Parasitic coupling in magneto-inductive cable

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Parasitic coupling in magneto-inductive cable

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Abstract
Magneto-inductive (MI) waveguides are linear arrangements of magnetically coupled \( L-C \) resonators that propagate electrical energy at radio frequency without direct connection. To achieve the strong magnetic coupling needed for low-loss propagation, adjacent elements must be in such close proximity that electric coupling arises. In contrast to electric coupling in split ring resonators, the coupling occurs between the inductive tracks of adjacent resonant loops. Parasitic capacitance is demonstrated in flexible magneto-inductive cable, and shown to introduce additional propagation bands above the MI band. Simple models are developed to predict this effect, and strategies discussed to improve high-frequency isolation.

Keywords: metamaterial, magneto-inductive waveguide, parasitic

1. Introduction
Since the 1940s, there has been considerable interest in periodic arrangements of small numbers of coupled \( L-C \) networks for use as filters [1–3]. The coupling can be electric or magnetic, with the former generally providing the best performance. However, magnetically coupled systems have still attracted attention as band-pass filters, and designs have been optimized to yield a given response even in the presence of loss [4–7]. Interest has continued to modern times [8].

More recently, current waves propagating in larger periodic arrangements of magnetically coupled \( L-C \) resonators have been re-christened ‘magnetoinductive’ (MI) waves [9–11]. Such waves have been observed in many magnetically coupled systems [12–15], and applications have been proposed for 1D and 2D systems in communications [16–21], power transfer [22–26] and sensing [27, 28], particularly for magnetic resonance imaging (MRI) [29–33]. However, since the losses in MI systems are much higher those of conventional solutions such as coaxial cables, particular circumstances are required to justify their use. In communications and power transfer these include the ability to transfer energy through free space and dielectric walls. In internal MRI, these include the enhancement to patient safety that follows from segmentation, since this eliminates extended linear conductors on which common-mode standing waves may be excited by the electric field of the scanner’s powerful transmitter.

Propagation loss is minimized at the resonant frequency, when group velocity is at its highest, and high performance requires large quality factors and strong magnetic coupling. MI cables have been developed to obtain large propagation distances from a limited number of resonant elements, in a flexible format compatible with probes for internal MRI [33]. The element design (which uses two capacitors, rather than one) allows printing of cables without the need for via connections [34]. It also allows the formation of a simple resonant transducer from a halved element, which can provide exact matching to real impedance at two different frequencies, and improved matching between [35]. The use of identical elements prevents the local optimization used to control pass-band shape in filters, but this is relatively unimportant for narrow-band applications such as MRI. To achieve immunity to bending, the element shape is long and thin, and adjacent elements are strongly overlapped [36]. However, the price is an increase in parasitic effects compared with filters based on square elements on rigid substrates.

It is generally assumed that the coupling in MI systems is exclusively magnetic. However, the possibility of additional
electric coupling has been investigated in studies of planar-coupled microwave split ring resonators (SRRs) [37, 38], and exploited in arrangements with alternating types of coupling [39]. The coupling was attributed to an inhomogeneous charge distribution in the SRR gap region, resulting in a high electric field that can modify the charge in adjacent elements. However, for sufficiently high magnetic coupling, the tracks of adjacent elements in a magneto-inductive cable must be close together, this time resulting in electric coupling between the inductors. A preliminary discussion of this effect was presented in [40]. The aim here is to present a complete description, and show that it represents a significant and unavoidable non-ideality. In section 2, we demonstrate strongly coupled cables, and show they exhibit parasitic pass-bands above the MI band that cannot be explained by conventional theory. In section 3, we consider the physical basis for electric coupling. In section 4, we develop equivalent circuit models capable of explaining the observed response. Conclusions are presented in section 5.

2. Magneto-inductive cable

Magneto-inductive cable is a high performance form of MI waveguide [34]. Each loop is formed from printed copper inductors on a thin, flexible substrate, together with printed or surface mount capacitors. Figure 1(a) shows a plan view near a single element. The loops are divided into two halves, each of inductance $L/2$. The capacitors are also divided into two, each of value $2C$. Figure 1(b) shows a section view. Alternate loops are formed on either side of a substrate and overlaid with a lateral offset of one conductor width. High nearest neighbour magnetic coupling is then obtained, while non-nearest neighbour coupling is negligible. Despite the planar layout, the arrangement leads to axial coupling, with a positive magnetic coupling coefficient [9]. Figure 1(c) shows an approximation to the cross-sectional arrangement, which will be discussed later.

