Research paper

Pore-scale simulation of NMR response

Olumide Talabi *, Saif AlSayari, Stefan Iglauer, Martin J. Blunt

Department of Earth Science and Engineering, Imperial College London, SW7 2AZ, UK

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ABSTRACT

The random walk method is used to simulate magnetization decay in porous media. The simulations were performed on images of the pore space obtained using micro-CT scanning and in topologically equivalent networks extracted from these images using a maximal ball algorithm. The simulation results were validated through comparison with experimental measurements of $T_2$ distribution, absolute permeability and resistivity in two sand packs and from comparing predictions on images and networks of Fontainebleau sandstone. In all cases, the comparisons were good, although the networks gave a slightly narrower $T_2$ distribution, implying that some fine detail of the pore structure was lost. This work suggests that imaging, network extraction and pore-scale simulation can be used to predict single-phase transport properties successfully. It serves as a validation for pore-network models and the methods used to generate networks.

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1. Introduction

The pore space of a rock can be imaged directly at a resolution of a few microns using X-ray microtomography (micro-CT scanning) (Dunsmuir et al., 1991; Spanne et al., 1994; Coker et al., 1996). Alternatively, a three-dimensional voxel-based representation of the pore space can be generated from two-dimensional thin section images using statistical (Adler and Thovert, 1998) or object-based (Bakke and Øren, 1997) reconstruction techniques. These images can provide a basis for predictions of single-phase flow and transport properties (Øren et al., 2002; Knackstedt et al., 2004; Sakellariou et al., 2007). Rock properties predicted from three dimensional (3D) digitized images of sandstones have been shown to be in good agreement with conventional laboratory measurements, (Arns et al., 2001, 2002, 2004).

3D pore-scale micro-CT imaging of multiple fluid phases during drainage experiments have been used to compute relative permeabilities and capillary pressures directly on the digitised images and these have been found to be in good agreement with laboratory measurements (Turner et. al., 2004; Olafuyi et. al., 2006; Knackstedt et al., 2007). Although, multiphase flow properties have been predicted from 3D micro-CT images, simulations of multiphase flow with respect to the effects of wettability and capillary-controlled displacement are still a challenge (Hazzlett, 1995).

An alternative approach to the prediction of transport properties directly on 3D images is the use pore-scale network models (Fatt, 1956; Chatzis and Dullien, 1977; Bryant et al., 1993; Blunt et al., 2002) where the rock is represented as a lattice of pores connected by throats. Small samples, a few mm across, are represented with networks comprising several thousand pores and throats. To make accurate predictions, it is essential that the networks must be topologically representative of the porous medium of interest. A topologically equivalent network can be extracted from a three-dimensional image by a variety of methods (Lindquist et al., 1996; Bakke and Øren, 1997; Delerue and Perrier, 2002). Individual network elements are given properties such as inscribed radius, volume, length and shape factor, which are derived from the original image. Networks extracted in this manner had been used to predict rock properties such as capillary pressure, relative permeability, formation factor and wettability index (Bakke and Øren, 1997; Øren and Bakke, 2003).

Øren et al., 2002 reconstructed the three-dimensional microstructure of Fontainebleau sandstone with different porosity using a process-based reconstruction method. The results were validated by comparing their simulated magnetization decays with those computed directly on micro-CT images of Fontainebleau sandstones of similar porosity. Previous work on the simulation of NMR response in pore scale models have concentrated mainly on 3D images (Ramakrishnan et al., 1998; Øren et al., 2002; Toumelin et al., 2003; Hidajat et al., 2003), in our work, we simulate NMR response in both 3D images and extracted networks and the results are compared with experimental measurements. Successful comparisons of the 3D image simulation results with the extracted networks provide confidence that the networks are representative of the porous media from which they are derived. It also provides a basis for the extension of the algorithm developed for the simulation of NMR of single-phase fluid to two-phase flow.

Magnetization decay is primarily influenced by the surface area, the volume of the fluid and the surface relaxivity of the porous medium. In this work, we will develop a method to simulate magnetization decay in networks and micro-CT images from which the networks were extracted. Sand packs were used as the porous media because of their homogeneity.

