A Rigorous Pore-to-Field-Scale Simulation Method for Single-Phase Flow Based on Continuous-Time Random Walks

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Summary
We propose a pore-to-reservoir simulation approach for single-phase flow. Transport is modeled as a continuous-time random walk (CTRW). Particles make a series of transitions between nodes with a probability \( \psi(t)/dt \) that a particle will first arrive at a node from a nearest neighbor in a time \( t \) to \( t+dt \). A top-down multiscalar approach is used to find the flow field. At the micron scale, \( \psi(t) \) for particle transitions from pore to pore are found from modeling advection and molecular diffusion in a geologically representative network model. This \( \psi(t) \) is used to compute transport on the millimeter-to-centimeter scale. At larger scales, we represent the reservoir as a network of nodes connected by links. For each node-to-node transition, we compute an upscaled \( \psi(t) \) from a simulation of transport at the smaller scale. We account for small-scale uncertainty by interpreting \( \psi(t) \) probabilistically and running simulations for different possible realizations of the reservoir model. To make the number of computations manageable, \( \psi(t) \) is parameterized in terms of sub尺度 heterogeneity and Péclet number, meaning that only a few representative simulations are required.

We apply this method by finding \( \psi(t) \) for pore-scale flow and using it in a million-cell reservoir model. We show that the macroscopic behavior can be very different from that predicted by assuming that the advection/dispersion equation (ADE) operates at the small scale. Small-scale structure does affect macroscopic transport; increasing the pore-level heterogeneity delays breakthrough and leads to longer late-time tails of the production because the solute spends more time in slow-flowing regions of the domain. We discuss extensions to multiphase flow and the development of a novel network-based probabilistic reservoir-simulation approach.

Introduction
Current research in reservoir simulation—on improved discretization of the governing conservation equations, the use of unstructured grids and parallelization—reinforces the notion that the underlying principles that we use are correct (e.g., deBaun et al. 2005). However, we assume that the partial-differential equations we use, applying Darcy’s law and Fickian dispersion, are correct. Furthermore, even if we use the right equations, we can only estimate the parameters in these equations, such as permeability and porosity. This leads to uncertainties in our predictions of transport and oil recovery.

The approach we propose to handle these problems, while being conceptually simple and easy to implement, does not presuppose the form of the governing transport equations and automatically accommodates uncertainty. In this paper, we will apply it to single-phase flow. We first discuss the traditional approach to modeling transport and illustrate its limitations. We then introduce continuous-time random walks and apply them to a pore-to-core analysis of dispersion. We then show how to upscale transport to the field scale, and we demonstrate that variability at every scale affects the macroscopic behavior; a heterogeneous field-scale reservoir description does not remove the effects of inhomogeneity at smaller scales. Last, we discuss possible extensions to multiphase flow.

CTRWs
In this paper, we will simulate transport using a CTRW. Since the first application of CTRWs to model electron transport in semiconductors (Scher and Montroll 1975), they have been applied to a variety of other problems, including transport in porous media [see the recent review by Berkowitz et al. (2006)].

In a CTRW, particles hop between discrete nodes or sites with a probability \( \psi(t,i,j)dt \) that a particle that first arrives at site \( i \) will move to site \( j \) in a time \( t \) to \( t+dt \). CTRW does not make any assumptions about the governing transport equations. Instead, physical modeling of the process of interest is used to derive \( \psi \) from which the resultant macroscopic behavior can be found analytically or numerically.