Experimental cables have been constructed from copper-clad Kapton® HN (Dupont High Performance Films). Inductors measuring 66 mm × 5 mm were formed with a track width of 0.5 mm and a thickness of 35 μm on a dielectric of thickness 25 μm and arranged with a period $a = 35$ mm. The printed circuit board contained arrays of cables, together with single and paired elements used to extract circuit parameters. Figure 2 shows a panel of elements and a completed cable. Surface mount capacitors were added in the locations shown to tune the resonance to the frequency for which the mid-band impedance $Z_{0m} = \omega_0M$ (see below) equalled 50 Ω. Broadband transducers (which provide impedance matching at two frequencies [35]) were constructed from halved elements and SMA connectors. Ten elements and two transducers were overlaid to form a cable 385 mm long.

Parameters were estimated using inductive probes and a network analyzer. The self-inductance $L$, and Q-factor were found by measuring the resonant frequency $f_0 = 1/(2\pi \sqrt{(LC)}) = 227.5$ MHz and half-power bandwidth of a single element with known capacitance $C = 4.7$ pF, as shown in figure 3. The mutual inductance $M$ and coupling coefficient $\kappa = 2\sqrt{M/L}$ were found from the split resonant frequencies $f_{12} = 1/(2\pi \sqrt{(L \pm M/C)})$ of a coupled pair, namely $f_1 = 201.6$ MHz and $f_2 = 280.8$ MHz. The following values were obtained: $L = 104$ nH, $Q = 110$, $M = 34.4$ nH and $\kappa = 0.66$. The frequency giving $Z_{0m} = 50$ Ω was estimated from the values of $\omega_0 = 2\pi f_0$ and $M$ as 231.5 MHz, which required $C = 4.55$ pF. As described earlier, the closest available value ($C = 4.7$ pF) was used, reducing $f_0$ slightly. However, unexplained high frequency resonances were noted in the mode spectrum. One is shown at $f_3 = 1.08$ GHz; the next, less distinct resonance lay at $f_4 \approx 3.3$ GHz.

Transmission was also measured using the network analyzer, with the cable in a U-shape between the measurement ports. The full line in figure 4 shows the frequency variation of $S_{21}$ up to 3 GHz. The MI band extends over the frequency range 200–400 MHz. The minimum insertion loss is 2.2 dB, corresponding to a propagation loss of $\approx 5.7$ dB m$^{-1}$. However, there is also propagation a much wider band between 750 MHz and 2.5 GHz, where there are multiple resonances due to standing waves. Clearly, the effect is associated with the additional resonance at $f_3$, but despite its wide spacing from $f_0$ the parasitic band extends very close to the MI band. We have noted similar phenomena in seven cable designs with centre frequencies ranging from 20–300 MHz. In fact, a second parasitic band was visible in all such cables. Higher bands were visible in some cables, but their existence was often masked by increased loss at high frequency.

The measurements were compared with standard theory [9] as follows. Figure 5 shows a link consisting of N resonant elements between a source and a load. Ignoring loss, non-nearest neighbour coupling and other parasitic effects, the current $I_n$ in the nth element of the periodic section at angular frequency $\omega$ is related to the currents $I_{n-1}$ and $I_{n+1}$ by the recursion equation:

$$j\omega L + \frac{1}{j\omega C}I_n + j\omega M(I_{n-1} + I_{n+1}) = 0. \quad (1)$$