* Corresponding author. Tel.: +44 2075947140; fax: +44 2075947444.
E-mail addresses: olumide.talabi@imperial.ac.uk (O. Talabi), saif.alsayari@imperial.ac.uk (S. AlSayari), s.iglauer@imperial.ac.uk (S. Iglauer), m.blunt@imperial.ac.uk (M.J. Blunt).

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We also measured the magnetization decay experimentally. For each sand pack considered, three micro-CT images were obtained in order to ensure the consistency of our simulation results. A maximal ball algorithm (Silin et al., 2003; Silin and Patzek, 2006; Al-Kharusi and Blunt, 2007; Dong, 2007) was used for network extraction and a random walk method (Ramakrishnan et al., 1998; Øren et al., 2002; Hidajat et al., 2003) was used to simulate magnetization decays in both the micro-CT images and extracted networks. These magnetization decays were then inverted into \( T_2 \) distributions by using a curvature-smoothing regularization method (Chen et al., 1999; Toumelin et al., 2003).

We also computed the permeabilities and formation factors of these networks (Bakke and Øren, 1997; Øren and Bakke, 2003) which were compared with those computed on the micro-CT images (Øren et al., 2002; Knackstedt et al., 2004) and experimental values. In all cases we found good agreement between the values, which serves to validate the network extraction algorithm and provides a basis for the reliable prediction of multiphase properties (Valvatne and Blunt, 2004; Pir and Blunt, 2005a,b; Øren et al., 1998; Al-Futaisi and Patzek, 2003).

2. Theory

2.1. NMR theory

During NMR measurement, the magnetization of the protons in the fluid relaxes exponentially with the constant of proportionality being the transverse relaxation time, \( T_2 \). For a single pore the magnetization decay as a function of time is given by:

\[
M(t) = M_0 \exp \left( -\frac{t}{T_2} \right)
\]

where \( M(t) \) is the magnetization at time \( t \), \( M_0 \) is the magnetization at initial time \((t=0)\) and \( T_2 \) is the transverse relaxation time. The transverse relaxation time is a combination of three relaxation mechanisms (Bloembergen et al., 1948) and is defined as:

\[
\frac{1}{T_2} = \frac{1}{T_{2B}} + \frac{1}{T_{2D}} + \frac{1}{T_{2S}}
\]

where \( T_{2B} \), \( T_{2D} \) and \( T_{2S} \) are the transverse relaxation time due to bulk relaxation, relaxation due to diffusion in magnetic gradients and surface relaxation respectively. The transverse relaxation time due to surface relaxation is given by (Loren and Robinson, 1970; Brownstein and Tarr, 1979),

\[
\frac{1}{T_{2S}} = \frac{S}{\rho V}
\]

where \( \rho \) is the surface relaxivity and \( S/V \) is the surface area to volume ratio of the pore, which is a measure of the pore size. A porous medium is comprised of interconnected pores of different size and shape; as a consequence a multi-exponential expression representing a distribution of \( T_2 \) constants is used to represent the magnetization decay (Kenyon, 1997):

\[
M(t) = M_0 \sum_{i=1}^{n} a_i \exp \left( -\frac{t}{T_{2i}} \right)
\]

where \( a_i \) is the volume fraction of pores of size \( i \) that decay with relaxation time \( T_{2i} \). Usually a logarithmic mean is used to determine the mean transverse relaxation time, \( T_{2\text{mv}} \):

\[
T_{2\text{mv}} = \exp \left( \frac{\sum a_i \log T_{2i}}{\sum a_i} \right)
\]

2.2. NMR simulation model

The NMR response of a porous medium assuming no magnetic gradient can be modeled by the reaction–diffusion equation (Senturia and Robinson, 1970):

\[
\frac{\partial [M]}{\partial t} = \nabla^2 [M] - \frac{[M]}{T_2 \delta}
\]

with initial and boundary conditions:

\[
[M] = [M_0] \quad \text{at} \ t = 0 \quad \text{and} \quad \nabla [M] = \rho [M]
\]

where \([M]\) is the magnetization per unit volume, \( \rho \) is the surface relaxivity and \( D \) is the diffusion coefficient. The total magnetization \( M(t) \) at a given time can be calculated by integrating the magnetization over the whole volume. The reaction–diffusion equation can be solved by finite-difference method or random walk method (Ramakrishnan et al., 1998).

