Pore-Scale Modeling: Stochastic Network Generation and Modeling of Rate Effects in Waterflooding

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Abstract

Pore scale network modeling has been used to predict transport flow properties for multiphase flow successfully. The prediction is based on having geologically realistic networks that are computationally expensive to generate and normally represent only a very small section of the rock sample. We present a new method to generate stochastic random networks representing the pore space of different rocks with given input pore and throat size distributions and connectivity – these distributions can be obtained from an analysis of pore-space images. The stochastic networks can be arbitrarily large and hence are not limited by the size of the original image.

The basic assumption made in the prediction of transport flow properties using most pore-scale models is that the flow is capillary dominated. This implies that the viscous pressure drop is insignificant compared to the capillary pressure. However, at the field scale, gravity and viscous forces dominate displacement processes. We develop a rate-dependent network model that accounts for viscous forces by solving for the wetting and non-wetting phase pressure and which allows wetting layer swelling near an advancing flood front. We propose a new time-dependent algorithm by accounting for partial filling of elements.

We use the model to study the effects of capillary number and mobility ratio on imbibition displacement patterns, saturation and velocity profiles. We also investigate the effects of capillary number and mobility ratio on the water fractional flow curve, cumulative oil production and residual oil saturation for water-wet and mixed-wet systems. By using large networks we reproduce Buckley-Leverett profiles directly from pore-scale modeling thereby providing a bridge between pore-scale and macro-scale transport.
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## Contents

ABSTRACT .................................................................................................................. 2

ACKNOWLEDGMENTS ........................................................................................... 3

CONTENTS .................................................................................................................. 4

LIST OF FIGURES ..................................................................................................... 7

LIST OF TABLES ..................................................................................................... 12

NOMENCLATURE ................................................................................................... 13

1 INTRODUCTION .............................................................................................. 15

2 LITERATURE REVIEW .................................................................................. 17

  2.1 REGULAR LATTICE-BASED NETWORKS .................................................. 17

  2.2 PORE SPACE IMAGE-BASED NETWORKS ........................................... 19

      2.2.1 3D Image Reconstruction Using Statistical Methods ............... 19

      2.2.2 X-ray Micro-tomography (Micro-CT) ......................................... 20

      2.2.3 Process-Based Reconstructions ................................................. 20

      2.2.4 Networks Extraction from Pore Space Images ......................... 22

  2.3 PORE GEOMETRY AND WETTING LAYERS ........................................... 26

  2.4 WETTABILITY EFFECTS, SATURATION HISTORY AND CONTACT ANGLE

      HYSTERESIS ................................................................................................. 27

  2.5 PORE SCALE MODELS ............................................................................. 30

      2.5.1 Quasi-static Models ................................................................. 31

      2.5.2 Dynamic Models ................................................................. 34

  2.6 PROJECT OVERVIEW .............................................................................. 46

      2.6.1 Network Type and Pore Geometry ........................................... 46

      2.6.2 Fluid Volume and Viscosity ..................................................... 47

      2.6.3 Wetting Layer Conductance and Flow Rate ............................ 47

      2.6.4 Non-Wetting Phase Pressure Gradient ................................. 48

      2.6.5 Computational Efficiency and Network Size ....................... 48
3 STOCHASTIC NETWORK GENERATION ALGORITHM ...................... 49

3.1 CURRENT ALGORITHM .................................................................................................. 50
3.2 PROPOSED NEW ALGORITHM .................................................................................. 50
3.3 DESCRIPTION OF THE BEREA NETWORK ................................................................. 59
3.4 GENERATED EQUIVALENT STOCHASTIC BEREA NETWORKS ................................. 60
3.5 COMPARISON BETWEEN THE ORIGINAL AND AN EQUIVALENT BEREA NETWORK ..................................................................................................................................................................... 63
  3.5.1 Geometrical and Topological Data ........................................................................ 63
  3.5.2 Correlational Data ................................................................................................. 65
  3.5.3 Relative Permeability Curves ............................................................................. 67
  3.5.4 Pore-space correlations ..................................................................................... 71
3.6 CONSISTENCY CHECK ............................................................................................... 72

4 DYNAMIC PORE-NETWORK MODEL ................................................................. 74

4.1 DISPLACEMENT FORCES – COMPUTATION OF CAPILLARY PRESSURE .......... 74
4.2 DISPLACEMENT FORCES – COMPUTATION OF VISCOS PRESSURE DROP ...... 76
4.3 COMPUTATION OF GLOBAL PRESSURE FOR IMBIBITION .................................... 78
4.4 PROPOSED NEW MODEL ........................................................................................... 79
  4.4.1 Maximum Local Capillary Pressure .................................................................. 80
  4.4.2 Minimum and Optimum Time Step .................................................................. 80
  4.4.3 Updating Water Layer Volume ......................................................................... 81
  4.4.4 Water Corner Conductance Approximation ..................................................... 81
4.5 FILLING SEQUENCE ................................................................................................. 83
4.6 TIME-DEPENDENT PARTIAL FILLING ALGORITHM .............................................. 84
4.7 TRAPPING AND UPDATING THE SORTED LIST ..................................................... 88
4.8 OIL SUB-NETWORK .................................................................................................. 88
4.9 SCALING AND ADJUSTING OF OIL PRESSURE ....................................................... 89
4.10 MODEL ASSUMPTIONS / APPROXIMATIONS ......................................................... 90
  4.10.1 Outlet Water Flow Rate and Maximum Injected Water Volume ...................... 90
  4.10.2 Complete Filling of Elements ........................................................................... 90

5 WATER-WET BEREA SANDSTONE RESULTS .............................................. 92

5.1 MODEL VALIDATION ................................................................................................. 93
List of Figures