Assumption of the travelling wave solution $I_n = I_0 \exp(-j\kappa a)$, where $k$ is the propagation constant and $a$ is the period, then leads to the dispersion equation [10]:

$$\cos(ka) = -\left(1 - \frac{\omega_0^2}{\omega^2}\right)\kappa. \quad (2)$$

Here $\omega_0 = 1/\sqrt{(LC)}$ is the angular resonant frequency and $\kappa = 2\sqrt{M/L}$. In the lossless case, propagation is band-limited. Resistive loss $R$ in the inductor may be incorporated, by replacing $j\omega L$ with $j\omega L(1 - j\omega_0/\omega Q)$, where $Q = \omega L/R$ is the quality factor of the elements. Its effect is to render $k$ complex (so that $k = k' - jk''$), introduce propagation loss and allow out-of-band propagation. For low loss, $k'$ is approximately as in the lossless case, while $k'' \approx 1/(\kappa Q \sin(k' a))$. At resonance, $k'' \approx 1/\kappa Q$, so that strong coupling and a high Q-factor are required for low loss.

The first element is excited using a source with output impedance $Z_0$ and voltage $V_1$, while the last is terminated using the same impedance. The characteristic impedance of a MI
The waveguide is $j \omega M \exp(-jka)$ [11]. At resonance, this expression reduces to the real value $Z_{0M} = \omega_0 M$. If $Z_{0M}$ matches $Z_0$, there will then be no reflections. However, broadband transducers allow a considerable improvement in performance, since they provide an exact impedance match at two frequencies [35]. Their implementation merely requires the inductance of the terminating elements to be halved and their capacitance doubled, as shown in figure 5. Ignoring line losses again, Kirchhoff’s voltage law then gives for the first and last element:

$$\left( j \omega L + \frac{1}{2j\omega C} + Z_0 \right) I_1 + j \omega MI_2 = V_i$$

$$\left( j \omega L + \frac{1}{2j\omega C} + Z_0 \right) I_N + j \omega MI_{N-1} = 0.$$  \hspace{1cm} (3)

Equations (1) and (3) form a set of $N$ equations that may be written in a matrix form as $Z I = V$. Here $Z$ is an $N \times N$ impedance matrix while $I$ and $V$ are $N$-element vectors of currents and voltages. Only the first element of $V$ is non-zero, and given by $V_1$. The currents $I_n$ can be found by inverting the matrix equation, and the transmission scattering parameter $S_{21}$ extracted by standard methods. The dotted line in figure 4 compares the predictions of this theory with the previous experimental results, assuming $N = 12$ to correspond to a 10-element line with 2 transducers. There is good agreement within the MI band, but the theory cannot explain the propagation at high frequency.

3. Parasitic electric coupling

We now develop a model to explain the observed effects. Estimation of equivalent circuit parameters can be extremely complicated for rectangular loops and rectangular wires. We therefore use an approximation based on infinite, parallel cylindrical wires, which yields analytic results easily. In this model, the cable cross-section in the overlap region is as shown in figure 1(c). Here, two elements based on wires of
radius \( r \) and separation \( S_x \) overlap with offsets \( O_x \) and \( O_y \) in the two perpendicular directions. Ignoring the ends of the loops, the per-unit-length self inductance \( L_{PUL} \) and the mutual inductance \( M_{PUL} \) between elements can be found by calculating the magnetic flux due to currents in a first pair of wires and integrating the perpendicular component of flux passing through the first or second pair, respectively. The results are:

\[
L_{PUL} = \frac{(\mu_0/\pi) \log_e{(S_x - r)/r}}{4}
\]

\[
M_{PUL} = \frac{(\mu_0/4\pi) \log_e{|A/B|}}{2}
\]

with

\[
A = \left\{ (O_x + S_x - r)^2 + O_y^2 \right\} \left\{ (O_x - S_x + r)^2 + O_y^2 \right\}
\]

\[
B = \left\{ (O_x + r)^2 + O_y^2 \right\} \left\{ (O_x - r)^2 + O_y^2 \right\}.
\]

(4)

(5)

However, it is well known that significant capacitance exists between closely spaced cylindrical conductors. The per-unit-length capacitance \( C_{PUL} \) between the closest wires can again be found by calculating the total electric field due to line charges on a pair of wires with centres chosen to yield equipotentials co-located with the real wires, and integrating this field to find the voltage between the real wires. The result is:

\[
C_{PUL} = \frac{(\pi \varepsilon_0) \log_e\left\{ \frac{d}{2a} + \sqrt{\left( \frac{d^2}{4r^2} - 1 \right)} \right\}}{4}
\]

with \( d = \sqrt{O_x^2 + O_y^2} \).

