Indication for Primordial Anisotropies in the Neutrino Background from the Wilkinson Microwave Anisotropy Probe and the Sloan Digital Sky Survey

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We demonstrate that combining cosmic microwave background anisotropy measurements from the 1st year Wilkinson Microwave Anisotropy Probe observations with clustering data from the Sloan galaxy redshift survey yields an indication for primordial anisotropies in the cosmological neutrino background.

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Introduction.—Recent cosmological data coming from measurements of the cosmic microwave background (CMB) anisotropies (see, e.g., [1]), on galaxy clustering (see, e.g., [2]) and, more recently, on Lyman-alpha Forest clouds (see, e.g., [3]) are in spectacular agreement with the expectations of the so-called standard model of structure formation, based on primordial, purely adiabatic inflationary perturbations and a cosmological constant (see, e.g., [3,4]).

Cosmology is therefore becoming more and more a powerful laboratory where physics not easily accessible on Earth can be tested and verified. An excellent example of this comes from the new cosmological constraints on neutrino physics. According to the standard model, a cosmic neutrino background (NB), similar to the CMB, should exist in our universe with a density of approximately \(n_{\nu} \approx 57/\text{cm}^3\) per flavor. Neutrinos of mass \(\ll 10^{-3}\) eV would still be relativistic today with a Fermi-Dirac spectrum at a temperature \(T \approx 2\) K, while more massive neutrinos would be nonrelativistic and clustered around galaxies with a typical velocity of \(v_{\nu} \approx 200\) km/s. Therefore, despite the high density of cosmological neutrinos a direct detection is virtually impossible due to their extremely low energy and cross sections (see, e.g., [5]).

However, cosmological neutrinos have a profound impact on cosmology since they change the expansion history of the universe and affect the growth of perturbations (see [6] for a detailed account). As a consequence, recent cosmological data do provide strong—albeit indirect—evidence for the presence of a NB (see, e.g., [7]) and have been used to put upper limits on absolute neutrino masses competitive with those from laboratory experiments (see, e.g., [3,8]).

In this Letter we show that current cosmological data now provide for the first time an interesting indication for primordial anisotropies in the NB. Although inflationary anisotropies in the NB at the level of \(\sim 10^{-5}\) are expected in the standard scenario, a direct detection is clearly impossible. However, anisotropies in the NB background affect the CMB anisotropy angular power spectrum at level of \(\sim 20\%\) through the gravitational feedback of their free streaming damping and anisotropic stress contributions [9] and an indirect detection is indeed possible.

Data analysis.—A way to parameterize the anisotropies in the NB has been introduced in [10] with the “viscosity parameter” \(c_{\text{vis}}^2\), which controls the relationship between velocity/metric shear and anisotropic stress in the NB. A value of \(c_{\text{vis}}^2 = 1/3\) is what one expects in the standard scenario, where anisotropies are present in the NB and approximate the radiative viscosity of real neutrinos (see Fig. 1). The case \(c_{\text{vis}}^2 = 0\), on the contrary, cuts the Boltzmann hierarchy of NB perturbations at the quadrupole, forcing a perfect fluid solution with no NB anisotropies but only density and velocity (pressure) perturbations. Furthermore, the generation of polarization is also modified, due to changes induced in the polarization source term. Cosmological data which would discriminate \(c_{\text{vis}}^2 = 0\) in favor of \(c_{\text{vis}}^2 = 1/3\) would therefore provide a strong