2.1: Comparison of the 3D pore space of Fontainebleau sandstone ......................21
2.2: A network extracted from a carbonate image.............................................24
2.3: A network generated from a reconstructed microstructure .......................24
2.4: A skeleton of a pore network ......................................................................25
2.5: A mixed-wet pore ......................................................................................28
2.6: Oil and water in a triangular pore or throat..............................................29
2.7: A typical unit cell of the porous medium ..................................................35
2.8: A plot of the residual oil saturation, $S_{or}$ vs. the capillary number, $N_{cap}$ ......35
2.9: The predicted effect of the viscosity ratio, $x$ on the residual non-wetting phase
    saturation, $S_{or}$ for various values of the capillary number $N_{cap}$ ..................37
2.10: The experimental effect of the viscosity ratio, $k$ on the residual non-wetting
      phase saturation, $S_{or}$ for various values of the capillary number, $N_{cap}$ ......37
2.11: Comparison between two-dimensional (2-D) model simulations results
      (residual oil saturation vs. porosity divided capillary number) with three-
      dimensional coreflood experimental results .................................................39
2.12: Residual saturation curves for three-dimensional simulations ....................42
2.13: Simulation results of non-wetting phase saturation and velocity profiles ....43
2.14: Comparison between predicted and experimental relative permeability curves .
      ..................................................................................................................45
3.1: A regular network of size 20 x 10 x 2 .......................................................51
3.2: Randomized network with variable coordination number ..........................51
3.3: A flowchart for the stochastic algorithm .................................................54
3.4: Randomly placed pores in a 2D network showing pore index for each pore .56
3.5: A 2D stochastic network showing the randomly placed pores and the
      connected throats. .......................................................................................58
3.6: Correlation between pore coordination number and average pore volume .....60
3.7: Pore and throat radius distributions for the Berea network ......................61
3.8: Correlation between radii of pores and connecting throats for the reconstructed
      Berea ...........................................................................................................61
3.9: A ball and stick representation of the original Berea network. ..................62
3.10: A ball and stick representation of an equivalent Berea network. ..........62
3.11: Comparison between the original and stochastic Berea network aspect ratios. 67
3.12: Comparison between relative permeability curves for the diluted stochastic and Fontainebleau rock networks. .................................................................69
3.13: Comparison of predicted primary drainage relative permeability curves........70
3.14: Comparison of predicted secondary imbibition relative permeability curves. 70
3.15: Comparison of primary drainage relative permeability curves for \((3\times3\times3)mm^3\) and \((12\times6\times6)mm^3\) stochastic networks. .........................................................72
3.16: Comparison of secondary imbibition relative permeability curves for \((3\times3\times3)mm^3\) and \((12\times6\times6)mm^3\) stochastic networks..............................................73

4.1: Piston-like displacement mechanism; this can only occur if there is an adjacent element whose center is filled with oil. ...............................................................75
4.2: Pore-body displacement mechanism; (a) \(I_1\) – only one throat is filed with oil and (b) \(I_2\) – two throats are filled with oil.................................................................75
4.3: Snap-off displacement mechanism; it can only occur if there is no adjacent element whose center is filled with water.............................................................76
4.4: A flow chart for the partial filling algorithm ..................................................87

5.1: Relationship between receding and advancing contact angles ...............92
5.2: Initial water distribution at \(S_{wi} = 0.2\).........................................................94
5.3: Displacement patterns for the quasi-static model of Valvatne and Blunt with average \(S_w = 0.5\) ........................................................................................................94
5.4: Displacement patterns for the Hughes and Blunt model. .........................95
5.5: Displacement patterns for the dynamic model .........................................95
5.6: Comparison of predicted relative permeability curves for the dynamic and Hughes and Blunt (H&B) models .................................................................96
5.7: Comparison of 2D displacement pattern projections at \(N_{cap} = 3.0\times10^{-7}\) .......97
5.8: Comparison of 2D displacement pattern projections at \(N_{cap} = 3.0\times10^{-6}\) .......97
5.9: Comparison of 2D displacement pattern projections at \(N_{cap} = 3.0\times10^{-5}\) .......97
5.10: Comparison of the effects of capillary number, \(N_{cap}\) on residual oil saturation, \(S_o\) for the dynamic model with the Hughes and Blunt model .........................98
5.11: 2D projection of displacement patterns for a water-wet system .............99
5.12: Effects of rate on average saturation profiles .............................................. 100

5.13: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-8}$ and $M = 1$ for a water-wet system. .......................................................... 102

5.14: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-8}$ and $M = 5$ for a water-wet system. .................................................. 102

5.15: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-8}$ and $M = 10$ for a water-wet system. ........................................... 103

5.16: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-8}$ and $M = 20$ for a water-wet system. ............................................... 103

5.17: Comparison of (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-8}$ .............................................................................. 103

5.18: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-7}$ and $M = 1$ for a water-wet system. ................................................. 104

5.19: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-7}$ and $M = 5$ for a water-wet system. .................................................. 104

5.20: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-7}$ and $M = 10$ for a water-wet system. ................................................. 105

5.21: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-7}$ and $M = 20$ for a water-wet system. ............................................... 105

5.22: Comparison of (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-7}$ .............................................................................. 105

5.23: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-6}$ and $M = 1$ for a water-wet system. ............................................. 106

5.24: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-6}$ and $M = 5$ for a water-wet system. ................................................. 106

5.25: Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-6}$ and $M = 10$ for a water-wet system ................................................... 107

5.26: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-6}$ and $M = 20$ for a water-wet system. ................................................. 107

5.27: Comparison of (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-6}$ ................................................................. 107

5.28: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{\text{cap}} = 3.0 \times 10^{-5}$ and $M = 1$ for a water-wet system. ........................................... 108
5.29: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 5$ for a water-wet system. ................................................. 109

5.30: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 10.0$ for a water-wet system. ............................................ 109

5.31: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 20.0$ for a water-wet system. ............................................ 109

5.32: Comparison of (a) saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ ....................................................................................... 110

5.33: Fractional flow curves for $N_{cap} = 3.0 \times 10^{-5}$ for different mobility ratios $M$, for water-wet Berea sandstone. ................................................................. 112

5.34: Effects of network size on saturation profiles. .............................................. 114

5.35: Effects of contact angle on saturation profiles ............................................. 115

5.36: Effects of contact angle on saturation profiles for a water-wet system ...... 117

5.37: Effects of initial water saturation on saturation profiles .............................. 118

5.38: Effects of initial water saturation on saturation profiles for a water-wet system with mobility ratio, $M = 10$. ................................................................. 119

5.39: Predicted relative permeability curves at different values of initial flowing water saturation ............................................................................................. 121

5.40: (a) Predicted relative permeability curves compared with the quasi-static prediction and experimental data ................................................................. 122

5.41: Effects of using different fractions of the network size, $f$ in the computation of the relative permeability curves at $N_{cap} \leq 10^{-8}$ ................................................. 123

5.42: Saturation profiles at initial flowing water saturation $S_{wi} = 0.03$ and $C_{vol} = 0.22$ at $N_{cap} = 3.4 \times 10^{-6}$ ....................................................................................... 124

5.43: Computed relative permeability curves for different fractions of the network size. ........................................................................................................... 125

5.44: Effects of $N_{cap}$ on the computed relative permeability curves .................... 126

5.45: Effects of contact angles on the computed relative permeability curves ..... 127