Unfortunately, since \( L_{PUL}, M_{PUL} \) and \( C_{PUL} \) all depend on the geometric constants \( r, S_x, O_x \) and \( O_y \), it is difficult to obtain high values of the first two without the third also being high. For the high-frequency cable of the previous section, this model suggests equivalent circuit parameters \( L \approx a'L_{PUL} \) and \( M \approx a'M_{PUL} \) where \( a' = 66 \text{ nm} \) is the inductor length and \( a'' = 31 \text{ mm} \) is the overlap length. Assuming an equivalent conductor radius \( r = 225 \text{ \mu m} \) (slightly reduced from the value of 250 \( \mu m \) suggested by the track width), a conductor separation \( S_x = 5 \text{ mm} \), and offsets of \( O_x = 0.5 \text{ mm} \) and \( O_y = (25 + 37.5) = 62.5 \text{ \mu m} \) yields \( L = 80.7 \text{ nH} \), \( M = 29.1 \text{ nH} \) and \( \kappa = 0.72 \). These values are broadly in agreement with measurements (104 nH, 34.4 nH, and 0.66, respectively), but there are discrepancies attributable to the approximations made to the inductor and track shapes. An alternative and possibly more realistic model might be constructed using filamentary currents, as was done in [31]. More importantly, the model implies a parasitic capacitance \( C_P \approx 2a''C_{PUL} = 3.6 \text{ pF} \) between elements, comparable to the value \( C = 4.7 \text{ pF} \) used to achieve resonance.

There is limited scope to improve matters. Figure 6 shows the variation of \( M_{PUL} \) and \( C_{PUL} \) with \( O_x \), assuming that \( r = 225 \text{ \mu m}, S_x = 5 \text{ mm} \) and \( O_y = 62.5 \text{ \mu m} \). Both fall as \( O_x \) increases. \( M_{PUL} \) is positive when \( O_x/S_x = 0 \), but falls to zero when \( O_x/S_x \approx 1/\sqrt{2} \) and becomes negative as the geometry alters from axial to planar. \( C_{PUL} \) falls monotonically, and initially rather faster. Importantly, if \( S_x \) is increased, the variation of \( M_{PUL} \) approximately scales, while the variation of \( C_{PUL} \) stays unchanged. These characteristics imply that it should be possible to reduce \( C_P \) by increasing \( O_x \). However, additional loss will then be caused by the accompanying reduction in \( M \). Similar effects follow from a reduction in the axial overlap of adjacent elements. An alternative strategy would be to leave the geometry unaltered, but reduce the resonant frequency by increasing \( C \). This approach will space the MI band further from the spurious pass-band. However, additional loss will again arise, this time from the reduction in Q-factor. Thus, it appears that there are intrinsic limits to the performance of MI cables.

4. Equivalent circuit models

We now develop equivalent circuit models capable of simulating the measured response. Figure 7(a) shows the simplest possibility. One complete element is shown, together with parts of near neighbours. Electric coupling is represented using discrete capacitors shunting the tracks of adjacent elements. Here we use two capacitors \( C_P/4 \) connecting the extremities of each pair of tracks. The total inductance...
Consequently, there is a mutual inductance $M$ magnetic fields in detail, but still generates realistic results.

Develop circuit equations. © 2013 IEEE. Reprinted, with permission, from [40].

Magnetic coupling only takes place between the induc-

tors $L_{C/4}$. This approximation avoids the need to consider magnetic fields in detail, but still generates realistic results. Consequently, there is a mutual inductance $M/2$ in each shaded region.

