The sweep stick mechanism of heavy particle clustering in 2d and 3d homogeneous, isotropic turbulence

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Several mechanisms proposed for the clustering of inertial particles.

- Ejection from vortical regions and concentration in strain regions - single scale flows. (Maxey 1987)
- Smoothing and filtering of fast timescales - acceleration statistics. (Ayyalasomayajula et. al 2006)
- Dissipative dynamics - multifractal attractors in phase space. (Bec et. al 2003)
- For high St formation of caustics - Falkovich et al. (2002), Wilkinson and Mehlig (2005)

For high Re turbulence, sweep-stick mechanism has been proposed by Goto and Vassilicos (2006) for St in the inertial range.
Clustering of inertial particles

- Locations of near empty spaces same for all Stokes numbers - size of these regions changes.

**Fig.:** Goto and Vassilicos, PoF, 18, 115103, (2006)
Consider small ($\ll \eta$), heavy ($\rho_p \gg \rho_f$) particles. Neglecting collisions, feedback on fluid

\begin{align}
\dot{\mathbf{r}} &= \mathbf{v} \\
\dot{\mathbf{v}} &= -\frac{1}{\tau_p} (\mathbf{v}(t) - \mathbf{u}(\mathbf{r}(t), t))
\end{align}

Effect of the drag is determined by the Stokes number $S_\eta = \tau_p / \tau_\eta$.

In the limit of small $S_\eta$ - Maxey (1987)

\[ \mathbf{v} \approx \mathbf{u}(\mathbf{r}(t), t) - \tau_p \mathbf{a}(\mathbf{r}, t) \]

When $\mathbf{a} = 0$, particles move with local fluid velocity $\mathbf{v}_p \approx \mathbf{u}$, when $S_\eta$ is small.

Time needed for dissipative eddies to sweep past an Eulerian observer is $\eta/u'$. Much smaller than timescale of dynamics of these eddies $\eta/u_\eta \Rightarrow$ small scales are swept by energy containing eddies.

Implies persistence of small scale structure when frame of reference moves with fluid.

Propose that acceleration is nearly constant along a fluid trajectory. Equivalently, acceleration in a turbulent flow moves on average with velocity equal to the fluid velocity
Sweep-stick mechanism

- When \( a = 0 \), particles move with local fluid velocity \( v_p \approx u \), when \( S_\eta \) is small.
- Furthermore the acceleration field is swept by the local fluid velocity when \( |a| \ll a_{rms} \)
- So particles move with \( a = 0 \) points once they have stuck to them.
- They will eventually stick to them because particles move away from points where \( a \neq 0 \) with relative velocity \( \tau_p a \).
$a = 0$ points and inertial particles in 2D HIT

- $4096^2$ pseudospectral DNS $L/\eta = 30$ - (Susumu Goto Kyoto University)- Inverse cascading HIT

**Fig.**: Goto and Vassilicos, PoF, 18, 115103, (2006)
\[ \nabla \cdot \mathbf{v_p} \approx -\tau_p \nabla \cdot \mathbf{a} \] (4)

- E-vectors \((e_i)\) and e-values \(\lambda_i (\lambda_1 > \lambda_2 > \lambda_3)\) of symmetric part of \(\nabla \mathbf{a}\)
- Particles will only converge along directions \(e_i\) when \(\lambda_i > 0\)
- \(\mathbf{v_p} \cdot e_i \approx \mathbf{u} \cdot e_i - \tau_p \mathbf{a} \cdot e_i\)
- If \(\mathbf{a} \cdot e_i = 0\) surfaces/lines are swept with local fluid velocity then particles will move with these surfaces in all directions but only converge along directions which \(\lambda_i > 0\)
\( \mathbf{e}_1 \cdot \mathbf{a} = 0 \) points and inertial particles in 3D HIT

- 512\(^3\) pseudospectral DNS \( R_{\lambda} = 187 \) - (Susumu Goto Kyoto University)

**Fig.:** Goto and Vassilicos, PRL, 100, 054503, (2008)
Open issues

- Which mechanism - $a = 0$ or $e_1 \cdot a = 0$?
- Quantification of Stokes number dependency.
- Quantification of similarity of clusters.
- Is the zero acceleration picture a significant improvement over vorticity/strain picture?
- Are $a = 0$ or $e_1 \cdot a = 0$ swept?
- Validity of the Maxey relation at higher Stokes numbers.
\( \mathbf{e}_1 \cdot \mathbf{a} = 0 \) points are lines in 2D - extremely numerous

- How to validate sweeping of \( \mathbf{e}_1 \cdot \mathbf{a} = 0 \) points?
- Box size 4 integral scales.
$e_1 \cdot a = 0$ are lines

- $e_1 \cdot a = 0$ points are lines in 2D - extremely numerous
- How to validate sweeping of $e_1 \cdot a = 0$ points?
- Box size 0.25 integral scales.
Have zero acceleration points been missed in 3D?
Pair correlation and fractal dimensions