5.46: Predicted relative permeability curves at $N_{cap}$ of $3.4 \times 10^{-6}$ compared with the quasi-static prediction and experimental data ........................................... 128

5.47: A plot of run times against number of pores with a polynomial trend line. ... 129

6.1: Saturation profiles for mobility ratio, $M = 5$ and $N_{cap} = (a) 3.0 \times 10^{-8}$; (b) $3.0 \times 10^{-6}$; and (c) $3.0 \times 10^{-5}$ for a mixed-wet Berea sandstone..................... 132
6.2: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 5$ for a mixed-wet system. .......................................................... 133

6.3: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 10$ for a mixed-wet system. .................................................. 133

6.4: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 20$ for a mixed-wet system. .............................................. 133

6.5: Comparison of (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ ....................................................................................... 134

6.6: Fractional flow curves for $N_{cap} = 3.0 \times 10^{-5}$ for different mobility ratios $M$, for a mixed-wet Berea sandstone. .......................................................... 135

6.7: Saturation profiles for a mixed-wet system with mobility ratio $M = 10$, $N_{cap} = 3.0 \times 10^{-5}$ .................................................................................................................................. 137

6.8: Saturation profiles for a mixed-wet system with mobility ratio, $M = 10$, $N_{cap} = 3.0 \times 10^{-8}$ .................................................................................................................................. 138

6.9: Saturation profiles for a mixed-wet system with mobility ratio, $M = 10$, $N_{cap} = 3.0 \times 10^{-5}$; for initial water saturations, $S_w = (a) 0\%; (b) 10\%$, (c) 20\%; and (d) comparison of saturation profiles for the different initial water saturations at an average water saturation of 40\%. ....................................................................................... 139

7.1: Comparison of low and high rates experimental relative permeability curves . .................................................................................................................................. 142

7.2: Transition from initially stable to unstable infiltration where gravity-driven fingers formed. .............................................................................................................. 144
List of Tables

3.1: Pore geometric data for the 2D network .......................................................... 55
3.2: Unsorted and sorted throat radius .................................................................... 55
3.3: Sorted branch weight and the assigned throat radius ................................. 58
3.4: Comparison between the original and stochastic Berea network data ......... 65
3.5: Comparison between the original and stochastic Berea network aspect ratio. 67

5.1: Flow duration for injecting 0.4 pore volume, PV and optimum time step for
different flow rates ........................................................................................ 101
5.2: Cumulative oil production for different mobility ratios ............................... 113
5.3: Residual oil saturation, $S_0r$ and end-point water relative permeability, $K_{rw}^{max}$
for different fractions of the network used in the computation of the relative
permeability curves .......................................................................................... 125
5.4: Residual oil saturation, $S_0r$ and end-point water relative permeability, $K_{rw}^{max}$
for different $N_{cap}$ .......................................................................................... 126
5.5: Residual oil saturation, $S_0r$ and end-point water relative permeability, $K_{rw}^{max}$
for different contact angles at $N_{cap}$ of $3.4 \times 10^{-6}$ ...................................... 127

6.1: Cumulative oil production at breakthrough for different mobility ratios for
$N_{cap} = 3.0 \times 10^{-5}$ for mixed-wet Berea sandstone ....................................... 136
Nomenclature

- $G$: pore or throat shape factor
- $N_{cap}$: capillary number
- $M$: mobility ratio
- $z$: average coordination number
- $NAPL$: non-aqueous phase liquid
- $\gamma$: genus
- $P_i$: weight of pore $i$
- $D_i$: diameter of pore $i$ (m)
- $n$: correlation parameter
- $T_i$: weight of throat $i$
- $Kr$: relative permeability
- $S_w$: average water saturation
- $P_c$: threshold capillary entry pressure (N/m$^2$)
- $\sigma$: interfacial tension (N/m)
- $\theta$: contact angle (degrees)
- $r$: radius (m)
- $\beta$: corner half angle (degrees)
- $A_i$: input parameters
- $x_i$: random numbers between zero and one
- $q$: volumetric flow rate (m$^3$/sec)
- $P_{global}$: global capillary pressure (N/m$^2$)
- $P_{local}$: local capillary pressure (N/m$^2$)
- $g$: conductance
- $\Delta p$: pressure drop (N/m$^2$)
- $\mu$: viscosity (kg/m sec)
- $A$: cross-sectional area (m$^2$)
- $l$: length (m)
- $Q$: flow rate (m$^3$/sec)
- $t$: filling time (sec)
- $T$: total flow duration (sec)
- $V$: volume (m$^3$)
- $v_{ew}$: volume of water in element $e$ (m$^3$)
\( v_{enw} \)  volume of non-wetting phase in element \( e \) (m\(^3\))

\( v_{et} \)  total net volume of element \( e \) (m\(^3\))

**Subscripts**

- \( a \)  advancing
- \( r \)  receding
- \( p \)  phase \( p \)
- \( w \)  water
- \( o \)  oil
- \( nw \)  non-wetting
- \( cur \)  current
- \( max \)  maximum
- \( min \)  minimum
- \( lpc \)  local capillary pressure
- \( lpce \)  local capillary pressure equilibrium
- \( lsw \)  layer swelling
- \( dsp \)  displaced
- \( mlv \)  maximum layer volume
- \( tstep \)  time step

**Superscripts**

- \( n \)  previous time step
- \( (n + 1) \) current time step
Chapter 1

Introduction

Multiphase flow occurs during the primary, secondary and tertiary phases in the producing life of a reservoir. While primary recovery is oil recovery by natural drive mechanisms: solution gas drive, gas cap expansion, water influx, and gravity drainage; secondary recovery refers to gas or water injection to maintain reservoir pressure and enhance the oil recovery factor. Tertiary recovery is any technique applied after secondary recovery. Multiphase flow also occurs in transportation of non-aqueous phase liquid (NAPL) through unsaturated soils or rocks in environmental engineering as well as in hydrology. Another application is geological storage of carbon dioxide, when it is injected into deep saline aquifers, gas fields or oil reservoirs.

In order to understand and model fluid flow in porous media, one needs to know the constitutive relationships between the macroscopic properties of the system. While experimental measurements do give some macroscopic properties such as relative permeabilities and capillary pressure, they have limited scope and may be difficult to perform or time consuming. Physically-based models that are based on pore-scale physics and realistic characterization of the structure of the porous medium on the other hand can give not only reasonable estimates of macroscopic properties and their likely variation in a reservoir setting but also the effects of injection rate and reservoir fluid properties on waterflooding displacement efficiency.