The random walk method is based on simulating Brownian motion of a diffusing particle called the random walker. Initially, the walkers are placed randomly in the pore space. At each time step, they are moved from their initial position \([x(t), y(t), z(t)]\) to a new position \([x(t+\Delta t), y(t+\Delta t), z(t+\Delta t)]\) on the surface of a sphere with radius \( s \) centred on their initial positions. The time step \( \Delta t \) is given by:

\[
\Delta t = \frac{s^2}{6D}
\]

where the new position after each time step is given by:

\[
x(t+\Delta t) = x(t) + s \sin \phi \cos \theta
\]

\[
y(t+\Delta t) = y(t) + s \sin \phi \sin \theta
\]

\[
z(t+\Delta t) = z(t) + s \cos \phi
\]

and the angles \( \theta \) and \( \phi \) are randomly selected in the range \( 0 \leq \theta, \phi \leq 2\pi \). If the walker encounters a solid interface, it is killed with a finite killing probability, \( \delta \) (Bergman et al., 1995).

\[
\delta = \frac{2\rho s}{3D}
\]

If the walker survives, it simply bounces off the interface and returns to its previous position (Ramakrishnan et al., 1998; Øren et al., 2002; Hidajat et al., 2003) and time is advanced by the time step given in Eq. (8). At each time step, a record of the ratio of the walkers alive to the initial number of walkers is kept; this is the normalized magnetization decay due to surface relaxation mechanism alone.

\[
P(t) = \frac{N_1}{N_0}
\]

\( N_1 \) is the number of walkers alive at time \( t \) and \( N_0 \) is the initial number of walkers at time \( t=0 \). Bulk relaxation is simulated by multiplying the decay due to surface relaxation in Eq. (13) by the exponential decay, \( \exp(-t/T_{2B}) \). The relaxation due to diffusion in magnetic gradients \( g(t) \) is modeled by:

\[
g(t) = \exp \left( -\gamma^2 G_0 (\Delta \tau)^2 t \right)
\]

where \( t \) is time, \( G_0 \) is the molecular self-diffusion coefficient of the fluid, \( \gamma \) is the gyromagnetic ratio of the proton, \( G_0 \) is the spatial gradient of the magnetic field intensity and \( \Delta \tau \) is the inter echo time. Since all the relaxation mechanisms occur simultaneously, the normalized
magnetization amplitude as a function of time is given as (Kenyon, 1992):

\[
\frac{M(t)}{M_0} = P(t)g(t) \exp \left(-\frac{t}{\tau_{29}}\right) \tag{15}
\]

In this work we will not consider relaxation due to magnetic field gradients.

3. Experimental

3.1. Sand preparation

Quartz sands, LV60 (Leaveseat sand, WBB Minerals, UK) and Ottawa F42 (unground silica, US Silica Company), were used for the NMR experiments and micro-CT imaging. The grain size distributions of the sands were determined by sieving them for 80 min using British standard meshes on an electric shaker. The mass fraction of grains that passed through each sieve size is plotted against their respective sieve standard meshes on an electric shaker. The mass fraction of grains that passed through each sieve size is plotted against their respective sieve standard meshes on an electric shaker.

The sands were then prepared for the NMR experiments by carefully pouring into a 10 cm length of a thermo-plastic heat shrink sleeve of diameter 3.81 cm with fitted plastic end caps with one piece of circular filter paper to ensure a tight fit. The sleeve was filled with sand to about two thirds of its length, the sample was then tapped and vibrated using an electric vibrator to ensure compaction, the sleeve was filled to the top and the compaction process is repeated. The sand pack sample is then placed in an oven at 100 °C to seal the end caps after which the sample is then weighed. Two samples of LV60 sand pack and three samples of F42 sand pack were made for the NMR experiments to ensure good reproducibility of our packing procedures and the measurements were repeated thrice.

