments conducted in the ONERA F2 tunnel is used. \textsection VI presents preliminary theoretical receptivity results for a laminar flow experimental wind-tunnel test campaign (figure 15) representative of a civil transport outer wing. A high fidelity transition prediction methodology is devised based on the UQ approach, and a systematic integration of the various stages in the analysis process is defined.

II. Linearised Navier Stokes Model

We use a harmonic form of the linearised Navier-Stokes equations, obtained by splitting the velocities \((u, v, w)\), pressure \((P)\) and temperature \((T)\) fields, \(q = (u, v, w, P, T)^T\), as follows

\[
\ddot{q}(x, y, z, t) = \bar{Q}(x, y) + \epsilon q(x, y)e^{i(\beta z - \omega t)}. \tag{1}
\]

The \(\bar{Q}(x, y)\) field is computed with a compressible infinite swept boundary-layer solver, while the order \(\epsilon\) problem defines the LNS equation set that we solve.\textsuperscript{18,23} \(\beta\) defines the spanwise wavenumber and \(\omega\) the unsteady frequency for time dependent forcing problems. Incorporation of geometrical curvature is via a body-fitted coordinate system, which is well suited for analysis when confined to the scales associated with boundary-layer flows, since disturbance structures of interest are confined to the vicinity of the viscous region.

II.A. Boundary Conditions

In the far field \(y \rightarrow \infty\), Dirichlet conditions are imposed, where we simply state that disturbances outside the regions of interest have decayed to negligible magnitude. At the wall \((y = 0)\) no-slip conditions are generally applied, namely the velocity, temperature disturbance states \((u, v, w) = T = 0\), and the pressure disturbance

\[
-0.4 \quad -0.2 \quad 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0
\]

\(x/c\)

\(Cp\)

Expt. measured

\(0.0\)

\(0.08\)

\(0.16\)

\(0.24\)

\(0.32\)

\(0.40\)

\(0.48\)

\(0.56\)

\(0.64\)

\(0.72\)

\(0.80\)

\(0.88\)

\(0.96\)

\(1.04\)

Figure 1. AERAST test campaign ONERA F2 tunnel; experimentally measured Cp data for \(Re = 3.7 \times 10^6\). The small inset, shows the upper surface geometry of the wing panel.
is discretised and solved for exactly at the wall. Wall slip conditions where the roughness surfaces have a non-zero size off the nominally smooth base surface, are satisfied by the use of first-order Taylor series expansions to transfer no-slip conditions to \( y = 0 \), thus

\[
\begin{align*}
    u(x, 0) &= -\epsilon_R \hat{H}(x, z, t) \frac{\partial \hat{U}}{\partial y}, \\
    v(x, 0) &= -\epsilon_R \hat{H}(x, z, t) \frac{\partial \hat{V}}{\partial y}, \\
    w(x, 0) &= -\epsilon_R \hat{H}(x, z, t) \frac{\partial \hat{V}}{\partial y}, \\
    T(x, 0) &= -\epsilon_R \hat{H}(x, z, t) \frac{\partial \hat{T}}{\partial y}.
\end{align*}
\] (2)

In the above \( \hat{H}(x, z, t) \) denotes the surface actuation (steady or unsteady) at the wall (i.e. vibrating membrane say) and defines the precise shape of the DRE or distributed surface roughness field. In the work below, \( \epsilon_R \) or \( \hat{H}(x, z, t) \) is assumed small (\( \leq 5\% \) of the local boundary-layer thickness \( \delta \)) and steady thus modelling surface roughness. In the DRE modelling \( \epsilon_R \ll 1 \), is a scaling parameter defining the height of the DRE’s, with \( H_{max} = 1 \). In distributed roughness modelling \( \epsilon_R = 1 \) and the smallness is introduced directly by the prescribed surface roughness field, i.e. dictated by the \( \text{rms} \) root-mean-square measure.

We define roughness in the present case as that due to a micron scaled surface bump, cylinder or rectangular element among others. In all cases, assuming that the steady roughness element can be discretised in the real physical form \( \hat{H}(x, z) \), its Fourier representation can be quite easily computed either analytically (for cylindrical elements) or via fast Fourier transform (FFT) methods (for more general arbitrary shaped roughness elements e.g. a 3d bump say). As an illustration, analytic expression for a cylindrical DRE is given by

\[
H_k(x) = \frac{\sin(kx)k \beta}{k \pi},
\] (3)

here the parameter \( d(x) \) represents, the varying with streamwise coordinate \( x \), spanwise half width and at the centre of streamwise placement of the cylindrical DRE, \( d(x) \) would then denote its radius. The subscript \( k \) denotes the \( k \)th mode, on operation of a FFT on the \( \hat{H}(x, z) \) field, to extract out the streamwise variation of the wall forcing, thus \( \hat{H}(x, z) = \hat{H}_k(x) e^{ikz} \) for the \( k \)th mode.

In the \( x \)-streamwise direction, our computational box, requires inflow and outflow conditions. In the former we simply set the disturbance quantities to have zero magnitude. At the outflow boundary downstream of the roughness element a radiation condition is specified with the streamwise wavenumber \( \alpha \) estimated by a linear non-parallel PSE computation and used in the approximations \( \partial q / \partial x \sim i \alpha q \).