In network modeling, the pore space of a porous medium is described as a network of large pores connected by narrower throats. Topologically equivalent networks are extracted from pore space images obtained either through statistical methods, X-ray micro-tomography or process-based reconstructions. Rules that describe the pertinent physical processes and arrangements of fluid within each pore and throat are then developed and combined to simulate multiphase flow.
The different methods of obtaining pore space images and extracting topologically equivalent networks, previously developed pore scale models as well as project overview are reviewed in the next chapter. Image-based networks in general and networks extracted from process-based reconstruction in particular have some limitations – computationally very intensive; fixed dimensions; and impractical to scale up to core scale and beyond. To overcome these constraints, a stochastic network generation algorithm that can be used to generate a network of arbitrary size with given input pore and throat size distributions and connectivity is presented in chapter 3.

Chapter 4 describes the new dynamic pore scale model and the incorporated time-dependent partial filling algorithm. A computer code developed based on the algorithm described in chapter 3 is used to generate large stochastic networks that are used as inputs in the second rate-dependent computer code developed based on the mathematical details given in chapter 4. Simulation results and reproduction of Buckley-Leverett displacement profiles for water-wet and mixed-wet systems are presented in chapters 5 and 6 respectively.

A summary of the main results and findings in this work is presented in chapter 7. Suggestions on future directions and applications of the developed model are also presented in this chapter.
Chapter 2

Literature Review

Network models have been used to understand the physics of fluid flow at the pore scale and to predict macroscopic transport properties such as capillary pressure and relative permeability [1-7]. The predictions are based on the representation of the void space of a rock in terms of wide pores connected by narrower throats – a network. Realistic network models must provide the following essential information in order to replicate exactly the microstructure of the porous medium being modeled.

1. Geometrical Information: - comprises the pore and throat size distributions, porosity, shape and pore-wall roughness.
2. Topological Information: - provides information on the skeleton of the pore network and is expressed mainly by the coordination number distribution and the genus. Mathematically, genus per node [8], \( \gamma \) is related to the mean coordination number, \( z \) by;

\[
\gamma = 0.5z - 1
\]

(2.1)

3. Correlational Information: - provides information on the spatial correlation, either on pore to throat basis only or on pore to pore as well as pore to throat, of the network.

In pore-scale modeling, networks are based on two major premises: either a regular topology is assumed or the network is derived from pore space images.

2.1 Regular Lattice-Based Networks

Starting from a regular square lattice with a coordination numbers, \( z \) of 4; a cubic lattice with a \( z \) of 6; a cubic close/hexagonal packing with a \( z \) of 12 e.t.c. several networks have been generated [8-10]. The geometric parameters of the resulting networks can be tuned to match available experimental data such as mercury intrusion capillary pressure data, the topological information may be altered by removing
throats randomly thereby reducing the coordination number to match a desired coordination number distribution and the resulting networks may assume spatially uncorrelated pore and throat sizes [8-10].

Fischer et al. [11] tuned a regular cubic lattice to match capillary pressure–saturation data for repacked and undisturbed soils. They then used the tuned network to predict absolute and relative permeabilities. For the repacked porous media, the predicted water relative permeabilities varied from good to fair when compared with experimental data. For the undisturbed soil samples however, the predictions were poor.

While Fischer et al. [11] tuned a regular lattice to match experimental data, Dixit et al. [12] randomly removed throats from a cubic network with cylindrical pores and generated a network with a desired coordination number. They then used the network to study the effects of wettability on waterflood oil recovery. Some other authors [13-15] have also used tuned regular lattices to predict macroscopic properties with various degree of success and accuracy.

Though regular lattice-based networks can be tuned to match desired experimental data or a coordination number, they have many limitations. Firstly, pores and throats are assumed, in most cases, to be cylindrical and hence can only contain one phase. However, Lenormand et al. [16] have shown through their micromodel experiments that in pores and throats with rough and angular cross-sections, the wetting phase resides in the crevices and corners while the non-wetting phase occupies the center.

Secondly, while the pore and throat sizes are assumed to be spatially uncorrelated in most regular lattice-based networks, Jerauld and Salter [17] have shown that correlations between the sizes of neighboring pore-throats affect the shape of the relative permeability curves. Spatial pore-space correlations also have a large impact on imbibition capillary pressure and relative permeability curves [18]. Knackstedt et al. [19] have also demonstrated that correlated heterogeneities exist down to the pore scale and showed that even small scale correlations had a profound effect on the structure of the fluid clusters at breakthrough and at residual saturation.
Lastly, real pore space does not have a regular topology. It has a random topology with a broad distribution of coordination numbers [9, 20]. Even when a desired average coordination number has been achieved by randomly removing throats from a regular based-network or the network has been tuned to match a particular experimental result, the random removal or matching process is generally non-unique and essential features of the pore space such as local and spatial correlation; and coordination number distribution are missing. Hence, spatially correlated networks with realistic geometry and (or) topology are extracted from pore space images. Various methods that can be used to obtain or reconstruct pore space images are discussed in the section below.

2.2 Pore Space Image-Based Networks

A three-dimensional image of the pore space of a real porous medium can be obtained through stochastic reconstruction using statistical methods with information obtained by analyzing 2D thin sections, X-ray micro-tomography and process-based reconstructions.

2.2.1 3D Image Reconstruction Using Statistical Methods

3D images can be stochastically reconstructed from information obtained by analyzing 2D thin sections using statistical methods based on a truncated Gaussian random field. Geometrical properties such as the one-point correlation function (porosity) and two-point correlation function (probability of finding two points separated by a certain distance within the same phase) measured on the original pore space (2D thin sections) can be compared with those obtained from the reconstructed 3D images. Adler et al [21, 22] generated fictitious Fontainebleau sandstones which shared the same geometrical properties as the real ones and the predicted permeabilities were found to be in acceptable agreement with the experimental ones. However, the one- and two-point correlation functions are insufficient to replicate the topological information of the porous media needed to predict multiphase properties [23].
Okabe and Blunt [24, 25] generated three-dimensional images using multiple-point statistics based on two-dimensional thin sections. Their method predicted long-range connectivity, measured through the local percolation probabilities, of the structures better than standard two-point statistical methods. They applied the method to a Berea sandstone and carbonate samples. The predicted permeability on the reconstructed microstructure was in good agreement with experimental value for the Berea sandstone but was overestimated by a factor of 3 for the carbonate rock [25]. However, in their study as well as many others that are based on statistical methods, equivalent networks were not actually constructed and flow simulations were carried out on the generated images of the pore space.