Anomalous or non-Gaussian transport is easily described using the CTRW method (Berkowitz et al. 2006). Reservoir heterogeneity on many scales often leads to a power-law dependence of the hopping probability at late time: \( \psi(t) \propto t^{-\beta} \) with some exponent \( \beta \geq 2 \). This results in an outlet concentration (the concentration of solute in the produced water or the breakthrough curve), at late time, that scales as \( C(t) \propto t^{1-\beta} \). For \( 1 > \beta > 0 \), both the average plume displacement and its standard deviation scales as \( l(t) \propto t^{1-\beta} \), while for \( 2 \geq \beta > 1 \), the average displacement scales normally as \( l(t) \propto t \). However, the spread is still anomalous: \( \sigma(t) \propto t^{\beta-2} \). For \( \beta > 2 \), the system displays Gaussian behavior.
In many applications in contaminant transport, only statistical properties of the permeability field are known. In such cases, the CTRW method has been applied to find the behavior averaged over an ensemble of all possible realizations of the subsurface (Dentz et al. 2004; Berkowitz et al. 2006). However, in petroleum engineering, the use of seismic surveys, well-test analysis, log and core data, and analysis of analog outcrops enables the determination of the permeability—although still with some uncertainty—at the large (meter) scale while smaller-scale structure is often treated statistically. In such cases, we need a simulation technique that considers transport deterministically for, say, the field-scale reservoir description, while accommodating subgridblock heterogeneity statistically, incorporating the effects of uncertainty in the reservoir description on the resultant predictions of transport. While the CTRW method has been applied to heterogeneous media, the implementation has been limited to 2D systems with a small number of gridblocks in which the solution involves the numerical inversion of a multidimensional Laplace transform (Cortis et al. 2004). Our method will be much simpler and will involve a generalized particle-tracking algorithm, in which we move particles from site to site with some known \( \psi(t;i,j) \).

We employ a multiscale methodology (Arbogast and Bryant 2002; Juanes and Patzek 2004; Audigane and Blunt 2004; Jenny et al. 2006; Tyagi et al. 2008) in which, at each stage, transport is considered conceptually as a series of transitions from node to node (Fig. 1). All the physics of the process is contained in the transit-time distribution \( \psi(t) \). At the very smallest scales—the pore and subpore level—we do know how to describe transport. It is Stokes flow in a spatially varying flow field with molecular diffusion. Hence, we start at the pore scale and then describe how to upscale these results to determine transport in the field.

**Pore-to-Core Simulation**

The basis of our small-scale simulation is the work of Bijeljic et al. (2004), who simulated dispersion through a 2D diamond lattice of throats connecting volumeless pores (nodes or sites). The throat-radius distribution matched that of Berea sandstone, while the cross section was assumed to be square, which allowed for a semianalytical description of the velocity field within each throat. Transport was simulated as a combination of advective steps following the flow field followed by random motion to represent diffusion. A physically based algorithm was used to assign particles to a new throat once a node had been reached. This method made accurate predictions of literature measurements of the dispersion coefficient \( D_L \) as a function of Péclet number, \( Pe = vL/D_m \), where \( v \) is the average flow speed in the porous medium, \( L \) is the throat length, and \( D_m \) is the molecular-diffusion coefficient.

Another way to simulate transport would be to move the particles from pore to pore with a known \( \psi(t;i,j) \). A direct method is to compute \( \psi \) for each pore-to-pore transition; this \( \psi \) will be a function of both the local flow speed in the throat and the Péclet number. We use a single explicit realization of the network in which the velocity in each throat is different. The best representation of Berea sandstone is as a topologically disordered 3D network; in our case, the network represents a sample 3 mm across with 12,349 pores and 26,146 throats. We apply a pressure drop across this network and compute the flow rate in each throat. Ten thousand particles are launched at the inlet weighted by the flow rate in each inlet throat, and these are tracked as they move across the network. One approach, which we will show works well, is to assume that the particle transport obeys the ADE (Eq. 1) within each throat (although, not necessarily on the macroscale). In this case, it is possible to derive \( \psi \) analytically (Rhodes and Blunt 2006). The cumulative transit-time distribution is given by:

\[
Y(t) = \int_0^t \psi(t)\,dt. \tag{2}
\]

where, by definition, \( Y(t = \infty) = 1 \). For each hop from pore to pore, we find a random number \( z \) between 0 and 1. We then invert \( Y \) to find the transit time \( t_z \) such that \( Y(t_z) = z \). The particle is then moved to the next pore, and the time counter associated with each particle is incremented by \( t_z \).