### II.B. Numerical Method

The algorithm devised discretises the \( x \)-streamwise direction derivative terms directly in real space, using high-order finite differences (4, 6 or 8), while a pseudo-spectral approach is utilised in the wall normal direction (we typically find with an optimally defined stretched grid, 51 polynomials in \( y \) suffice to give grid resolved solutions). For the streamwise direction, we typically use about 2000 points for a reasonably long domain. The LNS equations once discretised require the inversion of a large lower-upper (LU) block factorisation of a complex algebraic system of equations of the form

\[
\mathcal{L}\tilde{q} = \tilde{r}.
\] (4)

In the above \( \tilde{q} \) is the solution vector for all points in the field and the right hand side vector \( \tilde{r} \) contains the boundary conditions to be satisfied, representing the discretised roughness or actuation field vector.

With the LU factorisation stored in memory the back-substitutions to solve for parametric variation of the \( \tilde{r} \) fields can be accomplished very quickly (\( \sim 99\% \) of the CPU time is used in performing the LU step while the back-solve for a given \( \tilde{r} \) vector field is relatively rapid i.e. typically \( \sim 8 \) seconds per parametric variation of \( \tilde{r} \)). This observation, allows the uncertainty quantification work that we demonstrate below to be carried out relatively efficiently. Moreover, this similarly allows a rapid optimisation or parametric

*nonlinear receptivity effects are expected to become prominent when \( \epsilon_R \sim 5\% \) and higher.
variation analysis to be constructed, similar to that possible with an adjoint\textsuperscript{11} approach. Thus a rapid optimisation procedure to investigate for example, location of the DRE’s or their precise diameter in relation to the spanwise wavelength $\lambda$, for optimal forcing can be quickly ascertained in matters of minutes. The direct matrix inversion strategy is a natural and optimal choice for the type of instability problem examined in this paper.

III. Roughness Model

The most important aspects of the work have been firstly to implement a model for a general distributed 3d-surface roughness field and secondly to devise a strategy for modelling the uncertainty arising from the field. In this section we describe the issue of how the roughness amplitudes derived from power spectral density computations of real surface roughness data are determined and how they are used.

Characterisation of surface roughness and texture in metrology at the simplest level is generally done by way of defining surface $\textit{rms}$ value $Rq$ and $Ra$ the mean departure from some reference line namely

$$Ra_k = \frac{1}{L_k} \int_0^{L_k} |H| \, dx; \quad Rq_k = \sqrt{\frac{1}{L_k} \int_0^{L_k} H^2 \, dx}.$$ \hspace{1cm} (5)

In a 3d-roughness context, for a roughness strip of dimension $(L_x, L_z)$, we use the expression

$$Ra_{3d} = \frac{1}{\sqrt{L_x L_z}} \left( \int_0^{L_x} \int_0^{L_z} \hat{H}^2(x, z) \, dz \, dx \right)^{1/2};$$ \hspace{1cm} (6)

to ascertain the general roughness level of a material. If even greater roughness characteristics are required a plethora of measures are defined in the metrology field, the most widely used of these being measures of skew (measure of symmetry of the height distribution of the profile) and kurtosis (measure of peakedness of the surface height distribution); finally spacing between peaks is another measure. These features allow greater definition/detail to be attributed to the roughness fields as opposed to the single averaging feature defined in Eq.(6). In our work only this simple feature is used for the roughness specification, or rather as a guide to gauge the roughness of the material. In the numerical discretisation of the roughness content, we model and capture in a strict sense all the spectral content of the surface roughness field.

III.A. Uni-directional Surface Height (Roughness) Data

One-dimensional, uni-directional, surface height data measurements using a high specification Tally Surf profilometer, were carried out for two different test samples. Test sample 1 (painted) representative of the wing top cover; sample 2 (aluminium) unpainted anodised aluminium plate similar in roughness to the anodised leading edge surface of a single aisle aircraft.

For each sample two measurements in directions generally orthogonal to each other (longitudinal and transverse) were provided. The raw measured roughness data, known as primary profiles, for the two surfaces are shown in figure\textsuperscript{2}. It should be noted, however, that in general the surface roughness $\textit{rms}$ value is based on some filtering of the raw primary measured data. Hence the roughness profile is derived from the primary profile, by suppressing the long-wave component, using the so-called profile filter (lc, ISO-4287 standard); the profilometer is calibrated with in-built parameter settings for the filters. Also built in or standardised is the length over which the measurement is taken, namely $L = 4.7980 \text{ mm}$. Another quantity output or provided is the waviness profile, which is derived by subsequent further filtering, to suppress long-wave components and short wave components.

Importantly, the standard measures adopted in metrology, apply both long and short wave filters, and then compute surface roughness characteristics on the cleansed data, for industry accepted measures of roughness. It should be of note, in the work reported here, typical instability wavelengths arising lie precisely in the low pass filtered region. Thus from a receptivity analysis viewpoint, the most dangerous surface roughness (or spectral content) is actually extracted out of the roughness data, which would in practice, give rise to the most dominant crossflow disturbances. In view of this, purpose built filters\textsuperscript{3} were implemented in the LNS solver, which allowed complete control of what was being filtered off, for cleansing of noise induced by the measuring instrument.