2.2.2 X-ray Micro-tomography (Micro-CT)

Micro-CT can be used to provide three-dimensional images with a resolution in the range of a few microns. This is a non-destructive and direct three-dimensional imaging tool. Using synchrotron X-ray source, Dunsmuir et al. [26] measured the three-dimensional structures of fused bead packs and Berea sandstones with about 10 to 3 micron resolution and performed preliminary studies of flow in these images. Arns et al. [27] imaged a vuggy reservoir carbonate core plug using high resolution X-ray micro-tomography. They derived pore scale morphology and petrophysical properties directly on the highest resolution digitized tomographic images and the computed permeability was in agreement with the experimental values. Recently, Dong et al. [28] imaged rocks cuttings of poorly consolidated sandstone and vuggy carbonate using X-ray microtomography and validated the extracted topologically equivalent networks through comparisons with networks derived by a different method. However, due to high cost, synchrotron micro-CT is not a routine method for industrial application and core analysis. Bench-top machines can be used, although the quality of the images is poorer [28].

2.2.3 Process-Based Reconstructions

The use of geologically realistic networks derived from real porous media was pioneered by Bryant et al. [29-32]. Starting from a dense random packing of equal spheres, they simulated compaction (reduction of the bulk volume of the packing by
Figure 2.1: Comparison of the 3D pore space of Fontainebleau sandstone (a) a micro-CT image and (b) a process-based image [33].

moving the centers of the spheres closer together in the vertical direction and allowing the spheres to overlap) and grain growth (increase in grain radii without moving the grain centers). They then extracted network models that preserved the topology and geometry of the pore space from these structures. They predicted the trend of
permeability with porosity for Fontainebleau sandstone successfully and showed that spatial correlations in the pore size distribution were important for correct predictions. The major limitation of their method was that the networks were based on a pack of equally-sized spheres and could not be applied to more complex sandstones.

Øren et al. [1, 33-35] extended the process-based reconstruction method by including spheres of different size in the packing. They used petrographical information obtained from two-dimensional thin section images of the actual sandstone to stochastically model sedimentation, compaction and diagenesis in a similar manner to Bryant et al. They compared the reconstructed microstructure of Fontainebleau sandstone with microtomographic image of the actual sandstones as shown in Figure 2.1. They showed that the process-based reconstruction reproduced adequately important intrinsic properties of the actual sandstone [33]. They also showed that a statistical reconstruction of Fontainebleau sandstone that matched the porosity and two-point correlation function differed strongly from the actual sandstone in its connectivity properties [33, 36]. The reconstructed microstructure was transformed into a topologically equivalent network that was used directly as input to a network model [1, 34, 35].

While the reconstruction method of Bryant et al. used a packing of equal spheres and Øren et al. used spheres of different sizes as the basis, Pilloiti [37] proposed a method that allow the reconstruction of clastic porous media made up of irregular grains with controlled level of angularity, sorting and porosity. He argued that the intergranular void space resulting from this procedure provided a satisfactory reproduction of micro-geometry of several clean consolidated sandstones and could be used to explore the effect of void topology on the flow field properties. Thus, the process-based reconstruction technique could, in principle, be applied to irregular grains.

### 2.2.4 Networks Extraction from Pore Space Images

While single phase flow simulations can be performed directly on an image of the pore space, pore networks extracted from the pore space images are required for multiphase flow simulations. Medial axis based algorithm, maximal ball algorithm and Voronoi diagram based algorithm are some of the commonly used methods to
extract networks from images. Lindquist et al., Liang et al. and Sheppard et al. [38-40] used the medial axis transform, based on a thinning algorithm, to reduce the pore space to its one-dimensional line skeleton with the junction in the skeleton forming the pore bodies while the chains of voxels connecting them were taken as the pore throats. The skeleton was then processed by merging junctions that were in close proximity to generate the pore network. Thus, the medial axis method mathematically preserves the topology of the pore space but it is difficult to identify pores unambiguously.

In order to resolve the difficulties associated with pore identification using the medial axis algorithm, Silin et al. [41, 42] proposed a maximal ball algorithm which distinguishes between pores and throats to study the morphology of the pore space. No image thinning was applied and comparisons of their results with those obtained by a thinning procedure showed that their method produced more realistic estimates of the number and shapes of pores and throats and the pore coordination numbers. However, the information about the pore distribution was used to compute a dimensionless drainage capillary pressure rather than to extract a network.

Al-Kharusi and Blunt [43] extended Silin’s method and extracted topologically equivalent networks from a representation of Fontainebleau sandstone and carbonate image. They predicted absolute permeability successfully. However, a small portion (200³ voxels out of 300³ voxels) of the Fontainebleau image could only be processed due to processing time and memory requirements and the extracted network consisted of 848 pores and 2419 throats. For the same reasons the extracted carbonate network from a digitized 2-dimensional image of 200² voxels shown in Figure 2.2 contained 643 pores and 2623 throats.

Dong [44] refined Silin and Al-Kharusi’s methods by developing a two-step searching algorithm to find the nearest interface of void and solid to define a ball and also developed a clustering process to define pores and throats by affiliating the maximal balls into family trees according to their size and rank. He used the refined method to extract topologically equivalent networks from images of consolidated sandstones and vuggy carbonates [28] and validated the results through comparisons with networks
derived from a different method. Figure 2.3 shows the extracted network from a reconstructed Fontainebleau sandstone.

Figure 2.2: A network extracted from a carbonate image [43].

Figure 2.3: A network generated from a reconstructed microstructure of Fontainebleau sandstone using a refined maximal ball algorithm [44].
A Voronoi diagram based algorithm has been used to extract networks from reconstructed images of the pore space. Bryant et al. [29-32] constructed a Delaunay cell (a cell that joins \((d+1)\) nearest neighbor points in an arbitrary collection of points in \(d\)-dimensional space) as the basis for the pore network extraction. Each cell represented a pore and each cell face was a throat. Since each cell was a tetrahedron, every pore had four throats leading from it and hence the topology of the network was uniform and the coordination number was four.

Øren et al. [1, 33-35] used ultimate dilation of the grain network which is exactly the same as ultimate thinning of the pore network to extract the skeleton of the network. This approach gives a good estimate of the skeleton of a pore network and the skeleton provides both visual and quantitative information about the connectivity of the pore space. Figure 2.4 shows the skeleton of a pore network extracted using the ultimate dilation technique.

Figure 2.4: A skeleton of a pore network extracted using the ultimate dilation technique [34].
2.3 Pore Geometry and Wetting Layers

Real pores and throats have highly irregular and complex geometry that allow two or three phases to be simultaneously present in a single pore or throat. Lenormand et al. [16] have shown through their micromodel experiments that in pores/throats with irregular and rough cross-sections, the wetting fluid resides in grooves and crevices while the non-wetting fluid occupies the centers. However, most early work in pore scale modeling assumed that the throats were cylinders with circular cross-sections while pores were spherical or cylindrical in shapes and hence could only contain one fluid.