We also need to assign a particle to a connected throat when it reaches a pore. Again, for transport governed by an ADE, it is possible to find the probability of moving into a connected throat analytically as a function of local Péclet number (Rhodes and Blunt 2006).

During the simulation, we compute the mean particle location:

\[
l(t) = x(t) = \frac{1}{N_p} \sum_{k=1}^{N_p} x_k(t), \tag{3}
\]

where \( x_k \) is the displacement of particle \( k \) from where it was launched and \( N_p \) is the number of particles. The variance in location is given by

\[
\sigma^2(t) = \frac{1}{N_p} \sum_{k=1}^{N_p} (x_k(t) - \bar{x}(t))^2. \tag{4}
\]

Then we define the longitudinal dispersion coefficient:

\[
D_L = \frac{1}{2} \frac{d\sigma^2(t)}{dt}. \tag{5}
\]

**Fig. 2** shows the late-time dispersion coefficient, Eq. 5, as a function of Péclet number, where \( Pe \) is varied by changing the pressure drop across the network. We use a fixed value of the molecular-diffusion coefficient, \( D_m = 10^{-9} \text{ m}^2\text{s}^{-1} \). The results are in good agreement with experimental results in the literature and the network studies of Bijeljic et al. (2004) and Bijeljic and Blunt (2006, 2007) on 2D networks. We can represent transport as a series of hops between pores. The assumption of an ADE in each throat may not be strictly valid because there is a subtle interplay of advection and diffusion in a spatially varying flow field. Yet, because the wide distribution of average velocity between throats, rather than within throats, dominates the behavior at intermediate \( Pe \), the dispersion coefficient is accurately predicted.
When we model transport in an upscaling framework, it will be more convenient to use cubic networks. The first step is simply to employ the same simulation method but on a cubic network. We use a 100×100×100 cubic network but with the same distribution of throat radii as before. The throat length is 100 μm, so this represents a sample 1 cm across. Fig. 2 shows that the predicted dispersion coefficient is similar to that measured experimentally in Berea sandstone. This fit is a good indication that the cubic ADE model is appropriate for this range of lengths and that the data encompass a full range of transport behavior.

Fig. 2—The longitudinal dispersion coefficient as a function of Péclet number Pe. The points and vertical bars are experimental results on sand packs and sandstones in the literature collated by Bijeljic et al. (2004). The lines are predictions using different pore-scale modeling approaches. All the models assume that transport occurs between pores connected by throats. The squares show the 2D network model results of Bijeljic and Blunt (2006, 2007) in which advection and diffusion are modeled within throats. The asterisks represent a 3D simulation on a network representing the topology of Berea sandstone but in which pore-to-pore transitions are computed semianalytically assuming a 1D ADE in each throat. The crosses indicate an equivalent simulation in a cubic lattice, but again with a throat-size distribution representing Berea sandstone. The last—dots and line—shows a simulation on a homogeneous cubic lattice using an ensemble-averaged transit-time distribution (Eq. 6). In all cases, the models give accurate predictions of the data.

Core-to-Field-Scale Simulation

It is not practical to simulate field-scale transport using a pore-scale network model. We propose an upscaling strategy that emulates the CTRW approach and which, at any scale, we represent transport as a series of transitions between discrete nodes. We consider that we have a field-scale reservoir description, and we want to honor this heterogeneity explicitly. However, we will compute transport for all possible realizations of the reservoir below the gridblock scale. The approach we use borrows ideas from multiscale modeling (Arbogast and Bryant 2002; Juanes and Patzek 2004; Audigane and Blunt 2004; Jenny et al. 2006; Tsygai et al. 2008).