The porosity of F42 sand pack is 35.4% ± 1.3% while that of LV60 is 37.0% ± 0.2%. Formation factors were determined from electrical resistivity measurements. The experimental formation factor is 4.8 and 5.2 for the LV60 and F42 sand packs respectively. Permeabilities of the sand packs were measured on similar but longer columns of 2 cm diameter and 1 m length; the values are 42.0 D ± 4.0 D for F42 and 32.2 D ± 0.3 D for LV60.

3.2. Micro-CT imaging

Six small sand pack samples, three for each of sand pack type, F42A, F42B and F42C for the F42 sand pack and LV60A, LV60B and LV60C for LV60 sand pack were made for micro-CT imaging: the samples are 0.65 cm in diameter and 4 cm in length. They were packed in a similar manner as the plugs used in the experiments described above; analysis of the resultant images (see below) confirms that they have the similar porosity as the plugs used in the experiments. The micro-CT imaging was performed on a commercial XMT unit (Phoenix—X-ray Systems and Services GmbH). Scanned images of 750 × 750 × 450 voxels, with resolutions ranging between 8 µm and 10 µm were collected for each sample with the images cropped digitally to remove the edge artifacts.

3.3. NMR measurement

As mentioned previously, sand packs 3.81 cm in diameter and 10 cm in length saturated in brine were used for NMR measurement. The brine was a mixture of de-ionized water, (5 wt.% NaCl and 1 wt.% KCl solution), Potassium Chloride (KCl) was used to prevent the sand from swelling, while Sodium Chloride (NaCl) was added to increase the ionic strength of the water to levels found in oil reservoirs. The density of the brine was measured to be 1035 kg/m³. NMR relaxation measurements were performed using a MARAN2 bench top spectrometer (Resonance Instruments) at a temperature of 308 K operating at 2 MHz. The Carr–Purcell–Meiboom–Gill (CPMG) sequence was used for transverse relaxation measurements (\(T_2\)-measurements) with an inter-echo spacing of 200 µs. The short inter-echo time was selected to minimize relaxation due to diffusion in internal field gradients. A single data point was acquired at the center of each echo, with 32,000 data points being collected. The \(T_2\) relaxation time distributions were obtained from the magnetization decays by using the curvature-smoothing regularization method (Chen et al., 1999; Toumelin et al., 2003).

4. NMR simulation

4.1. Simulation of NMR response in micro-CT images

The micro-CT images were processed using ImageJ software (Abramoff et al., 2004): the image data were converted into binary format where 0 and 1 represent the pore and grain voxels respectively. The data were then filtered to remove noise in the image, central cubic sections of size 3³ mm³ were cut from the scanned images and used for micro-CT simulations and network extraction (Dong, 2007). At initial time, \(t = 0\) pore voxels are assigned with walkers; in order to ensure a uniform distribution of walkers in the image, one walker is substituted in Eq. (8) to determine the time step. This will ensure that walkers take a significant number of steps before encountering a grain surface. Each walker can either diffuse within its respective pore voxel or into any of

![Fig. 2](image-url)
For any network element having a shape factor between 0 and \( \sqrt{3}/3 \) (shape factor of an equilateral triangle), we determine the three half angles \( \beta_1, \beta_2, \beta_3 \) of the triangular cross-section of that element.