\textsuperscript{1}the technique we employ is crude from a strict metrology viewpoint, but effective for present.
III.B. Power Spectral Density (PSD) Based, Real Surface Roughness Model

With access to real measured rough surface characteristics a randomised surface roughness distribution which has the correct spectral content as the real measured surface, but still displaying some variance is prescribed by the formula

\[ \hat{H}(x, z) = \hat{\epsilon} \sum_{k=N_1}^{N_2} \sum_{j=N_1}^{N_2} a_{k,j} \exp \left( -i(k\hat{\alpha}x + j\hat{\beta}z + \chi_{k,j}) \right) + c.c. \] (7)

Here the complex coefficients \( a_{k,j} \) are assumed known from measurements, while the \( \phi_{k,j} \) phases are prescribed randomly and \( c.c. \) is the complex conjugate; \( \hat{\alpha} = 2\pi/S_x \) and \( \hat{\beta} = 2\pi/S_z \), with \( S_x, S_z \) lengths in \( x \) and \( z \) directions of the sampled material. An additional scaling parameter, \( \hat{\epsilon} \), is introduced purely to allow greater flexibility and scope in investigations since this allows the computed \( Ra_{3d} \) to be simply scaled by some amount. Setting values of \( \chi = (0, 1) \) is used as a device for various means of implementing a randomised field description of \( \hat{H}(x, z) \); \( \chi = 1 \) conveniently allows for sampled data sets, whereby phase variations in \( a_{k,j} \) generate variability in the roughness field, while the spectral content is kept maintained in all realisations.

III.C. Filtering

As remarked above, receptivity analysis dictates that disturbances are generated when a match of roughness length-scales with naturally occurring instability eigen-mode wavelengths occurs. For example the level of roughness detail, having an impact or role in the disturbance generation process, may be quite quickly estimated by a simple linear stability analysis, identifying the range of crossflow wavenumbers \( \beta \) giving rise...
to unstable modes. The largest relevant wavenumber from such an analysis may then be used to estimate roughness periodicity, or grain size $k_g$ namely

$$k_g = \frac{2\pi}{\beta_{\text{max}}},$$

which then gives an estimate of smallest roughness granular size to be included, and thus more importantly provides estimates of finishing/polishing standards that must be controlled for wind-tunnel model preparation. For example, it may appear obvious, but scale effects down to roughness scale clearly require careful control in model machining. For example a CF wave-number of $\beta = 1000/m$, gives a wavelength of 6.28 mm, while $\beta = 40000/m$ gives a wavelength of 0.157 mm. If the roughness granularity happens to have spectral peaks at similar values to the wavelengths of the predicted disturbance scales then this would trigger the generation of disturbances. Thus use must be made of the spanwise and streamwise wavenumbers in defining finishing standards in model manufacture as well as in setting bounds on any filtering of measured surface data and surface roughness fields for use in numerical simulations.

The expected wavelengths of instability modes, may thus be used as a useful guide to limit the truncation of the parameters $(N_1, N_2)$ in Eq. (7). $N_1$ serves effectively as a long wave filter, while setting an upper bound to $N_2$ serves as a short-scale filter. Truncation of $N_2$ to a particular value is used to filter out the very short-scale spike features arising in the raw profilometer measured data (note figure 4). For the painted case, reconstruction of the surface using as few as 21 Fourier components suffices, while for the anodised aluminium case, 81 Fourier components generally reproduces the original data as may be noted from the PSD shown in figure 3. Here we have converted the uni-directional PSD, measured in the two orthogonal directions, into a fully three-dimensional field by simply assuming a linear variation in the PSD coefficients in the interior. The re-constructed surface roughness fields for both cases are shown in figure 4. Table 1 lists values of $Ra_{3d}$ for the two materials, filter limits used and thus roughness grain size retained/resolved in the numerics. These parameters are used in the analysis in Section VI.

![Figure 4. Reconstructed natural distributed surface roughness fields used in the linearised Navier Stokes solver. Painted finish (left) and anodised aluminium unpainted surface (right); red shading represents the peaks and blue the valleys of the surface roughness variations.](image)

<table>
<thead>
<tr>
<th>Material type</th>
<th>Filter limits</th>
<th>Grain size, mm</th>
<th>$\mu$m</th>
<th>$Ra_{3d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Painted</td>
<td>$N_1$ = 1, $N_2$ = 61</td>
<td>$k_g^{(1)}$ = 4.80000, $k_g^{(2)}$ = 0.07869</td>
<td>0.2770</td>
<td></td>
</tr>
<tr>
<td>Aluminium</td>
<td>$N_1$ = 1, $N_2$ = 61</td>
<td>$k_g^{(1)}$ = 4.80000, $k_g^{(2)}$ = 0.07869</td>
<td>0.3149</td>
<td></td>
</tr>
</tbody>
</table>

### III.D. Reconstructed Roughness Fields From Fully 3D Laser Scans

Very high definition surface measurements using a laser scanning device were also carried out on a 30 mm anodised aluminium square panel. Figure 5(a) shows the surface roughness distribution, after appropriate cleansing and removal of measuring device induced noise and spike features by appropriate filtering, while...
the PSD is shown in figure 5. Here figure 5(b) shows the manner, albeit applied rather crudely, in which one may apply a long-wave filter. The results of doing so are shown in the sequence of plots in figure 5(b),(c). Figure 5(d) is an identical reproduction of plot (b) but here random phases have also been switched on (χ = 1). Roughness graininess scales appear to be very similar, however since the cut-off filter \( N_1 = 15 \), large scale features are absent and thus the impact of phase rotations is difficult to discern. A more striking example is that shown in figure 7, reproduced with \( N_1 = 3 \) filtering. Filtering, of course, also affects the final \( \text{rms} \) value of the surface as is tabulated in table 2 (see table 4 too).