First, we compute the flow field at the field scale with known boundary conditions (wells) and initial distribution of fluids. For single-phase flow, we assume that this is independent of the solute concentration. We then extract pairs of gridblocks, as represented in Fig. 3. We know the total flux across each face Q. We then perform a subblock simulation within each block, we represent the medium as a network of elements. At this scale, this will be an ensemble-averaged, homogeneous, cubic pore-level model, described in the previous section. This assumption that we can use a uniform network assumes that the system is homogeneous between the pore and the gridblock scales. We discuss later how this limitation may be overcome. We find the flow rate in each throat by solving for the flow field with the known, Neumann (flux), boundary conditions.

Particles enter the faces of the left-hand block in proportion to the flux across each face. Eq. 6 is used to transport particles from node (pore) to node; however, we solve for the average velocity in each throat now to account for the different face fluxes. This is taken into account in defining a local Péclet number in Eq. 6. We record the time between the particle first entering the right-hand block and it leaving. We compute this distribution of transit times for all the particles and for different pairs of blocks.

For this example, each gridblock was represented by a 50×50×50 homogeneous cubic network, 5 mm across. We launched 10,000 particles into the left-hand block.

At the gridblock scale, transport is advection-dominated in our examples. As a consequence, particles almost always moved macroscopically in the direction of the imposed flux. Furthermore, regardless of launch or exit face or which pair of blocks we considered, the transit-time distribution was exponential.

\[ \psi(t) = A e^{-t/t_2} \left(1 + t/t_1\right)^{-1/\beta} \] .......................... (6)

where \( A \) is a normalization constant, \( t_1 \) is an average advective transit time = \( l/v \), where \( l \) is the average pore-to-pore length, \( v \) is the average velocity in a throat, and \( t_2 \) is a typical diffusive transit time = \( D/v^2 \). This functional form for \( \psi \) can be interpreted physically (Dentz et al. 2004): Heterogeneity leads to an approximate power-law distribution of transit time but with a short-time and long-time cutoff. The short-time cutoff \( t_1 \) is the mean time to transit a throat by advection, while the long-time cutoff \( t_2 \) is the time to move through a throat by diffusion if there is no flow. The power-law exponent \( \beta \) is a function of the heterogeneity of the network. For the Berea-sandstone network, we find a best-fit value of 1.8.

We use a homogeneous cubic network to simulate transport. We do not need to compute the flow field; instead, for a given system-averaged Péclet number, we use Eq. 6 to transport particles from pore to pore. This method, as shown in Fig. 2, also predicts the experimental data accurately. This is the simplest simulation and is fully consistent with our desire to model transport while accommodating uncertainty. It is this approach that we will employ in the rest of the paper.

Apparent in Fig. 2 is an approximate power-law trend of the dispersion coefficient at intermediate values of \( Pe: D_l \approx Pe^\beta \) with \( \beta \approx 1.2 \). It is possible to show that this exponent is related to \( \beta \) by \( \delta = 3 - \beta \)—the macroscopic dispersion comes from the power-law distribution of throat velocities (Bijeljic and Blunt 2006).

The squares show the 2D network model results of Bijeljic and Blunt (2006, 2007) in which advection and diffusion are modeled within throats. The asterisks represent a 3D simulation on a network representing the topology of Berea sandstone but in which pore-to-pore transitions are computed semianalytically assuming a 1D ADE in each throat. The crosses indicate an equivalent simulation in a cubic lattice, but again with a throat-size distribution representing Berea sandstone. The last—dots and line—shows a simulation on a homogeneous cubic lattice using an ensemble-averaged transit-time distribution (Eq. 6). In all cases, the models give accurate predictions of the data.
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For our reservoir description, we used the SPE10 model that represents a North Sea oil reservoir. The model is a large-scale simulation, the flow field is computed so the fluxes across each block face are known. Pairs of gridblocks are extracted from the model. Then a simulation is performed within these blocks. In this example, the subgrid scale is a pore-level network model. As an ensemble average, the subgrid is homogeneous. The flow rate in each element is computed using the block-face, Neumann, boundary conditions. While the network is homogeneous, there is a distribution of velocity between throats because of the different fluxes at the block faces. Transport is modeled as before, by a series of pore-to-pore transitions using Eq. 6, with the known average velocity in each element. Particles are launched along the face of the left-hand block. When a particle first enters the neighboring, right-hand block, we record the time taken before the particle first exits that block. We then find this transit-time distribution for all the particles and for different pairs of blocks. At the larger scale, this transit-time distribution is used to represent, conceptually, a hop between two nodes indicating the centers of the gridblocks.