For a periodic line, we need only consider two currents $J_n$ and $K_n$ in the parasitic capacitors in addition to the main current $I_n$. Use of Kirchhoff’s voltage law around the three loops in figure 7(b) gives three equations relating the currents. Once again, we ignore losses, since they can be included later by modifying impedances. For the resonator loop itself we obtain:

$$\omega L + 1/jωC)J_n + jωM(J_{n-1} + J_{n+1})$$

$$- jω\left(\frac{L_{NC}}{2} + M\right)(J_{n-1} + K_{n-1}) - j\left(\frac{ωL}{2}\right)(J_n + K_{n-1}) = 0. \quad (7)$$

Similarly, for two loops including parasitic capacitors we get:

$$j\left(\frac{ωL}{2}\right)J_n + jω\left(\frac{L_{NC}}{2} + M\right)J_{n+1} - jω\left(\frac{ωL}{2} + 2L_{NC}\right)J_n = 0$$

$$- jω\left(\frac{L_{NC}}{2} + M\right)K_n = 0$$

$$- jω\left(\frac{ωL}{2} + jωL_{NC} + M\right)J_{n+1} + j\left(\frac{ωL}{2}\right)J_n - jω\left(L_{NC} + M\right)J_n$$

$$- j\omega\left(\frac{ωL + 2L_{NC}}{2}\right)K_n = 0. \quad (8)$$

Assumption of the travelling wave solutions $I_n = I_0 \exp(-jωn)$, $J_n = J_0 \exp(-jωn)$ and $K_n = K_0 \exp(-jnka)$ again allows a dispersion equation to be derived. The manipulations are relatively lengthy, and will not be detailed here; however, the result is:

$$\cos(κα) = -f_1/f_2$$

$$f_1 = (1 - ω_0^2/ω^2)\left(a_1 - \frac{ω_0^2}{ω^2}\right)^2 - a_2^2 - \frac{1 + a_3^2\left(a_1 - \frac{ω_0^2}{ω^2}\right)}{2} + a_2 a_3$$

$$f_2 = \kappa\left(a_1 - \frac{ω_0^2}{ω^2}\right)^2 - a_2^2 - a_1^2 - \frac{ω_0^2}{ω^2} + a_2(1 + a_3/2)$$

$$a_1 = \frac{(1 + α)}{2}, \quad a_2 = \frac{α + κ}{2}, \quad a_3 = α + κ; \quad a_4 = \frac{ω_0^2}{ω_0}; \quad α = \frac{L_{NC}}{L}; \quad ω_0 = \sqrt{\left(8/LC_p\right)}.$$

The model has therefore introduced two new parameters, $α$ (which depends on the geometric arrangement of the induc-
tors) and $ω_0$ (which depends on the parasitic capacitance $C_p$). However, when $C_p$ is small, $ω_0$ and $a_4$ are large, and we recover the MI dispersion equation near $ω = ω_0$.

Figure 8 shows dispersion characteristics calculated assuming the previous experimental parameters ($f_0 = 227.5$ MHz, $κ = 0.66$) and a value of $α$ obtained from the track lengths defining $L_{C}$ and $L_{NC}$, namely, $5/(66 + 5) = 0.07$. The parasitic capacitance has been assumed as $C_p = 4.3$ pF for agreement with experimental results. Five characteristics are presented. The simplest model (with no parasitic capacitors) uses equation (2). In this case, there is only one branch in the dispersion diagram. The next simplest (with two par-
asitic capacitors per track section) uses equation (9). Three branches may now be seen: a MI branch at low frequency, and two new branches at higher frequency. The MI branch is almost identical to the prediction of the simple model, and supports forward waves. In contrast, the first parasitic branch supports backward waves. Increasing $C_p$ forces it closer to the
MI branch, while increasing $\alpha$ mainly alters the separation between the first and second parasitic branches.

These findings are at least qualitatively correct. However, the circuit of figure 7(a) is an extreme simplification. More realistically, the self-inductance, mutual inductance and parasitic capacitance must all be distributed. We have therefore developed more detailed models in which the parasitic capacitance $C_P/2$ of each pair of tracks is separated into $N_C$ elements distributed along the shaded regions, and the corresponding self- and mutual inductance $L_C/4$ and $M/2$ are subdivided into $N_C - 1$ elements. The currents may then vary in position for $N_C > 2$. We have investigated models up to $N_C = 16$. Although simple to construct, the recurrence equations are tedious. We therefore present only the equations for $N_C = 4$, detailed in Appendix.