### Table 2. Fully 3d roughness data characteristics for the 30 mm aluminium panel. Impact of filtering parameters on roughness \( \text{Ra}_{3d} \ \text{rms}^2 \).

<table>
<thead>
<tr>
<th>Filter limits</th>
<th>Grain size, mm</th>
<th>( \mu m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_1 )</td>
<td>( N_2 )</td>
<td>( k_g^{(1)} )</td>
</tr>
<tr>
<td>a 3</td>
<td>221</td>
<td>10.00667</td>
</tr>
<tr>
<td>b 15</td>
<td>221</td>
<td>2.00133</td>
</tr>
<tr>
<td>c 30</td>
<td>221</td>
<td>1.00067</td>
</tr>
</tbody>
</table>

1 The raw unfiltered \( \text{Ra}_{3d} \) was \( \sim 0.82 \ \mu m \), with \( N_1 = 1, N_2 = 600 \).  
2 This roughness data is used in \( \chi \) the \( \text{Ra}_{3d} \) in this case is \( \sim 0.12\% \) of the local boundary-layer thickness.

### IV. Uncertainty Quantification (UQ) Analysis

The random variable used is the surface roughness field description \( \hat{H}(x, z) \). The real measured data outlined in figure 5 is can be exactly described by a FFT deconstruction and with known complex values of \( a_{k,j} \) in Eq. (7), the surface height at any \((x, z)\) field point on the measured roughness panel, can be precisely reproduced. A given data set will have a precise \( \text{rms} \) value. Thus randomness in this mathematical description has to be introduced either by some combination of uniformly distributed \((0 - 2\pi)\) phase rotations through the quantity \( \phi_{k,j} \), together with random variations in the magnitude of the spectral coefficients \( a_{k,j} \) (somehow), which still retains sufficient spectral content of the measured field, to represent the actual surface under investigation, but has sufficient variability to truly reflect the nature of distributed surface roughness; i.e. no two samples taken on different parts of a wing panel have identical spectral content, but one expects some degree of uniformity in statistical measure such as \( \text{rms} \) values, and retention of dominant machined in or manufactured features, between two differently sampled sets. This is the challenge, introducing sufficient enough variability into a roughness model, which retains sufficient characteristics of the original sample, but also has the variability, that would naturally exist.

To gain some insight, two strategies are investigated. For the \( \chi \) AERAST data set analysis, the roughness data set in III.D is large enough in spatial extent for variability to be introduced by randomly sampling different regions of the 30 mm square panel (note the rectangular strips outlined in figure 5(a), (b) & (d) and figure 7). Thus the roughness field variability does not arise through variability of the known complex valued \( a_{k,j} \) coefficients, which are held fixed in all realisations, but through the random regional selection of different parts of the roughness spatial field; phase variances (\( \phi_{k,j} = 0 \) in Eq. (7)) are not used. Secondly, in the EADS-ATC3 data set analysis, the effect of different material properties, that of the measured aluminium and painted surfaces is investigated. However, due to a very small measured roughness length \((4.8 \ \text{mm} \ \text{square patch extrapolated from 1-d profilometer})\), roughness variability is introduced by again fixing the the magnitude of the \( a_{k,j} \) coefficients, to the actual measured values, but random phase variations are applied, i.e. \( \chi = 1 \) for both the painted and anodised aluminium data sets (figures 9). Both procedures constrain the \( \text{rms} \) to within a small neighbourhood of the nominal base/reference values. A fully (uniformly distributed) randomised \( a_{k,j} \) prescription may also be used, however in this case a constraint then needs to be imposed on the randomly generated \( a_{k,j} \) coefficients, to give a sensible description of the spectral content (PSD) by imposing a target \( \text{rms} \), which must be maintained during each roughness field realisation. This too has been coded for in our solver, but the effects of employing this strategy are not covered in this paper.
Figure 5. Three-d roughness data, aluminium panel, effect of various long wave filtering (a) grain size modelled $k_g < 10$, (table 2a filter parameters); (b) grain size modelled $k_g < 2$, (table 2b)”; (c) grain size modelled $k_g < 1$, (table 2c); (d) grain size modelled $k_g < 2$, table 2b coupled with random phase variation to the PSD. The black outlined rectangular region has spanlength of 3.3 mm and is of variable streamwise extent; this essentially shows the MC sampling procedure used in §V for the UQ analysis in the production of results shown in figure 10.