We employed two sets of boundary conditions. In the first, we completed an injector well and a producer well at opposite corners of the model. We set the control on the injector to a flux of 800 m³/d and on the producer to a bottomhole pressure of 27 × 10⁵ kPa. In the second, we injected across one face of the model and produced from the opposite face with no flow across the other faces.

In the transport simulation, links joined centers of adjoining blocks with a face-flux Q associated with the link, as shown in Fig. 3. We then launched 10,000 particles along the injector by a flux-weighted scheme and monitored the time taken for each to reach the producer. In the second simulation, we injected particles along one face of the model and produced from the opposite face, with no flow on the other faces. We compute transport as before, as a series of transitions between nodes, using Eq. 7 for the transit-time distribution. This is an efficient method, comparable to streamline-based simulation in efficiency (Di Donato et al. 2003) because most of the computer time is taken to compute the flow field while the transport simulation is relatively fast.

While the transit time is counted for transport across a block and the macroscopic simulation is, conceptually, from block center to block center, the method does correctly track typical transit times because we consider movement block to block across specified faces.

It is remarkable to note how simple the macroscopic transit-time distribution is. In theory, an ensemble-averaged macroscopic transport algorithm needs to consider a convolution integral in time that accounts for the probability of particles arriving after sampling different paths (Cortis et al. 2004; Berkowitz et al. 2006). This subtlety is ignored in all petroleum upscaling applications—fortunately, without significant error for single-phase flow at least.

**Field-Scale Results**

For our reservoir description, we used the SPE10 model that represents a North Sea oil reservoir. The model is 366 × 670 × 52 m divided into 60 × 220 × 85 = 1,122,000 blocks of approximately 6 × 3 × 0.6 m, representing sand channels surrounded by lower-permeability shale with more than four-orders-of-magnitude variation in permeability (Christie and Blunt 2001). Each gridblock is represented—at the smaller scale—by identical homogeneous pore-networks derived from Berea sandstone, as described in the previous section. This network is only 5 mm across; we assume that we can scale the results by \( Q/V \) (Eq. 7) to represent a larger system. An improved approach that considers core-to-gridblock scaling is discussed later.
The late-time behavior is matched by an approximate power law $C(t) \propto t^{-(1 + \beta_m)/2}$ of approximately $0.66 \pm 0.02$ and $0.64 \pm 0.04$ for $s(t)$, indicative of anomalous transport with $\beta_m = 1.68 \pm 0.04$ and $1.72 \pm 0.08$ for face-to-face and well-boundary conditions, respectively.

While the boundary conditions alter the time-scale of the displacement, the scaling behavior is broadly similar for both well and face-to-face transport.

Fig. 6 shows the corresponding concentration at the producer where we find $C(t) = Ct^{-\beta_m/2}$, but this time, the apparent exponent is $\beta_m = 1.2 \pm 0.1$. The inconsistency with the exponent from the early-time behavior indicates long-range spatial correlation in the transport; the simple characterization with a single exponent assumes statistical homogeneity in the system, which is not appropriate for this highly structured reservoir description (Di Donato et al. 2003).

The macroscopic boundary conditions do not affect the late-time behavior of the plume, so the power-law scaling is not caused by near-well radial flow. Also note that the highly heterogeneous nature of the field leads to breakthrough in approximately 100 days, while it takes more than 100,000 days for all the particles to traverse the system.

Which Affects Field-Scale Recovery: Small- or Large-Scale Heterogeneity?

Traditionally, transport would be simulated directly on the field-scale model, without any upscaling, with some dispersion coefficient used to represent subgridblock heterogeneity. In this section, we test to see if the proper incorporation of small-scale transport affects the large-scale results.