- Pair correlation function $m(r)$ (similar to radial distribution function) as a function of scale
- $m(r) \sim r^{-D_f}$ indicates cluster is self-similar.
Quantify the similarity between clusters by comparing the density of zero acceleration points $\mathbf{e}_1 \cdot \mathbf{a}$ in box $i$ ($\rho_a^i$) with the density of inertial particles $\rho_p^i$ at different box sizes $\epsilon$.

$$c(\epsilon) = \frac{\sum_i (\rho_a^i - <\rho_a>)(\rho_p^i - <\rho_p>)}{[\sum_i (\rho_a^i - <\rho_a>)^2 \sum_i (\rho_p^i - <\rho_p>)^2]}$$

(5)
Both $\mathbf{a} = \mathbf{0}$ and $\mathbf{e}_1 \cdot \mathbf{a} = 0$ points show much higher correlation than uniform distribution.

$a = 0$ points are better correlated at length scales in the inertial range - $\eta = 5$ mesh sizes.

For smaller $S_\eta$ where the distribution is more uniform $\mathbf{e}_1 \cdot \mathbf{a} = 0$ points show better correlation.
In 3D zero acceleration points clearly show the highest correlation.
Low Vorticity vs. low acceleration

- Pdfs of vorticity and $a_x$ in the fluid and at particle positions.
- 2D inverse cascading DNS $4096^2 - L/\eta = 30$

At $S_\eta = 3.2$, $pdf(\omega)$ is indistinguishable from uniform points in the flow.
Particle distribution for $S_\eta = 3.2$

- Okubo-Weiss $Q = \frac{1}{2}(|S(x, t)|^2 - |\omega(x, t)|^2)$

- Centrifugal ejection from vortices cannot explain clustering for $S_\eta > 1$
The importance of sweeping

\[ u(x, t) = \sum_{n=1}^{N_k} A_n \cos(k_n \cdot x + \omega_n t) + B_n \sin(k_n \cdot x + \omega_n t) \] (6)

- \( k_n \cdot A_n = k_n \cdot B_n = 0 \)
- Prescribed energy spectrum \( E(k) \sim k^{-5/3} \) over two decades of wavenumber space
- No sweeping - box size 4 ‘integral’ length scales.
Zero acceleration points in KS

- Sweeping is crucial for observing clustering with zero acceleration points.
Reformulation of the sweeping hypothesis

- Position of zero acceleration point $z_a$ and define $V_a \equiv \dot{z}_a$

\[
\frac{\partial a}{\partial t} + V_a \cdot \nabla a = 0 \Rightarrow (7)
\]

\[
\frac{Da}{Dt} + (V_a - u) \cdot \nabla a = 0 \quad (8)
\]

- Applying Kolmogorov 41 scaling for $Da/Dt$ and $\nabla a$

\[
\langle (V_a - u)^2 \rangle^{1/2} \sim u' \left( \frac{L}{\eta} \right)^{-1/3} \quad (9)
\]

- As $L/\eta \to \infty$, zero acceleration points move on average with the local velocity $u$. 

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Scaling of $V_a - u$ in 2D HIT and KS

- In KS, zero accn points move increasingly slowly relative to large scale motions as $L/\eta \to \infty$.
Statistical analysis of $V_a$ and $u$ at $a = 0$ points - 2D HIT

- **2D HIT**

- **3D HIT**
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\textbf{Maxey relation}

- Stick part of the mechanism is based on $\mathbf{v} \approx \mathbf{u}(\mathbf{r}(t), t) - \tau_p \mathbf{a}(\mathbf{r}, t)$ for small $\tau_p$.
- Not expected to hold at higher $\tau_p$ where clustering with zero accn points is still observed.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{Graph showing the relationship between $v$ and $\tau_p$ for different values of $S_\eta$.}
\end{figure}
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Stickiness

- Crucial point is \( \nu \approx u(r(t), t) \) at \( a = 0 \) points
- To move away from \( a = 0 \) points need \( \tau_p |a| > u' \) - stickiness changes for different Stokes numbers.
- For low \( \tau_p \) whole field is ‘sticky’

**Fig.** Goto and Vassilicos, PoF, 18, 115103, (2006)
Conclusions

- Propose the sweep-stick mechanism to explain the similarity of heavy particle and inertial particle clusters in 2D/3D HIT in the inertial range.
- There is no need for a modified \((\mathbf{e}_1 \cdot \mathbf{a})\) sweep stick mechanism in 3D - zero acceleration points are present in 3D.
- The clustering of inertial particles mimicking that of zero acceleration points is stronger than that of inertial particles being in low vorticity regions.
- Centrifugal ejection cannot explain clustering at higher \(S_\eta\).
- Sweeping is crucial for obtaining the similarity between \(\mathbf{a} = 0\) clusters and heavy particle clusters - it cannot be observed in standard KS.