Figure 6. Laser scanned PSD of prepared, filtered, cleansed 3d-roughness data. (a) Unfiltered PSD; (b) Low wavenumber filtered.
IV.A. UQ Methodology

We employ a non-intrusive stochastic approach. For a given random $\tilde{H}(x, z)$ field, followed by the discretisation of the $\tilde{r}$ vector, Eq. (4) is solved, the quantity of interest $u_S$ from the solution (i.e. disturbance quantity at $X_S$, the sample point) is monitored. A sequence of computations of Eq. (4) for $m$ realisations of the $\tilde{r}$ field vector are computed and gives $m$ deterministic values of $u_S^{(m)}$. The LNS solver during run time monitors the $u_S^{(1,...,m)}$ solution vector, so that convergence (if any) to some expected mean ($\mu$) state can be monitored. Finally, a post-processing procedure sorts the $u_S^{(m)}$ into $p$ bins in the range $\min(u_S^{(1,...,m)})$ to $\max(u_S^{(1,...,m)})$ to compute the histogram of the probability density function (PDF).

V. AERAST DTI Experimental Analysis of DRE Control and LNS Simulations

The AERAST wind-tunnel test carried out in the ONERA F2 tunnel is described by Sunderland & Sawyers. Runs at a range of Reynolds number were conducted and pressure data measured. We focus on the $Re = 3.7$ million case which was determined to be optimal for the effectiveness of DRE control to be realised. The focus is on the $(0, 2), 3.3~mm$, naturally occurring mode. In all analysis here, we use the fully three-dimensional $30~mm$ square roughness distribution discussed in §III.D. To ensure all roughness contributions which influence the generation of the $3.3~mm$ mode, are included in the LNS model. The largest grain scale $k_g^{(1)} \sim 10~mm$ and smallest $k_g^{(2)} \sim 0.136~mm$ are allowed, while the grid resolution is $k_{grid} \sim 0.058~mm$ (note table 2). From a receptivity viewpoint, linear PSE analysis gives the streamwise wavelength $\sim 2.6~mm$, hence the most optimal roughness size/granularity which should generate the strongest CF disturbance will be of similar order. Thus any naturally occurring roughness grain scales, in the $x$-streamwise and $z$-spanwise directions, of this order must be included in the roughness field description, rather than be filtered off in the pre-processing stages.

V.A. Linear Stability Analysis

Figure 8 compares linear PSE computations for the two relevant modes, i.e. $(0, 2), 3.3~mm$ and the $(0, 3), 2.2~mm$ (the latter forced by the DRE for the stabilisation mechanism). Mode $(0, 3)$ is generated by the placement of cylindrical DRE’s $(1.1~mm$ diameter) at an optimal position, i.e. the neutral point $d$. The figure shows the importance of surface curvature effects, in that though the geometrical curvature is $(0, 1)$ mode is generated by the nonlinear interaction involving the $(0, 2)$ and $(0, 3)$ modes. In designing the wing geometry, one must ensure that this $(0, 1)$ mode does not happen to have a larger more rapid growth rate, or $N$-factor, than the $(0, 2)$ mode.

Our LNS receptivity code models this too, and optimises over location and diameter of DRE to give the largest response.
which shows the instability arising from the inflexional nature of the boundary-layer flow, thus once disturbances (generated by the wall roughness say) become strong enough, further small scale forcings at the wall are ineffectual.

relatively mild (see figure 1), N-factors are affected; a lowering in the N-factors of both modes arises with its inclusion.

V.B. Generation of Stationary CF Mode by Natural Distributed Roughness

The roughness spanwise width is set at 3.3 mm, an unknown is how long must the roughness strip, \( L_R \), be in the \( x \)-streamwise direction? The neutral location being the optimal site for disturbance generation\(^8\) with the strip prescribed to begin well upstream of this point (identified by linear PSE), three LNS simulations by randomly selecting two roughness patches of \( L_R = 35 \) mm by 3.3 mm and a third run of \( L_R = 5 \) mm by 3.3 mm to provoke the CF mode are shown in figure 9. The figures on the left show the \( u \)-disturbance generation and subsequent growth; the corresponding maximum of the \( u \)-disturbance field \( |u_{\text{max}}| \) variation with streamwise extent is shown in the figures to the right. Also shown is the equivalent linear PSE result for an arbitrary amplitude forcing. Observe that the distributed roughness strip extends about 9\% of chord downstream of the neutral point. Of the top two plots (both simulated with \( L_R = 35 \) mm), we observe a relatively stronger disturbance generated compared to the second; the only change in the two simulations being the extraction of the roughness field \( \hat{H}(x,z) \) sampled data from different areas of the 30 mm square roughness data patch. The third simulation (bottom plot), shows an equally strong CF disturbance generated, as the upper two plots, but this result is for a randomly selected \( L_R = 5 \) mm region of the full roughness data set (figure 9a).