In Fig. 6, we show breakthrough curves in which we use a $\psi$ assuming a 1D ADE in each link with $D = D_m$. This is equivalent to a traditional simulation with an infinitely resolved discretization between nodes. The late-time results are similar to those obtained using Eq. 7 for $\psi$, indicating that the large-scale heterogeneity dominates the overall behavior. This is to be expected because the pore-scale representation of the field as a relatively homogeneous Berea sandstone contrasts with the extreme variability in the large-scale permeability. Streamline-based simulation, again assuming advective transport on the same reservoir model, also gave the same macroscopic behavior with $\beta_m = 1.2 \pm 0.1$ (Di Donato et al. 2003).

However, assuming the ADE at the small scale underpredicts the breakthrough time, particularly for face-to-face transport. This is because it does not account for the fact that particles will, occasionally, encounter stagnant regions across which transport occurs only slowly by diffusion. This tends to slow down the solute, as shown in previous CTRW simulations in macroscopically heterogeneous media (Cortis et al. 2004).

We ran one further set of tests, in which we assumed more pore-scale heterogeneity. At the pore scale, we used Eq. 6 for transport in an ensemble-averaged (homogeneous) network, but now with $\beta = 1.1$ and $\beta = 0.5$. We reran the upscaling step to find an empirical form for the transit-time distribution (Eq. 7). For $\beta = 1.1$, Eq. 7 is still valid (Fig. 4) but with $\lambda = 1.8$, indicating slower transport. This is to be expected: As the medium becomes more heterogeneous, particles will encounter more slow-flowing domains, and this will, on average, increase the transit times.

For the most heterogeneous pore-scale network (representing, for instance, a vuggy carbonate) with $\beta = 0.5$, we can no longer use Eq. 7. Instead, we find Fig. 4:

$$
\psi_s(t) = \frac{\lambda Q P_{m1}^{\gamma-1}}{V} e^{-2P_{m1}\lambda Q/V},
$$

with an exponent $\gamma = 0.8$ and constant $\lambda = 1.6$.

Fig. 7 shows the breakthrough curves for macroscopic face-to-face transport with differing amounts of pore-scale heterogeneity. Although the macroscopic reservoir description, captured at the meter scale with more than a million gridblocks, is the same, the macroscopic behavior is very different, with breakthrough times that vary by a factor of approximately 4 and slightly different late-time exponents, decreasing from an apparent $\beta_m$ of 1.2 to approximately 1.1 as the pore-scale $\beta$ decreases. Increasing the pore-scale heterogeneity forces the solute to sample stagnant regions of the pore space more frequently. This slows down the overall transport and leads to a very long tail in the breakthrough curve (Cortis et al. 2004). Heterogeneity at all scales affects the macroscopic behavior. It is not correct to presume that simply because the meter-scale reservoir description is highly structured with more than four-orders-of-magnitude variation in permeability, it will dominate over any smaller-scale variability. Hitherto, there has been no tool to see this phenomenon: A direct simulation of this pore-to-field transport would require on the order of a trillion cells, and using fewer cells and some effective ADE is, as we have shown, inadequate.
Further Refinements: The Missing Scale

The current methodology could be refined in a number of ways. The most glaring problem with the results presented here is that the pore-to-core simulations represent a sample approximately 1 cm across, while the field-scale gridblocks are meter-scale. We need to include a third, intermediate core-to-grid-block upscaling step. This involves modeling typical submeter-block heterogeneity at the cm scale. To find an ensemble average, different macroscopic flux boundary conditions and small-scale realizations need to be considered. We have done this, using a layered sandstone-and-shale model at the meter scale; the transit-time distribution follows a truncated power law—Eq. 6 (Rhodes et al. 2008). The method was validated by comparing predictions at the meter scale with the heterogeneous sand-pack experiments of Levy and Berkowitz (2003). Then, the field-scale simulation is performed as before. We find that heterogeneity at all three scales—core, meter, and field—affects the breakthrough time and long-time tail of the production curve.