In order to simulate fluid flow in the extracted pore network, it is necessary to replace the irregular geometry of real pores and throats with an equivalent but simplified geometry which retains the essential features relevant to fluid flow with: maximum (inscribed) pore radius, minimum (inscribed) throat radius and pore/throat shapes [34]. The simplified shape is chosen to place the correct volume of wetting phase in layers in a pore whose center is filled by non-wetting phase and to give the right hydraulic conductivity to the layers [2].

Many authors have used different simplified shapes to represent pore and throat cross-sections. Network elements with equilateral triangles [45, 46], square [7, 14, 47, 48], spherical and cylindrical [49] and star-like grain boundary pore shape [50, 51] cross-sections have been used. The cross-sectional area, radius and perimeter length of pores and throats in the reconstructed images can be measured. Mason and Morrow [52] suggested that the appropriate normalized shape factor, $G$, for capillary actions is the ratio of the area of cross-section, $A$, to the square of perimeter length, $P$. Once the shape factors of real pores and throats in the reconstructed images have been measured network elements are then modeled as having circular, square or irregular triangular cross-sections with the same shape factor as the measured one [1, 33-35]. Square elements have a shape factor of 1/16, circular elements have a shape factor of $1/4\pi$ while shape factor varies from zero for slit-shaped triangles to $\sqrt{3}/36$ for equilateral triangles.
Most of the networks elements from the process-based reconstruction method have triangular cross-sections. Triangular and square elements allow two or more fluids to flow simultaneously through the same pore or throat with the wetting layers residing in corners while the non-wetting phase occupies the center of the elements. It is also assumed that all the wetting layers are connected to those in the adjacent pores and throats and hence, the wetting layers span the entire network. The connectivity and flow of the wetting layers ensure low residual wetting phase saturation.

2.4 Wettability Effects, Saturation History and Contact Angle Hysteresis

Wettability is the ability of one fluid to spread on a solid and form a wetting film in the presence of another fluid. It is generally classified as either homogeneous (an entire rock surface has a uniform molecular affinity for water or oil) or heterogeneous/mixed-wet (distinct surface regions that exhibit different affinities for oil or water) [51]. For an oil-water system, the oil-water contact angle is the angle of contact of oil and water on a solid surface measured through the denser (water) phase. Based on the contact angle, homogeneous wetting can be classified as: water-wet, oil-wet and intermediate-wet with contact angles of less than, greater than or close to 90° respectively.

Polar crude oil components can adsorb by several different mechanisms, depending on brine composition and asphaltene composition of the oil, on mineral surfaces of hydrocarbon reservoirs and alter their wetting properties [53]. Using translationally invariant star-shaped pores, Kovscek et al. [51] developed a pore-level picture of how altered wettability might form and evolve in a reservoir rock initially filled with water that was strongly water-wetting. Following primary drainage, oil invades the pore space displacing water from the central portion of the tube leaving water only in the corners and thick water films lie between the pore walls and the oil in the center. As more oil enters the pore space, the capillary pressure increases and the water films coating pore walls thin. The asphaltenes adsorb along the walls of the oil occupied pore where the thick protective water films are broken. The asphaltene coated portion of the pore wall alters its wettability and becomes oil-wet while the corners of the
pore that retain bulk water preserve their water wettability as shown in Figure 2.5. Hence, the pore becomes mixed-wet: some regions are oil-wet while others remain water-wet. Small pores and throats that are not invaded by oil and remain completely water-filled as well as pores with unbroken thick water films remain water-wet.

Figure 2.5: A mixed-wet pore showing the location of asphaltene deposition and the coexistence of oil-wet and water-wet regions within a single pore [51].

During waterflooding, the pore-scale displacement processes depend on wettability. A water-wet pore immediately refills with water provided that a capillary connection to a water supply exists. Due to the snap-off mechanism, oil may become disconnected (and isolated) from the continuous oil phase and isolated oil regions are no longer under the influence of the applied capillary pressure. For a mixed-wet pore, the fraction of water-wet surface is quite small and the pore appears oil-wet. During forced water invasion (negative capillary pressure), water re-enters the pore space as the non-wetting phase and occupies the centers of the pore space as shown in Figure 2.6 for a triangular pore [2]. Oil may reside as a layer sandwiched by water in the corner and water in the center, Figure 2.6(b). This provides connectivity of the oil and allows for very low residual saturation to be reached [51].
Prior to the work of Kovscek et al. [51], low residual oil saturation has been observed experimentally by Salathiel [54] in mixed-wet cores. He conducted flooding experiments on a range of cores with different wettabilities and found that oil production continued long after initial water breakthrough in mixed-wet cores with very low residual oil saturation obtained as a result. A wettability classification system that relates recovery from mixed-wet systems to aging and fraction of oil-wet pores was introduced by Dixit et al. [12, 15, 55] using simple network models. They developed analytical expressions relating different wettability indices to different mixed-wetting scenarios and compared these to network modeling predictions [15].

![Figure 2.6: Oil and water in a triangular pore or throat. (a) After primary drainage, the areas directly contacted by oil (shown by the bold line) have an altered wettability. (b) If wettability alteration is strong enough, oil might become sandwiched as a layer by water in the corner and water in the center during waterflooding [2].](image)

Using square elements for both pores and throats, Blunt [47, 56] explicitly modeled wetting layers and developed capillary pressure expressions, from simple geometrical considerations, for different filling events described by Lenormand et al. [16]. Multiple wetting conditions were allowed to exist in each element with the corners remaining water-wet while the center becoming oil-wet in a similar approach to that of Kovscek et al. [51]. The effects on relative permeability from a wide range of wetting conditions were investigated and although the main conclusions were the
same as those reached using simpler network models [13, 55] although an improved understanding of oil-wet systems was made possible.

The dependence of flow upon the sequence in which phases are introduced, or saturation history, and the direction of saturation change is called hysteresis [17]. The oil-water contact angle depends on the direction of displacement. Advancing contact angles are contact angles measured when wetting phase displaces non-wetting phase (imbibition) while those measured when non-wetting phase displaces wetting phase (drainage) are referred to as receding contact angles. Due to wettability alteration, saturation history, surface roughness and heterogeneity, advancing contact angles are typically found to be larger than receding contact angles.