Keeping all other parameters fixed there is clearly variance in the results introduced as a result of random regional selection, and as a result of roughness strip length. The latter is an issue, in that natural roughness is, to state the obvious, there over the entire length of the wing. The key question here is, is this required to be modelled and included along the entire \( y = 0 \) streamwise computational surface? i.e. is the disturbance field being continuously perturbed, modified and or enhanced by the continued distributed surface roughness forcing? Therefore do simulation codes require this to be prescribed at all points representing the surface? These issues are clarified by a series of runs within the framework of Monte-Carlo (MC) simulations and thus utilising uncertainty quantification tools for establishing expected disturbance magnitudes and to allow associated variances to be specified. Result of our extensive analysis is summarised in figure 11(a). Observe that for all \( L_R \) values for large enough number of MC realisations (\( \gg 1000 \)) a converged expected mean state is reached. Moreover, there appears to be an upper maximum value for \( u_{\text{max}} \sim 0.0022 \) at the sample point \( X_S \) given a sufficiently long enough roughness strip. Figure 11(b) shows the PDF for the \( L_R = 50 \) mm roughness strip and the expected variance \( \langle \sigma \rangle \); PDF curves for \( L_R = 40 \) & 30 mm, though not shown were found to be nearly coincident with the \( L_R = 50 \) mm PDF. This result is shown more clearly in figure 11 which shows the \( u_{\text{max}} \) variation at the sample point for the varying values of \( L_R \). This is explained in that given a long enough roughness strip forcing, the generated CF disturbance, provided the base flows allows it to, will grow as it convects and at some stage the disturbance becomes fully established and sizable enough, such that further wall based forcing has little effect. The CF disturbance is primarily governed by an inviscid instability arising from the inflexional nature of the boundary-layer flow, thus once disturbances (generated by the wall roughness say) become strong enough, further small scale forcings at the wall are ineffectual.
Figure 9. AERAST data set: Comparison of the LNS receptivity, distributed roughness based computations with linear PSE solutions (black). Top two plots are for simulations with roughness strip length $L_R = 35 \text{ mm}$; bottom simulation results with $L_R = 5 \text{ mm}$. The vertical line is the sample point $X_S$ used to monitor the solutions in the MC sampling.
A sequence of varying $L_R$ MC runs for the roughness data set of figure 10(a) rotated by 90 degrees, to investigate effects of roughness direction/orienteering was also investigated. A summary of this is shown in figure 11 as the dashed lines. This suggests little impact of roughness grain orientation for this particular data set.

Fixing $L_R = 25 \text{ mm}$, figure 12 shows the crucial impact that retention of various roughness scales has on the generated CF disturbance. As a reference case, the filter parameters in table 2(a) are used and also thus refers to the roughness data sets shown in figures 5(a–c). Here the largest response is from the roughness data set which had retention of the longest roughness scales, the smallest response arises where roughness grain size is limited to being below $k_g < 1 \text{ mm}$. Adding in additional variability, by way of phase rotations to the PSD field (curves (b, d) compared) has little effect, with a nearly similar response predicted. The significant result here is to note that the largest variance in the expected disturbance field arises from retention of the large roughness grain sizes, i.e., with the data of table 2(a).

Figure 13 compares the effect of roughness field rotation and inclusion of geometrical curvature, keeping all other parameters fixed, within the LNS model. As already remarked above, we see little difference in the result of rotating the roughness data set, but curvature inclusion (curve (c)) gives a reduced response at the sample point $X_S$.

*in other parameter regimes this may not be the case, and this is a feature which needs to be investigated*
Figure 12. AERAST data set. a) Monte-Carlo simulations of the effect of long-wave filtering for the $L_R = 25$ mm roughness strip. Curve labelling is as in table 2; curve (a) $k_g^{(1)} < 10$ mm, curve (b) $k_g^{(1)} < 2$ mm, curve (c) $k_g^{(1)} < 1$ mm. Curve (d) is as curve (b) for the roughness grain size, but random phase variations $\chi = 1$ (see Eq. (7)), are also included here. b) PDF for the cases (a),(b) and (c).

Figure 13. AERAST data set. Roughness field rotation and curvature effects compared for the $L_R = 50$ mm roughness strip. Curve c (black) includes effect of surface curvature; curves b and c compares result of roughness field rotation by 90 degrees; keeping all other parameters fixed, i.e. $k_g < 10$ mm, $\chi = 0$. 

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V.C. Inclusion of Variance in Nonlinear PSE and Transition Location.

A culmination of the work is usage of the capability in establishing how nonlinear simulations are affected by being able to prescribe, firstly more precise initial amplitude seedings for the PSE simulations and secondly the ability provided by the UQ analysis to repeat the nonlinear simulations by allowing the prescription of upper or maximum bounds on most likely or expected disturbance size. Allowing simulations with variance, will lead to different nonlinear behaviour, and in final stages will lead to different positions where the flow becomes highly nonlinear to invoke secondary-instability\textsuperscript{26} breakdown process of the flow to turbulence. A demonstration of this new, physics based UQ inclusive, capability is summarised by way of figure\textsuperscript{14} which in addition also simulates control of the (0,2) mode by forcing a larger in magnitude (0,3) sub-dominant CF disturbance by DRE cylinders.