Another limitation of the current work is that we only consider homogeneous pore-scale networks. Ideally, we would consider different networks that properly represent the differences in pore structure between blocks of different macroscopic permeability and porosity.

Extensions to Multiphase Flow

The current approach relies on using a particle-tracking approach in which the flux is linearly related to the solute concentration. This allows us to treat each particle independently and makes ensemble averaging easy. In multiphase flow, however, the flux from node to node is nonlinearly dependent on saturation, making particle tracking and ensemble averaging problematic. While it is possible to extend particle tracking to multiphase flow (Tyagi et al. 2008), this method has yet to be fully developed. Instead, we propose that the node-to-node transport of saturation is treated as in single-point upstream weighting but with a probabilistic distribution of putative fractional flows, representing different possible smaller-scale structure.

The advantage of this approach for general petroleum-reservoir simulation is that the logic of the nodal structure does not have to be ordered, so it is easy to deal with unstructured grids representing large-scale geological features, such as faults, and to couple the reservoir transport with flow in wells and facilities.

Conclusions

We have proposed a pore-to-field transport simulation approach and applied it to single-phase flow accounting for advection and diffusion. We assume that transport occurs as a series of transitions between discrete sites governed by a transit-time distribution.

We can predict the Péclet-number dependence of the dispersion coefficient at the core scale by modeling transport as a series of pore-to-pore hops. This can be achieved either by explicitly representing the pore-scale heterogeneity of Berea sandstone or by using a homogeneous network with an ensemble-averaged transit-time distribution that is given by a truncated power law.

We developed a multiscale upscaling method to simulate transport. We showed how to find the transit-time distribution at the core scale and how to use this in macroscale simulation.

For advective-dominated transport, the transit-time distribution at the core scale is exponential in time, with a time scale related to the time for a particle to advect across the block with a typical velocity. At the field scale, with a finely resolved, highly heterogeneous reservoir model, the overall transport behavior is anomalous with power-law scaling of the mean plume location, its standard deviation, and the breakthrough curves.

The macroscopic behavior is affected by the small-scale transport even when a very heterogeneous field-scale reservoir description is used. Increasing the pore-level heterogeneity delays the particle transport because advective trapping in slow-flowing regions becomes more common with significantly increased tailing of the breakthrough curves. It is erroneous to assume that small-scale heterogeneity is, in some mysterious way, only controlled by larger-scale geology. We are now able to perform rigorous pore-to-field simulation, and it is evident that such an assumption cannot be sustained.

Nomenclature

\[ A = \text{area, } L^2, \text{m}^2 \]
\[ c, C = \text{concentration (density), } ML^{-3}, \text{kgm}^{-3} \]
\[ D = \text{diffusion or dispersion coefficient, } L^2T^{-1}, \text{m}^2s^{-1} \]
\[ l, L = \text{length, } L, \text{m} \]
\[ n, N = \text{number} \]
\[ Pe = \text{Péclet number, dimensionless, } ML^{-1}T^{-2}, \text{Pa} \]
\[ Q = \text{flux, } L^3T^{-1}, \text{m}^3s^{-1} \]
\[ t = \text{time, } T, \text{s} \]
\[ v = \text{velocity, } LT^{-1}, \text{ms}^{-1} \]
\[ V = \text{volume, } L^3, \text{m}^3 \]
\[ Y = \text{cumulative probability, dimensionless} \]
\[ \beta = \text{exponent} \]
\[ \gamma = \text{exponent} \]
\[ \delta = \text{exponent} \]
\[ \lambda = \text{coefficient, dimensionless} \]
\[ \sigma = \text{standard deviation} \]
\[ \phi = \text{porosity, dimensionless} \]
\[ \psi = \text{transition probability, dimensionless} \]

Subscripts

\[ g = \text{grid-block (core scale)} \]
\[ L = \text{longitudinal} \]
\[ m = \text{macroscopic or molecular} \]

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References


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