By making travelling wave substitutions, these equations may be converted into coupled dispersion relations, which may then be solved numerically. The three additional characteristics in figure 8 shows the predictions of these more complicated models, assuming $N_C = 4, 6$ and 8, respectively. The effect of the subdivision is to modify the diagram still further. For example, as $N_C$ rises, the ranges of the lower parasitic bands alter. There are large discrepancies between the bandwidths predicted with $N_C = 2$ and $N_C = 4$; however there is steady convergence, and little difference between results with $N_C = 6$ and $N_C = 8$. For example, the upper edge of the second parasitic branch is predicted to lie at 4.11, 4.77, 4.81 and 4.81 GHz by models with $N_C = 2, 4, 6$ and 8, respectively. The number of bands also alters, and at high frequency every increment in $N_C$ inserts an additional band (not shown). Importantly, the first parasitic band lies close to the MI band, and has a much larger bandwidth.

Similar models may be used to simulate end-to-end transmission in a line. However, the equations must be modified at the input and output transducers, to account for the different local circuit in these loops. For an $N$-element line, a total of $(N_C + 1)N - N_C$ circuit equations must therefore be constructed. Although cumbersome, the matrix $Z$ is simple to define, since the impedance values follow a regular pattern. Matrix inversion may then be used to calculate the current $I_N$ and the power transmission coefficient.

Loss must also be considered. Here we have ignored loss in the surface mount capacitors, focusing on the higher loss expected in the inductors and parasitics. These are estimated as follows. At low frequency, the inductors have resistance $R$. At high frequency the skin effect will increase resistance to $Rt/2\delta$, where $t$ is the conductor thickness, $\delta = \sqrt{(2/\omega\mu_0\sigma)}$ is the skin depth in a material of conductivity $\sigma$ (5.8 × 10$^7$ S m$^{-1}$ for copper), and $\mu_0 = 4\pi \times 10^{-7}$ H m$^{-1}$ is the permeability of free space. The parasitic capacitance involves electric fields that exist partly in air and partly in the substrate. We have included dielectric loss by writing this term as $C_P\{1 - j\tan(\delta)\}$, where $\tan(\delta)$ is an effective loss tangent.

The dotted line in figure 9 shows a comparison between the earlier experimental data and this model, assuming the previous parameters and $\tan(\delta) = 0.012$, $N_C = 16$. Here the frequency range is extended to include the second parasitic band. The lowest parasitic band is correctly predicted, as are the peaks and troughs due to standing waves. The agreement between experiment and theory is worse for the second band. The most
likely explanations are the capacitance between the inductor tracks of the same element (which leads to self-resonance) and radiation, both of which are neglected. In other measurements (not shown) we have observed as many as four parasitic bands over a wider frequency range, in qualitative agreement with the model. However, losses are much higher, and both the data and the quantitative agreement appear less reliable.

The same theory can be used to predict the resonances of coupled elements. Rather than simulate the details of excitation, we have simply assumed a voltage source with 1 Ω output impedance at the extremity of one element and a similar load at the extremity of the other. Figure 10 shows the resonance spectrum thus obtained. The three resonances at 3 GHz are correctly predicted. However, the resonance at 3 GHz is low by comparison with experiment (3.3 GHz). We believe this discrepancy to be due to further parasitic capacitance. In each case, large variations in the currents distributions are as expected. Modes 3 and 4 are new symmetric and anti-symmetric resonances arising from the parasitic capacitance. In any case, large variations in the currents can be seen in the coupled track sections.

Figure 11 shows the normalised currents along the long conductors at the four frequencies. Here the index is a normalised position, which (since each coupled track is divided into 16 sections) ranges from 0 to 16. Mode 1 (occurring at f₁) is the symmetric resonance of a pair of coupled L–C resonators. With no parasitic capacitance, the currents should be uniform and equal. Similarly, Mode 2 (at f₂) is the corresponding anti-symmetric resonance, and the currents should be equal and opposite. To a reasonable approximation, both distributions are as expected. Modes 3 and 4 are new symmetric and anti-symmetric resonances arising from the parasitic capacitance. In any case, large variations in the currents can be seen in the coupled track sections.