2.5 Pore Scale Models

The idea of using a network to predict macroscopic transport properties was pioneered by Fatt [57-59] in the 1950s. He used a regular two-dimensional network of tubes, sequentially filled them in order of inscribed radius using the Young-Laplace equation and measured capillary pressure and relative permeability curves. The dynamic properties (relative permeability and resistivity curves) were obtained using an equivalent network of electrical resistors and were compared with experimental data. The curves had the same shape as and closely resembled those obtained experimentally from real porous media. He concluded that a network was a valid model of porous media and represented real porous media more closely than a bundle of tubes model.

Two-dimensional networks have fewer interconnections compared with three-dimensional networks. Chatzis and Dullien [60] extended the electrical-resistor network approach to three-dimensional networks. Their results showed that the properties of two-dimensional networks differed substantially from those of three-dimensional networks. Their results also showed that only one phase can be continuous in two dimensional networks, the second phase must be discontinuous and they suggested that two-dimensional network models should not be used to simulate two-phase flow phenomena.
While Fatt used a conventional Wheatstone bridge to measure the total resistance of the analogue networks of electrical resistors to find the network flow properties (because the numerical solution of the network problem was not available then), Koplik [61] reduced the flow problem on a two-dimensional network of 10×10 to an analogue linear-network problem and solved it by numerical matrix inversion and an “effective medium theory” (EMT - the replacement of random microscopic parameters with a certain mean value, chosen so that the mean field produced by the random parameters is the same as that produced when all parameters have this mean value) and computed the network conductance. He compared conductances calculated numerically with those calculated using EMT for various lattices and found out that the agreement was well within the statistical standard deviation in each case.

Pore network models can be classified into two main types: quasi-static and dynamic models. Quasi-static models assume that the capillary pressure dominates the displacement and that the viscous pressure drop is insignificant; pores and throats change their fluid configurations one at a time and hence are only applicable to laminar flow. The displacement events are dominated by layer swelling and snap-off. The computation of the pressure field, although necessary to determine the relative permeability, does not affect the displacement sequence. However, both viscous and capillary pressures contribute, in dynamic models, to the displacement and the fluid configurations can change in many pores and throats at every time step. The effects of viscous forces are modeled through the explicit computation of the pressure field in the network.

### 2.5.1 Quasi-static Models

There are two dimensionless numbers that characterize the different properties of immiscible displacements in porous media, namely the capillary number \( N_{\text{cap}} \) and mobility ratio \( M \). \( N_{\text{cap}} \) is defined as the ratio of viscous forces to capillary forces between two immiscible fluids.

\[
N_{\text{cap}} = \frac{q \mu}{\sigma} \tag{2.2}
\]

where \( q \) is the Darcy velocity, \( \mu \) is the viscosity and \( \sigma \) is the surface or interfacial tension between the two fluid phases. \( M \) is the ratio of the viscosities of the two fluids (defending fluid viscosity / invading fluid viscosity).
The effect of pore structure on hysteresis of two-phase relative permeability and capillary pressure of strongly wetting systems at low capillary number was investigated by Jerauld and Salter [17]. They used both regular cubic networks and random Voronoi networks with an average coordination number of six. They found that the pore to throat aspect ratio was the most important structural determinant of the hysteresis behavior. Their results also showed that spatial correlation between pore-throat sizes affected the shape of the relative permeability curves.

Many other authors [11, 13, 15, 48, 56, 61, 62] have also used quasi-static models based on regular lattices or random networks to predict absolute and relative permeability; investigate relative permeability hysteresis, wettability and heterogeneity effects; and to study different displacement processes. While all these authors used regular lattices or random networks that do not reflect the random nature of real porous media, Bryant et al. [29-31] used networks extracted from process-based reconstruction method in their pore-scale models. The predicted absolute and relative permeability, capillary pressure and electrical conductivity were compared successfully with experimental results from water-wet sand packs, sphere packs and a simple sandstone.

Øren and co-workers [1, 33-35] at Statoil also used quasi-static models based on geologically realistic networks to predict transport properties for various sandstones. Their predicted drainage and waterflood permeability curves for a water-wet Bentheimer sandstone were in good agreement with experimental data. For a reconstructed Fontainebleau sandstone, the predicted permeabilities and formation factors agreed well with published data over a wide range of porosities. For mixed-wet reservoir rocks, the computed waterflood residual oil saturation and oil relative permeabilities were in fair agreement with experimental measurements.

Starting with a network generated from a reconstructed Berea sandstone, Valvatne and Blunt [4] successfully predicted flow properties for water- and oil-wet datasets using quasi-static models. However, for mixed-wet datasets, the calculated Amott water and oil indices reasonable matched the experimental values and they reproduced the same oil recovery trend but the quantitative match with experiment
was poor. Maximum recovery was predicted to occur at lower initial water saturation, $S_{wi}$ and the variation in recovery with $S_{wi}$ was predicted to be less than what was observed experimentally. Their results showed the difficulty associated with determining the wettability variation at the pore scale.

Behbahani and Blunt [63] adjusted the contact angles and oil-wet volume fraction parameters in a quasi-static model and matched experimental waterflood recovery and wettability index for Berea samples aged in crude oil for different times. The computed relative permeability and capillary pressure were then used in a conventional simulator to predict oil recovery from countercurrent imbibition in mixed-wet Berea. Their simulated results reproduced the experimental oil recovery trends of Zhou et al. [64] but the quantitative agreement was poor. Their results also showed the difficulty in determining, for mixed-wet media, the wettability variation at the pore scale.

Suicmez et al. [65] extended the quasi-static three-dimensional network model of Piri and Blunt [5, 6] by adding two new double displacement processes that involved trapped gas. Using a reconstructed Berea sandstone network, they generated relative permeabilities for water alternating gas (WAG) flooding. Their results showed the variations in gas, water and oil relative permeabilities under different wetting conditions and saturation history. They also showed how pore-scale model could be used to develop a physically-based empirical model to predict three-phase relative permeability.

Svirsky et al. [66] used the three-phase quasi-static network model of van Dijke and Sorbie [67] to predict the three-phase relative permeability experimental data of a water-wet Berea sandstone obtained by Oak [68] using a $15 \times 15 \times 15$ network. They anchored the network to the experimental two-phase relative permeabilities and capillary pressure data through a trial and error approach and obtained a match of the simulated and experimental data. They then used the anchored/tuned network to predict the three-phase relative permeability data and obtained reasonable quantitative predictions. Comparisons of their predicted results with those of Piri and Blunt [6] also showed reasonable quantitative agreement. However, the simulated and experimental saturation paths did not agree very well.
2.5.2 Dynamic Models

There are several circumstances where the approximation of quasi-static displacement is not valid. Examples include fracture flow, where flow rate may be very large – often of the order of hundreds of meters a day; displacements with very low interfacial tensions that substantially reduce capillary forces, such as near-miscible gas injection, gas condensate reservoirs and surfactant flooding; near well-bore flows; flows involving polymers, gels and foam where very large pressure gradients are found; and some cases where wetting layer flow and formation are significant, such as spontaneous wetting into a dry soil. In all these cases, capillary and viscous forces both control the fluid configuration at the pore scale [2].