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(color online). The effect of NB anisotropies on the CMB temperature angular power spectrum. The standard model with parameters which provide the maximum likelihood best fit to WMAP is plotted (see [4]) against the same model but with no NB anisotropies. The 1st year WMAP data is also plotted for comparison.}
\end{figure}
We show that this is indeed the case by computing the constraints on $c_{\text{vis}}^2$ obtained by combining several cosmological data sets. The method we adopt is based on the publicly available Markov Chain Monte Carlo package COSMOMC [12,13], which has been modified to allow for values of $c_{\text{vis}}^2 \neq 1/3$. We sample the following 7 dimensional set of cosmological parameters, adopting flat priors on them: $c_{\text{vis}}^2$, the physical baryon and CDM densities, $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$, the ratio of the sound horizon to the angular diameter distance at decoupling, $\Theta_s$, the scalar spectral index and the overall normalization of the spectrum, $n_s$, and $A_s$, and, finally, the optical depth to reionization, $\tau_r$. Furthermore, we consider purely adiabatic initial conditions, we impose flatness and we do not include gravitational waves. We first restrict our analysis to the case of 3 massless neutrino families, thereby fixing the background neutrino density, but we proceed to relax this assumption at the end. Introducing a neutrino mass in the standard case with $c_{\text{vis}}^2 = 1/3$ has a $\chi^2 = 1482.9$, while for the $c_{\text{vis}}^2 = 0$ case we obtain $\chi^2 = 1490.2$, clearly favoring the standard case ($\Delta \chi^2 = 7.3$). Furthermore, the best fit model with $c_{\text{vis}}^2 = 0$ has a rather tilted spectrum ($n_s = 0.90$), while the combined effect of a red index and a slightly smaller amplitude of the fluctuations reduces the

![FIG. 2 (color online). Joint 2-dimensional posterior probability contour plots in the $c_{\text{vis}}^2 - n_s$ (top) and $c_{\text{vis}}^2 - A_s$ (bottom) planes, showing the 68% and 95% contours from the WMAP + other CMB data alone (cyan/light gray) and adding Sloan matter power spectrum information (black). Degeneracies between these parameters are evident and no lower limit on $c_{\text{vis}}^2$ can be placed with CMB data alone.](image)

![FIG. 3 (color online). Illustration of the role of galaxy clustering data in constraining $c_{\text{vis}}^2$. With CMB data only (scatter plot), lower values of $c_{\text{vis}}^2$ can be compensated by a redder spectral index and a lower amplitude of the fluctuations. Inclusion of the Sloan data disfavors models with low $n_s$, thereby cutting away most samples with low $c_{\text{vis}}^2$. The solid black contour is the joint 2$\sigma$ posterior from CMB and Sloan.](image)
optical depth to $\tau_r = 0.07$. As already noticed in [11], it is in fact possible to identify a correlation between the amplitude $A_s$ and spectral index $n_s$ of the primordial fluctuations and the value of $c_{\text{vis}}^2$. This arises because the absence of NB anisotropies boosts the amplitude of the CMB acoustic peaks, see Fig. 1. This can be compensated, for example, with a lower value for the spectral index $n_s$. The degeneracy is nearly exact for CMB anisotropies alone and we found that no strong constraint can be placed from the CMB data only: in that case, the best fit value for the standard case is $\chi^2 = 1452.5$, while the $c_{\text{vis}}^2 = 0$ case has $\chi^2 = 1454.5$. However, the latter case has as best fit values $h = 0.79$ and $n_s = 0.89$, which are somewhat at odds with the HST measurement and with the combined WMAP + Sloan analysis, which indicates a spectral index close to scale invariance. The value of $c_{\text{vis}}^2$ has very little impact on the matter power spectrum, but inclusion of the Sloan data does reduce the allowed range of the other cosmological parameters, and especially of the tilt, thereby allowing one to place more stringent bounds on $c_{\text{vis}}^2$, as illustrated in Fig. 3. The 1D marginalized posterior distribution for $c_{\text{vis}}^2$ yields a lower limit $c_{\text{vis}}^2 > 0.12$ (2$\sigma$ c.l., 1 tail). The value $c_{\text{vis}}^2 = 0$ is found to lie about 2.4$\sigma$ away from the standard case, $c_{\text{vis}}^2 = 1/3$. Therefore we can conclude that the case where the NB does not have anisotropies above the first moment is quite clearly disfavored.

Finally, it is interesting to relax the assumption on the number of massless neutrino species, $N_\nu$, in order to test the prediction of the standard model that $(c_{\text{vis}}^2, N_\nu) = (1/3, 3)$. Departures from this point would constitute a strong indication for the presence of new physics. By treating $N_\nu$ as a free parameter in the range $0 \leq N_\nu \leq 10$ we obtain the constraints depicted in Fig. 4. Joint contours in the $c_{\text{vis}}^2 - N_\nu$ plane enclose the predicted value (1/3, 3), and we do not find a significant degeneracy between the two parameters, since their impact on the CMB is almost orthogonal. As a consequence, the marginalized bound on $c_{\text{vis}}^2$ is only slightly weakened, and we obtain that $c_{\text{vis}}^2 = 0$ is disfavored at just above the 2$\sigma$ level (CMB and Sloan data). Further imposing a conservative big bang nucleosynthesis (BBN) constraint on $N_\nu$ in the form of a Gaussian likelihood centered around $N_\nu = 3$ and with spread $\Delta N_\nu = 1$ [23] does not change the result for $c_{\text{vis}}^2$ in a significative way (see Fig. 4).