5. Conclusions

We have shown that strongly coupled magneto-inductive cable suffers from parasitic electric coupling, which introduces additional pass-bands that degrade isolation at high frequency. The mechanism is different from an apparently similar electric coupling noted in SRRs, since it involves coupling between the inductor tracks. Furthermore, since it is indivisibly linked with magnetic coupling, it is much more significant. The effects cannot be described using the standard model of magneto-inductive propagation. However, the addition of distributed parasitic capacitance to the equivalent circuit provides an excellent explanation. The model ignores self-resonance and radiation. Both are likely to modify the response further at high frequency. However, since the main interest of designers is likely to be at frequencies close to the MI band, the first parasitic band will be most important, and in this range the model appears realistic. If the electric coupling is relatively small, this band will lie above the MI band. However, if the electric coupling is much higher—which might occur, for example, if the tracks of adjacent elements are exactly overlaid—the two may merge. Since there appears to be no method of avoiding electric coupling, care should therefore be taken in designing strongly coupled MI cables to ensure a suitable balance between transmission efficiency and out-of-band signal rejection.

Appendix

If four parasitic capacitors are allowed per coupled track section, the equivalent circuit is as shown in figure A1(a). The circuit equations for the five loops in figure A1(b) are then as follows:

A.1. Main loop

\[ \omega M_{n+1} + \left\{ \frac{L_C + L_{NC}}{2} j \omega C + \frac{1}{j \omega C} \right\} I_n + j \omega M_n = 0. \]  

A.2. Parasitic loop 1

\[ \frac{L_C}{2} I_n + j \omega \left( \frac{L_C}{2} + L_{NC} \right) M_n = 0. \]
A.3. Parasitic loop 2

\[
\begin{align*}
\omega \left( \frac{L_C}{3} + \frac{L_{NC}}{2} + \frac{M}{3} \right) J_n + \omega \left( \frac{L_C}{6} + \frac{L_{NC}}{2} + \frac{2M}{3} \right) J_{n+1} \\
- \omega \left( \frac{L_C}{3} + L_{NC} + \frac{M}{3} \right) J_n \\
- \left\{ \omega \left( \frac{L_C}{2} + L_{NC} \right) + \frac{16}{\omega C_p} \right\} K_n \\
- \omega \left( \frac{L_C}{3} + L_{NC} + \frac{M}{3} \right) L_n \\
- \omega \left( \frac{L_C}{6} + L_{NC} + \frac{2M}{3} \right) M_n = 0. 
\end{align*}
\] (A.3)

A.4. Parasitic loop 3

\[
\begin{align*}
\omega \left( \frac{L_C}{6} + \frac{L_{NC}}{2} + \frac{2M}{3} \right) J_n + \omega \left( \frac{L_C}{3} + \frac{L_{NC}}{2} + \frac{M}{3} \right) J_{n+1} \\
- \omega \left( \frac{L_C}{6} + L_{NC} + \frac{2M}{3} \right) J_n \\
- \omega \left( \frac{L_C}{3} + L_{NC} + \frac{M}{3} \right) K_n \\
- \left\{ \omega \left( \frac{L_C}{2} + L_{NC} \right) + \frac{16}{\omega C_p} \right\} L_n \\
- \omega \left( \frac{L_C}{3} + L_{NC} + \frac{M}{3} \right) M_n = 0. 
\end{align*}
\] (A.4)

A.5. Parasitic loop 4

\[
\begin{align*}
\omega \left( \frac{L_{NC}}{2} + M \right) J_n + \omega \left( \frac{L_C}{2} + \frac{L_{NC}}{2} \right) J_{n+1} \\
- \omega \left( L_{NC} + M \right) J_n \\
- \omega \left( \frac{L_C}{6} + L_{NC} + \frac{2M}{3} \right) K_n \\
- \omega \left( \frac{L_C}{3} + L_{NC} + \frac{M}{3} \right) L_n \\
- \left\{ \omega \left( \frac{L_C}{2} + L_{NC} \right) + \frac{16}{\omega C_p} \right\} M_n = 0. 
\end{align*}
\] (A.5)

References

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