For a given shape factor, the corner half angles \( \beta \) can take a range of values (Patzek and Silin, 2001) where \( \beta_1 < \beta_2 < \beta_3 \); first \( \beta_2 \) is chosen randomly within the allowed range:

\[
\beta_{2,\text{min}} = \arctan \left( \frac{2}{\sqrt{3}} \cos \left( \frac{\arccos \left( -12\sqrt{3}G \right)}{3} + \frac{\pi}{3} \right) \right)
\]

and

\[
\beta_{2,\text{max}} = \arctan \left( \frac{2}{\sqrt{3}} \cos \left( \frac{\arccos \left( -12\sqrt{3}G \right)}{3} \right) \right)
\]

with \( \beta_1 \) given by:

\[
\beta_1 = \frac{-\beta_2}{2} + \frac{1}{2} \arcsin \left( \frac{\tan \beta_2 + 4G}{\tan \beta_2 - 4G \sin \beta_2} \right)
\]

and \( \beta_3 \) determined from:

\[
\beta_3 = \pi/2 - \beta_1 - \beta_2
\]

For each network element with the shape factor of a triangle, we determine its three geometrical properties, these are the half angles \( \beta_1, \beta_2, \beta_3 \), the height \( H \) and the base length of the triangular cross section. For a network element having the shape factor of a square, we divide its volume by its cross sectional area using Eqs. (16) and (17) in order to determine its length.

4.2.2. Initialization

We define a walker density; this is the number of walkers to be assigned to a given volume of the element. This will ensure a uniform distribution of walkers in all the network elements at initial time, \( t = 0 \). A walker is assigned two data sets, the first is its positional data, which includes the element type (throat or pore) that the walker is embedded in, the index number (which differentiates each network element), and the \( x, y \) and \( z \)-coordinate of the walker.

4.2.3. Diffusion

It is assumed that a pore with a coordination number (number of throats connected to the pore) \( n \), has half of the throats connected to the pore at \( z = 0 \) and the other half connected to the opposite face at \( z = L \) as shown in Fig. 3. A walker in this pore will diffuse into any of the connected throats (pore-to-throat diffusion), if the \( z \)-coordinate of the walker is negative or greater than the length of the pore; the walker then diffuses into a randomly selected throat connected to that pore. The \( z \)-coordinate of the walker in this new throat is determined from its previous value to ensure continuity along the diffusion paths. The shape factor of this new throat is then checked to determine its cross section since it will determine how the walker will be placed randomly in its XY plane.

The positional data of this walker in a new throat now becomes: throat index, \( x, y \) coordinates (determined randomly) and \( z \)-coordinate (determined from its previous value). This pore-to-throat diffusion of a walker is
also valid for throat-to-pore diffusion except that a throat is now connected to only one pore each at its opposite ends as shown in Fig. 4.

4.2.4. Relaxation

When a walker comes in contact with a solid surface, a random number is generated to determine if the killing probability is honored or not. If the walker survives, it is returned to its previous position. The fraction of walkers alive at selected time intervals is then recorded. The bulk relaxation component is then applied to determine the magnetization decay due to surface and bulk relaxation mechanisms. A flow chart showing the algorithm for the simulation of NMR response of single-phase fluid in a network is described in the Fig. 11.

4.3. Simulation parameters

The diffusion coefficient and the bulk relaxation time of brine used in the simulation were determined from the correlations (Vinegar, 1995):

\[ D_{b(\text{brine})} = \left( \frac{2T}{298(\eta)} \right) 10^{-12} \]  

and

\[ T_{2B(\text{brine})} = \left( \frac{3T}{298(\eta)} \right) 10^{-3} \]

where \( D_{b(\text{brine})} \) is the diffusion coefficient of the brine in m²/s, \( T \) is the temperature of the brine in K, \( \eta \) is the viscosity in Pa s and \( T_{2B(\text{brine})} \) is the bulk relaxivity of brine in seconds. Since the experiment was performed at 308 K, this corresponds to a diffusion coefficient of 2.07 \( \times 10^{-9} \) m²/s and bulk relaxivity of 3.1 s using the experimental brine viscosity of 0.001 Pa s. Relaxation due to diffusion in magnetic gradients is neglected in the simulations because of the small inter echo spacing used in the NMR experiments, as such, only surface and bulk relaxations were accounted for. The required inputs into both micro-CT and network NMR simulation models are, 3D image (in binary format) or the network, resolution of the 3D image, number of walkers, unit time step, diffusion coefficient of the fluid, surface relaxivity of the rock, bulk relaxivity of the fluid, inter-echo time during NMR experiment, magnetic gradient of the NMR equipment and the gyromagnetic ratio of the diffusing proton in the fluid. These input model parameters have all taken into consideration all the quantities included in the modeling Eqs. (3), (8), (12)–(15).