From a Bayesian model selection point of view, we can compare the evidence in favor of $c_{\text{vis}}^2 = 1/3$ as opposed to $c_{\text{vis}}^2 < 1/3$ using the Savage-Dickey density ratio and taking a flat prior in the range $0 \leq c_{\text{vis}}^2 \leq 1/3$ (see [24] for an explanation of the method and precise definitions). We obtain that WMAP + SDSS data favor $c_{\text{vis}}^2 = 1/3$ with odds slightly larger than 2:1, irrespective of the assumptions on the neutrino background density, constituting positive (if only weak) evidence in favor of the standard model value.

We also considered models with $c_{\text{vis}}^2 > 1/3$, but in this case we found that the current data does not provide any relevant constraint up to $c_{\text{vis}}^2 = 1$.

Conclusions—In this Letter we show that current cosmological data are providing for the first time an interesting indication for primordial anisotropies in the cosmological neutrino background. Our result shows indication for the existence of a NB presenting anisotropies as predicted by the standard scenario, in which higher order multipoles in the neutrino distribution function are generated by free streaming. The significance is still small (slightly larger than 2$\sigma$, quite independently on the assumptions on the radiation content) and possibly plagued by systematics.

We have analyzed a limited set of models and our conclusions are valid only in the theoretical framework we consider. Enlarging the cosmological model by increasing the number of parameters could lower the significance of our result. For instance, a running of the spectral index would make the degeneracy between $c_{\text{vis}}^2$ and $n_s$ worse, thus weakening our constraints. However, a measurable running is not expected in the most common inflationary models and there is no strong indication for it from the data we considered, so it is conservative to exclude this possibility from our analysis. Another possibility would be to consider a dark energy model different from a cosmological constant. The impact of dark energy is mainly in the

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FIG. 4 (color online). Relaxing the assumption on the background neutrino density by including the number of neutrino families $N_\nu$ as a free parameter: joint 2-dimensional posterior probability contour plots in the $c_{\text{vis}}^2 - N_\nu$ plane, enclosing 68% and 95% probability from the WMAP + other CMB data alone (cyan/light gray), adding Sloan matter power spectrum information (black, solid) and further imposing a conservative BBN constraint on $N_\nu$ (black, dashed). The indication that $c_{\text{vis}}^2 > 0$ is largely independent on prior knowledge of $N_\nu$, since the joint WMAP + SDSS data can constrain $c_{\text{vis}}^2$ and $N_\nu$ simultaneously. The prediction of the standard model is shown by the cross.
position of the acoustic peaks and on the large scale CMB spectrum, which is rather orthogonal to the power suppression signature due to the viscosity parameter. Hence we do not expect a significant degeneracy between such models and $\sigma^2_{\nu_s}$. Finally, we have made use of a very limited number of data sets. Inclusion of Lyman-alpha data from SDSS, for example, would probably improve our constraints on the spectral index $n_s$ and therefore provide a tighter constrain on $\sigma^2_{\nu_s}$. The latest Ly-$\alpha$ analysis from SDSS [3] finds $n_s = 0.98 \pm 0.02$ which would further close our likelihood contours around the $\sigma^2 = 1/3$ region. However, the precise statistical errors to be attached to such measurements are still under debate among specialists, so we preferred not to include this data at the moment.

One should note that the method we have adopted provides the only way to detect primordial anisotropies in the neutrino background in the foreseeable future. Furthermore, this work illustrates the power of current cosmological data sets to test the hot Big Bang model and constrain subtle details of the energy density components of our universe. For instance, one might speculate that detecting $\sigma^2_{\nu_s} \neq 1/3$ for the NB would hint to interaction of neutrinos with other species (one such example has been recently introduced by [25] and investigated in [26]), including exotic components like dark energy. Future measurements and data releases should be able to strengthen our result, possibly shedding new light on the physics of the early universe.

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