Nonlinear self-interactions of the (0,2) mode are firstly carried out, by adjusting the initial amplitudes assigned to it, such that at the $X_S$ sample point the disturbance has the size $u_{\text{max}} = \mu = (\sum_{l=1}^{m} u_{S}^{l})/m$ predicted from the UQ analysis (i.e. vertical line in figure). The value of expected $\mu$ is based on the $L_R = 50$ mm MC, LNS simulations, i.e. $\mu \sim 0.0022$. The PSE simulations are then be repeated, with $u_{\text{max}} = \mu + \sigma$ and $u_{\text{max}} = \mu + 2\sigma$ at $X_S$. The results are shown in figure\textsuperscript{14} as the dashed lines and marked 'un-controlled (0,2)'; all three PSE simulations have convergence problems as the amplitudes grow rapidly to beyond 25\% of freestream amplitudes at about 35\% chord position. Importantly observe the point of breakdown, $X_T$ of the PSE simulations shifts from about 42\% ($u_{\text{max}} = \mu$ initialisation) to 34\% chord ($u_{\text{max}} = \mu + 2\sigma$ initialisation); a variance of 8\% uncertainty in transition location is predicted\textsuperscript{4} In a conservative, or zero-risk, approach estimates could be made by initialising PSE amplitudes based on the 95\% confidence value, based on nearly the maximum amplitude value predicted by the PDF.

Next, the above simulations are repeated with the additional feature of including the DRE forced (0,3) mode, whose amplitude is generated by the LNS code, and which is then used to initiate the corresponding (0,3) mode in the forced PSE evolution. The mode is generated by a DRE of 1.1 mm diameter, 35 $\mu$m height placed at about 2.5\% chord (AERAST test point). The stabilisation of the (0,2) mode is clearly seen. Here the (0,2) mode initialised by the amplitude $u_{\text{max}} = \mu + 2\sigma$ at $X_S$, shows a shift in $X_T$ of 14\% downstream (red curve) over the un-controlled position; while the $u_{\text{max}} = \mu$ initialisation at $X_S$, predicts sufficient enough control such that the PSE simulations proceed to well beyond 53\% chord, with the (0,2) mode amplitude remaining below 0.1\% levels.

In the AERAST experiment, natural CF transition was observed at about 25\% chord; the standard of surface finish in model manufacture was requested to be no more than order of 0.5 $\mu$m\textsuperscript{8}

\textsuperscript{4}or where secondary-instability would no doubt set in to initiate the breakdown process.

\textsuperscript{8}detailed surface roughness measurements of the type used here (3d PSD) were not done.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure14.png}
\caption{Nonlinear control using DRE for the AERAST data set, showing the new capability. Here the effect of expected variance, from the uncertainty quantification is used to initiate nonlinear PSE simulations. This shows sensitivity of transition location. Note the forced (0,3) mode is arranged to give a situation where there is extreme sensitivity in the control and massive movement of transition point as the initial amplitudes attributed to (0,2) mode is increased by two counts from $\mu$, i.e. $\mu + 2\sigma$. The red vertical line is the sample $X_T$ point in MC simulations. The dashed green line, which merges with the solid green curve (computed by PSE) is the result from the LNS for the DRE cylinder.}
\end{figure}
VI. EADS-ATC3 Wing Geometry, Results

The ATC3 wind tunnel test was carried out by EADS Innovation Works in the Airbus Filton low speed wind tunnel as described by Simmons & Browaeys and Alderman. The model was of 45 degree sweep with a span of 2.5 m and chord of 2 m. The surface shape was designed by EADS Innovation Works with an aerofoil representative of a civil transport outer wing and with endplates and washout incorporated to maintain infinite swept flow characteristics. The leading edge was of large radius to ensure a naturally turbulent attachment line as the primary purpose of the test was to investigate attachment line relaminarisation through a leading edge device. Model instrumentation and visualisation included, amongst other things, surface pressure taps at two span stations and China clay visualisation of laminarity. Data was collected at a tunnel speed of 75 m/s and incidences ranging at 1/2 degree intervals from −2 degrees to 1 degree. RANS computations of flow about the model in the tunnel were performed and following validation against experimental data, were used as a source of sectional pressure data for stability calculations at a Reynolds number $11 \times 10^6$. Of interest here is the case for 1 degree incidence which provides conducive conditions on the lower surface for sustained cross flow instability. The geometry along with the RANS computed pressure data for this case is shown in figure 15, where it can be seen that there is an extended favorable pressure gradient region over the first 25% chord. Tunnel turbulence levels were such that travelling CF modes dominated over stationary modes, thus transition at 10% chord was observed – rather earlier than would be expected for the stationary modes of interest here.

![Figure 15. EADS-ATC3 aerofoil geometry and computed pressure field. Receptivity and surface roughness experiments are to be conducted on the lower surface (shown in blue).]

Linear stability analysis for stationary CF, identified the wavenumber $\beta = 1500/m$, corresponding to a spanwise wavelengths of 4.188 mm, as the strongest growing mode; the streamwise wavelength being $\sim 1.7 \text{ mm}$. Figure 16 shows a LNS simulation of this mode, generated by placing a cylindrical DRE of 2 mm diameter at about the neutral location. Compared and showing excellent agreement, are results of the linear PSE and LNS with and without geometrical curvature effects in the modelling. Note that in the ATC3 geometry, figure 15, the nose section has quite pronounced curvature on both the lower and upper surfaces. Curvature, as seen, causes the predicted disturbance magnitude to be a factor 10 less compared to the no-curvature model at about 7% chord.