5. Results

5.1. Experimental results

The experimental results shown in Fig. 5 demonstrate reproducibility between different packs of the same sand. The magnetization of LV60 decays faster than F42 and hence the mean transverse relaxation time, \( T_2 \) for F42 is higher than that of LV60. The chemical compositions of the sands are similar; the difference in their magnetization decays arises from their different pore size distributions: the faster decay of the LV60 sand implies smaller pores — this is consistent with the smaller grain size distribution shown in Fig. 1. In the \( T_2 \) distributions, the relative distribution of the porosities within the rock in arbitrary units, a.u. (these do not reflect absolute porosities) is plotted against their respective \( T_2 \).

5.2. Simulation results

Simulations were carried out on the six micro-CT images, F42A, F42B and F42C for the F42 sand pack and LV60A, LV60B and LV60C for LV60. Simulations were also performed on networks extracted from these images; Fig. 6 shows example images and networks for the two sands. The micro-CT images and extracted networks cover a rock volume of 3³ mm³. Table 1 shows the resolution, voxel size, porosity,
total grain surface area per unit volume of each micro-CT image. It also shows the number of network elements (pores and throats) and coordination number of the networks extracted from each micro-CT image. The porosity of the packs inferred from the images is similar to that obtained by direct measurement on larger packs, although we tend to under-estimate the porosity of the F42 sand (33% from the images as opposed to 35% measured directly).

The surface relaxivity \(41 \times 10^{-6} \text{ m/s}\) used in our simulations was obtained by adjusting the value until a match was obtained with the magnetization decay of the experimental data. The same surface relaxivity was used to match the experimental data for the two sand packs since they are made up of the same mineral (quartz) and they have similar chemical composition. The relatively high value of the surface relaxivity used, compared to literature values in the range of 9–46 \(\mu\text{m/s}\) for sandstones (Roberts et al., 1995) and 2.89–3.06 \(\mu\text{m/s}\) for silica sands (Hinedi et al., 1997) is due to two reasons. First, the sand packs used in this work has minute proportions of iron oxide.

Table 1
Shows the micro-CT and the extracted networks properties.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Resolution (μm)</th>
<th>Voxel size</th>
<th>Porosity (%)</th>
<th>Surface area (\text{M}^2/m^3)</th>
<th>Pores</th>
<th>Throats</th>
<th>Coordination number</th>
</tr>
</thead>
<tbody>
<tr>
<td>F42A</td>
<td>9.996</td>
<td>300</td>
<td>33.0</td>
<td>43,770</td>
<td>1246</td>
<td>2856</td>
<td>4.4</td>
</tr>
<tr>
<td>F42B</td>
<td>10.002</td>
<td>300</td>
<td>33.3</td>
<td>44,930</td>
<td>1323</td>
<td>3262</td>
<td>4.8</td>
</tr>
<tr>
<td>F42C</td>
<td>10.002</td>
<td>300</td>
<td>33.1</td>
<td>45,760</td>
<td>1434</td>
<td>3777</td>
<td>5.2</td>
</tr>
<tr>
<td>LV60A</td>
<td>10.002</td>
<td>300</td>
<td>37.7</td>
<td>57,670</td>
<td>3135</td>
<td>7818</td>
<td>4.9</td>
</tr>
<tr>
<td>LV60B</td>
<td>8.851</td>
<td>338</td>
<td>36.8</td>
<td>61,090</td>
<td>3502</td>
<td>9044</td>
<td>5.0</td>
</tr>
<tr>
<td>LV60C</td>
<td>10.002</td>
<td>300</td>
<td>37.2</td>
<td>61,590</td>
<td>3582</td>
<td>9001</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Table 2
Comparison of mean transverse relaxation time \(T_2\), permeability and formation factor of the experimental values with simulation results of micro-CT images and extracted networks.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Mean (T_2) (ms)</th>
<th>Permeability ((D)\times)</th>
<th>Formation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Micro-CT</td>
<td>Network</td>
</tr>
<tr>
<td>F42A</td>
<td>668</td>
<td>677</td>
<td>756</td>
</tr>
<tr>
<td>F42B</td>
<td>668</td>
<td>654</td>
<td>727</td>
</tr>
<tr>
<td>F42C</td>
<td>668</td>
<td>647</td>
<td>694</td>
</tr>
<tr>
<td>LV60A</td>
<td>496</td>
<td>512</td>
<td>565</td>
</tr>
<tr>
<td>LV60B</td>
<td>496</td>
<td>488</td>
<td>543</td>
</tr>
<tr>
<td>LV60C</td>
<td>496</td>
<td>471</td>
<td>530</td>
</tr>
</tbody>
</table>

