Consistent Recalibration of Yield Curve Models

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joint work with

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Yield curve modelling

Principles

- Absence of arbitrage.
- Robust calibration: the model is selected simultaneously from time series and prevailing market yields.
- Consistent recalibration: tomorrow’s market yield curve does not imply a rejection of today’s model.
- Analytic tractability: yield curve increments can be simulated accurately and efficiently.
Yield curve modelling
Difficulties with standard approaches

- Factor models: do not allow for robust calibration and consistent recalibration.
- HJM models: lack of analytic tractability.
- PCA models: absence of arbitrage and analytic tractability are issues.
- Filtered historical simulation: ditto.
Yield curve modelling

Our approach

- Use well-understood affine factor models as “tangent” models.
- The infinitesimal increments of our model belong to affine models with different coefficients.
- This allows us to fit the market dynamics better than in the case of affine models with fixed coefficients.
- The resulting models are called consistent recalibration (CRC) models.
- For each parameter vector $y$, consider a Hull-White extended affine factor model for the short rate.
- Each factor model admits a finite dimensional realisation in the space of yield curves.
• Concatenate yield curve increments, each belonging to a Hull-White extended affine factor model with possibly different $y$.

• CRC models are continuous-time limits of such concatenations.
CRC models

Setup

- \((\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})\) is a stochastic basis where \(\mathbb{P}\) is a risk-neutral probability measure;
- \(W\) is \((\mathcal{F}_t)_{t \geq 0}\)-Brownian Motion;
- for each parameter \(y\) and \(\theta \in C(\mathbb{R}_+)\) consider the factor model

\[
dX(t) = (\theta(t) - b_y(X(t)))\, dt + \sqrt{a_y(X(t))}\, dW(t), \quad t \geq 0,
\]

where \(a_y\) and \(b_y\) are admissible affine functions; and
- each factor model defines a short rate process by \(r = \ell(X)\), where \(\ell\) is a fixed affine map.
The HJM equation for the factor model with fixed parameter $y$ is

$$ dh(t) = \left( h'(t) + \mu_y^{\text{HJM}}(X(t)) \right) dt + \sigma_y^{\text{HJM}}(X(t)) dW(t), $$

$$ dX(t) = \left( C_y h(t)(0) + b_y(X(t)) \right) dt + \sqrt{a_y(X(t))} dW(t), $$

where $C_y$ is an operator which calibrates $\theta$ to the prevailing term structure.

CRC models replace $y$ by a Markov process $Y$. Thus, they are described by an SPDE for $(h, X, Y)$. 
By semigroup methods, one obtains convergence of the simulation scheme to solutions of the HJM equation.

- Increments of the HJM equation can be sampled accurately and efficiently.
• Quadratic covariations of forward rates satisfy

\[ d [h(\cdot, \tau_i), h(\cdot, \tau_j)] = \sigma^\text{HJM}_Y(X)(\tau_i)\sigma^\text{HJM}_Y(X)(\tau_j) dt, \quad \tau_{i,j} \geq 0. \]

• Estimate some of the components of \( Y \) fitting CRC covariation matrices to the dynamics of market yields.
• Calibrate the remaining components of \( Y \) to the prevailing market yield curve by regression methods.
• Select and fit a model for the estimated time series of \( Y \).
CRC Models

Consistent recalibration property

• The process $h$ does not leave a pre-specified set $\mathcal{I}$ of possible curves.
• The set $\mathcal{I}$ includes a large portion of possible market observables.
• The process $h$ reaches any neighbourhood of any curve in $\mathcal{I}$ with positive probability.
Example satisfying the consistent recalibration property

- Let $\mathcal{I}$ be the space of all possible forward rate curves. For each parameter $y \in \mathbb{R}$ consider the one-factor Vasiček model

\[ dh(t) = (h'(t) + \mu_y^{\text{HJM}}) \, dt + \sigma_y^{\text{HJM}} \, d\mathcal{W}(t), \]

where

\[
\mu_y^{\text{HJM}}(\tau) = -\frac{a}{\beta(y)} e^{\beta(y) \tau} \left( 1 - e^{\beta(y) \tau} \right),
\]

\[
\sigma_y^{\text{HJM}}(\tau) = \sqrt{ae^{\beta(y) \tau}},
\]

for $a > 0$ fixed and mapping $y \mapsto \beta(y)$.

- Parameter process: $Y = \sigma \tilde{\mathcal{W}}$ for $\sigma > 0$ and $\tilde{\mathcal{W}}$ independent of $\mathcal{W}$.

- Choose $\beta \in C_0^\infty$ such that $\sup_y \beta(y) < 0$ and $\beta'(y) \neq 0$ for all $y$. 

Numerical example
Zero-coupon yields estimated from Euro area government bonds by the ECB
Numerical Example
Calibration in the Vasiček and CIR cases: $a_Y$ estimated from the market dynamics
Numerical Example

Calibration in the Vasicek and CIR cases: $b_Y$ estimated from the market dynamics

- $a_Y$ and $b_Y$ vary significantly over time.
- Models with constant parameters $y$ do not satisfy the requirement of robust calibration.
Numerical Example
Calibration in the Vasiček and CIR cases: fitting the prevailing market yield curve

- Vasiček 1 and CIR 1: \( b_Y \) and \( a_Y \) are estimated from the yield curve dynamics.
- Vasiček 2 and CIR 2: \( b_Y \) and \( a_Y \) are fitted to the prevailing yield curve.
- \( \theta \) is calculated so that the initial model yield curve exactly matches the prevailing market yield curve.
• V and CIR: Hull-White extended Vasiček and CIR models.
• CRC-V and CRC-CIR: CRC versions of V and CIR.
• The consistent recalibration property of CRC models is reflected in the higher ranks of the covariation matrices.