VI.A. Generation of Stationary Most Amplified Mode by Natural Distributed Roughness

Based on the analysis above, the roughness spanwise width is set at 4.188 mm. We use the roughness data of §III.A and defined in table 1 for the analysis. The roughness rms is typically of $\sim 0.2\%$ of the local boundary-layer thickness for these surfaces, namely the painted and anodised aluminium sets. Since the overall width of the sampled roughness data ($\sim 4.8 \text{ mm}$) is of nearly the same order as the spanwise wavelength, variability in the roughness fields is introduced by way of phase rotations, i.e. $\chi = 1$. The analysis strategy is identical to that used in §V. In addition to confirming the findings from the AERAST data set the main issues to be explored here are whether different materials (painted and unpainted) and geometrical curvature affect the UQ analysis? Figure 17 is a summary of the quite exhaustive UQ analysis undertaken with our approach, to elucidate these issues. In all computations we observe that given a sufficient number of MC realisations a convergence of the numerics to some mean state arises. MC simulations for varying roughness strip length $L_R$ for the two surface are summarised in plots (a) and (b); while plot (c) is an identical simulation of
Figure 16. ATC3 data set. Comparison of generated CF wave with linear PSE (shown as dashed lines), with curvature model (b-curve) and without curvature model (a-curve). LNS result with and without curvature model is simulated with a roughness cylinder placed at roughly the neutral point. Red vertical line is the $X_{S}$ sample point in MC simulations.

plot (a) the painted roughness data set, but including in the surface curvature model in the LNS based UQ analysis. Observe the painted surface, without curvature model, is predicted to give the largest response at the sampling $X_{S}$ point followed by the aluminium panel, while the painted data set with curvature model included, generates the smallest in magnitude disturbance. In this latter case, note the variance or spread of uncertainty is also much reduced compared to the equivalent no curvature result of plot (a). This is seen clearly in the PDF which compares the three cases computed for the $L_{R} = 90$ mm strip in figure 18.

VI.B. Inclusion of Variance in Nonlinear PSE and Transition Location

With variances and expected mean values established above, nonlinear PSE simulations are next carried out to predict the impact on natural transition location due to the different roughness properties of the materials used. Figure 17 is the culmination of this analysis. Note the mean transition point moves from 20% chord to 25% chord i.e. by 5% chord between the painted and anodised aluminium cases. Curvature is seen to be strongly stabilising with a shift in transition to 40% chord. Importantly here note in cases (a) and (c), the uncertainty in the transition location $X_{T}$, only predicts a variability or movement of $\sim 5\%$ of $X_{T}$.

VII. Discussion

A unified and powerful approach has been devised, which provides more precise estimates of uncertainty in the laminar-turbulent transition point to be quantified. We have established a procedure to link the variability that would naturally exist in a material’s surface roughness, to the ensuing variability that this
Figure 18. ATC3 data set, summarising the findings in figure 17. MC simulations for $L_R = 90$ mm, curve (a) is the painted surface, (b) aluminium surface and (c) the painted surface with geometrical curvature effects included.

Figure 19. Nonlinear PSE results for the ATC3 data set, showing the new capability. Red vertical line is the sample point in the MC simulations, note figure 16. Nonlinear PSE simulations for painted, aluminium surfaces, while plot (b) is as plot (a) but with curvature effects included.
then introduces in the transition location. On convergence of the MC approach, with the aid of nonlinear PSE and secondary-instability breakdown models (eventually), the computed variance in the expected disturbance amplitudes may be used to ultimately draw a connection with the expected uncertainty in the transition location. This forms the basis of our methodology. The method is demonstrated for stationary CF transition, though the approach taken (with further development) is general enough to apply to unsteady transitional flows too.

From a receptivity viewpoint and intuitively, all the results display the expected behaviour. The most insightful is result of the painted and anodised aluminium panel, in that though the aluminium panel has a rms larger than the painted surface, the latter though gives a larger response; this of course results from fact that longer roughness grain scales are present here, more in tune with the expected naturally occurring wavelength of the CF disturbance. Filtering off these long-wave roughness features result in a weaker response (note figure 12).

In the analysis we have presented and highlighted issues of incorporating such micron-scaled details in the receptivity models. We have established that modelling of distributed roughness, and its inclusion is only required, at most over a few percentage points of the computational domain. Ultimately as disturbances grow and become large, surface roughness appears to then have little effect on the continued CF development. Nonlinear simulations with the computed variances suggest not too great a sensitivity of movement of transition point, for the surfaces examined in this paper. We have in both cases a maximum uncertainty of 5% predicted. To be conservative, as pointed out earlier, one might just use the maximally predicted possible amplitude from the PDF as the upper bound to be used in nonlinear PSE simulations.

Curvature effects are clearly important and have, as demonstrated, a significant effect on the predicted flow development. Geometrical curvature inclusion in the model was found to be stabilising, though this relates to the general long-scale effects of curvature on CF instability, and not the effects of curvature on receptivity itself, since this is a much more local effect, and it is felt in this case, only where surface curvature varies on the scale of the instability wavelength, will a significant difference be apparent. The comparison highlights the need to model precisely the instability process, and only then can agreement be expected with experiment, and correct inferences be drawn from computational simulations.

Further work is required, in improving the characterisation of surface roughness, its modelling and the standardisation of procedures for most precise filtering and cleansing of raw measured data.

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