\(1D = 9.86923 \times 10^{-13} \text{ m}^2\).
within the range of 0.02%–0.06% along with some other paramagnetic substances in minute proportions. Second, and more significantly, the overall surface decay is given by the apparent surface area multiplied by the surface relaxivity. In our work the surface area is determined at the resolution of the micro-CT image, around 10 microns whereas in other work (Hinedi et al., 1997) this is determined using nitrogen absorption method, which probes the surface at a much higher resolution, resulting in a larger measured surface area per unit volume, hence to obtain the same magnetization, a lower apparent surface relaxivity was obtained.

The differences between the magnetization decays of LV60 and F42 sand packs are as a result of their grain shapes and sizes which are responsible for their different pore size distributions. The micro-CT images shown in Fig. 6a and b indicate that the grains of the F42 sands are larger, more spherical and have smooth surfaces than those of LV60. The grains of the LV60 sand pack on the other hand, have rough surfaces with a wider variation of sizes, as shown by the grain size distributions in Fig. 1.

Table 2 shows the comparisons of experimental, image and network estimates of mean $T_2$, permeability and formation factor, the mean $T_2$ is calculated using Eq. (5). The LV60 sand packs have a much higher surface area and a smaller grain size than the F42 sand packs, leading to more pores and throats per unit volume, a faster magnetization decay (lower $T_2$) and lower permeability. The predictions of permeability are good, although the permeability of the F42 pack is slightly over-estimated, while that of the LV60 sand is underestimated. We will now examine the full spectrum of the magnetization decay.

### 5.3. F42 sand packs

The simulated magnetization decays and $T_2$ distributions of the F42 samples, (F42A, F42B and F42C) are compared with the experimental data of F42Y as shown in Fig. 7a–f. In all cases there is good agreement between the experimental measurement, the simulations on the micro-CT image and the network. In one sample-

![Fig. 7](image-url)
F42C—the network results tend to give a more rapid decay, indicating a lower estimated average pore size. This is due to the more constricted diffusion in slit-like triangular elements that allow a low mean-free-path before encountering grain. The main peak is narrower for the network than the experiment, implying that some features of full ranges of pore sizes and shape are lost in the extraction algorithm.

5.4. LV60 sand packs

The simulated magnetization decays and $T_2$ distributions of the LV60 samples, (LV60A, LV60B and LV60C) are compared with the experimental data of LV60Y as shown in Fig. 8a–f.

The magnetization of the networks representing LV60B and LV60C decay faster than the network from LV60A; this is as a result of an increase in the number of network elements extracted from their respective micro-CT images. These results again suggest that we under-estimate the mean-free-path of a random walker in the networks.

If bulk relaxation and relaxation due to magnetic gradients are negligible, the pore size ($V/S$) distribution can be determined from Eq. (24).

$$\frac{V}{S} = \rho T_2$$

(24)

The experimental pore size distribution of the F42Y sand pack using Eq. (24) is compared with the distribution of inscribed radius for the pores and throats in F42A network in Fig. 9.

The inscribed radius distribution of the network elements is wider than the experimental pore size distribution obtained from the $T_2$ distribution of F42 sand pack. This is because the pore size distribution from $T_2$ distribution can combine several throats to a given pore and see them as a single pore; it can even combine several pores together

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**Fig. 8.** a. Comparison of the experimental magnetization decay of LV60 with the micro-CT image and extracted network for LV60A. b. Comparison of the inverted $T_2$ distribution from the magnetization decays in a. c. Comparison of the experimental magnetization decay of LV60 with the micro-CT image and extracted network for LV60B. d. Comparison of the inverted $T_2$ distribution from the magnetization decays in c. e. Comparison of the experimental magnetization decay of LV60 with the micro-CT image and extracted network for LV60C. f. Comparison of the inverted $T_2$ distribution from the magnetization decays in e.
and see them as a single pore, while in the network, the elements are clearly distinguished from each other. However, the range of the pore size distribution obtained from the $T_2$ distribution lies within the range of the inscribed radius distribution of the pores.

5.5. NMR simulation of Fontainebleau sandstone

A reconstructed microstructure of Fontainebleau sandstone and an extracted network (Øren and Bakke, 2002, 2003) were used to validate the method used for simulating NMR response in networks. This microstructure and the extracted network have a porosity of 13.6%. The microstructure has a resolution of 7.5 µm with a cubic grid size of 300³ voxels. The extracted network has 8192 throats and 4997 pores with an average coordination number of 3.2. The maximal ball method was also used to extract a network of 6112 throats and 3101 pores with an average coordination number of 3.8 from the same reconstructed microstructure (Dong, 2007). Brine of diffusion coefficient $2.07 \times 10^{-9}$ m²/s, bulk relaxivity of 3.1 s a typical surface relaxivity value for Fontainebleau sandstone of $16 \times 10^{-6}$ m/s (Hurlimann et al., 1994) were used for the simulation.

The magnetization decay of the network extracted by Øren and Bakke (2002, 2003) is consistent with that of the reconstructed microstructure as shown in Fig. 10a, indicating that the network is representative of the microstructure from which it was extracted. The magnetization decay of the network extracted using the maximal ball algorithm decays faster than the reconstructed microstructure. This is because the maximal ball algorithm seems to slightly over estimate the grain surface area. The mean transverse relaxation times, $T_2$ lin is 570 ms for the microstructure and networks extracted by Øren and Bakke (2002, 2003); and 493 ms for the network extracted using the maximal ball algorithm.

6. Conclusions

We have been able to successfully compare magnetization decays and $T_2$ distributions of brine in networks extracted using the maximal ball method (Silin and Patzek, 2006; Al-Kharusi and Blunt, 2007; Dong, 2007) and micro-CT images of sand packs within a good degree of accuracy. Consistent results were also obtained in the comparison of the permeabilities and formation factors of the micro-CT images and networks with their experimental values. The narrower $T_2$ distributions observed in the simulation results are as a result of the information that is lost during the network extraction. In network extraction, complex pore spaces in the micro-CT image are represented by simple geometrically shaped elements whose properties are determined from the complex pore spaces. Realistically, these simple geometrically shaped elements may not accurately capture all the essential features of the actual geometry of the pore spaces, in this regard; some approximations and assumptions are made in the network extraction thereby eliminating some fine details of the complex pore spaces which leads to narrower $T_2$ distribution.

Numerical results for Fontainebleau sandstone were also compared. A network generated using a process-based algorithm gave a predicted magnetization decay similar to that computed on the image from which the network was extracted (Øren and Bakke, 2002, 2003). A network extracted using the same maximal ball algorithm used for the sand packs over-predicted the decay rate. These results suggest that a maximal ball extraction algorithm can be used to predict single-phase transport properties in unconsolidated media successfully, although may be inaccurate for consolidated sandstones. This work serves as a validation for pore-network models and the methods used to generate networks. It is a starting point for predictions of multiphase properties.

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References


Fig. 11. Flowchart of the algorithm used in the simulation of NMR response of single-phase fluid in networks.