Koplik and Lasseter [69] presented a dynamic network model of two-phase flow on a two-dimensional network with randomly sized cylindrical throats and randomly sized spherical pores. They used an appropriate electrical resistor network for the numerical computation of the flow problem and allowed for the advance of several menisci during a time step. They examined the dependence of residual oil saturations and interface shapes on network geometry and flow conditions. However, the model was computationally intensive and hence, they limited their simulations to networks not larger than 10×10. They also assumed fluids of equal viscosities and could not model flow in wetting layers due to the pore and throat geometry used.

Instead of cylindrical and spherical geometry, Dias and Payatakes [70, 71] developed a dynamic model of immiscible displacement of a non-wetting fluid by a wetting one using two-dimensional networks of randomly sized unit cells of converging-diverging geometry. Figure 2.7 shows a typical unit cell. All menisci were free to move in this model and a wide range of capillary number and viscosity ratio values could be investigated. They showed residual oil saturation in a 15×30 network as a function of capillary number for various values of mobility ratio as shown in Figure 2.8 and all the simulations results were modeled after a 100 - 200 mesh sandpack studied by Leverett [72]. However, they also neglected flow in wetting layers and their model was also computationally intensive.
Chen and Koplik [73] presented a dynamic model with emphasis on the mechanics of drainage and imbibition as a function of flow rate using small two-dimensional networks that were not fully random. Their micromodel experiments showed that in imbibition, the displacement occurred first via thin film spreading, followed by piston-like displacement at high flow rates or by snap-off of menisci and displaced...
phase trapping at low flow rates. Despite these experimental results, they did not allow flow in wetting layers and throat snap-off in their simulations. They also assumed that both fluids had the same viscosity for computational simplicity. The calculated permeability and electrical conductivity were in reasonable agreement with the experimental values but comparison of the two-phase flow characteristics had only qualitative similarities.

Using a regular two-dimensional square lattice with spherical pores and cylindrical throats of uniform length, Lenormand et al. [74] developed a dynamic model for drainage to study the effects of mobility ratio, $M$ and capillary number, $N_{cap}$ on displacement patterns. They showed the existence of three basic domains, namely capillary fingering, viscous fingering and stable displacement. Their results showed that there was a good agreement between the computer simulations and the experiments and both of them showed the existence of the three flow regimes.

While all the above authors used only two-dimensional networks in their models, Blunt and King [75] used both two- and three-dimensional topologically disordered, isotropic networks generated by Delaunay triangulation and containing up to 80,000 pores in their dynamic two-phase model of drainage. They assumed that all the fluid was contained in the pores but all the pressure drops occurred in the throats connecting them. They generated relative permeability curves that were shown to be functions of both capillary numbers at intermediate flow rate and viscosity ratios greater than unity. However, the contribution of flow in wetting layers was ignored in this work and they did not compare their simulation results with any experimental results.

Vizika et al. [76] used a 12×5×2 three-dimensional pore network model with pore size distributions obtained from measurements performed on a Berea sandstone [77]. They studied the effects of the viscosity ratio, $M$ on the pattern of displacement and residual non-wetting phase saturation in forced imbibition. Their results showed that $M$ affected the values of residual oil saturation even for very low $N_{cap}$ values and the predicted trends as shown in Figure 2.9 were in agreement with the experimental trends, Figure 2.10. However, the experimentally observed effect of $M$ on the residual
Figure 2.9: The predicted effect of the viscosity ratio, $x$ on the residual non-wetting phase saturation, $S_{or}$ for various values of the capillary number $N_{cap}$ [76].

![Graph showing predicted effect of viscosity ratio on residual non-wetting phase saturation](image1)

Figure 2.10: The experimental effect of the viscosity ratio, $k$ on the residual non-wetting phase saturation, $S_{or}$ for various values of the capillary number, $N_{cap}$ [76].

![Graph showing experimental effect of viscosity ratio on residual non-wetting phase saturation](image2)

Oil saturation in the region of small $N_{cap}$ values was substantially stronger than that predicted by the simulator because they did not allow flow in wetting layers.
Pereira et al. [78] extended the application of dynamic network modeling technique to three-phase flow and developed a dynamic network model for drainage-dominated three-phase flow that accounted for flow through both wetting and intermediate fluid films under strongly wetting conditions. The model was based on the pore-scale displacement mechanisms described by Øren and Pinczewski [79, 80] and Øren et al. [81, 82] and used a simple two-dimensional network of pores and throats. All the fluid volumes were allocated to the pores while all the pressure drops were allocated to the throats. The model correctly predicted all the important characteristics of three-phase flow observed in glass micromodel experiments.

Van der Marck et al. [83] developed a two-phase network model to describe fluid saturations in a porous medium and the pressure evolution during drainage displacements. Their networks consisted of pores that had volume but no resistance to flow while the throats did not have volume but had resistance to flow. They considered viscous, capillary and gravitational forces in their simulator and performed experiments using micro flow models to establish the validity of their simulator. Comparison of simulated and experimental results showed good agreement for a viscosity ratio of one. For higher viscosity ratio however, the agreements were poor. Their model did not take films or wetting layer flow into account.

Mohanty et al. [84] developed a dynamic imbibition model that kept track of the evolution of the displacement front and constructed an approximation of the entire pressure field. They used two square networks of $120 \times 40$ with identical throat radii distributions but different pore radii distributions – one network has a narrower distribution (narrow network) than the other one (wide network). The two networks were completely filled with oil before waterflooding and both fluids were assumed to have the same viscosity. Figure 2.11 shows the comparison of their two-dimensional simulation results with the results of Abrams [85] in (three-dimensional) porous rocks (Berea sandstone and Indiana limestone). The pore size distribution in the Berea sandstone was narrower than that in the limestone. The quantitative difference in the residual saturation between the simulations and the experiments was attributed to the difference in dimensionality. There are more interconnections and more ways out of a pore to the main oil bank in a three-dimensional network and thus a lower residual oil saturation than in a square network.
The effects of interconnections in two- and three-dimensional networks were also noted by Jerauld and Salter [17]. They stated that only one phase could percolate across a two-dimensional network at a time using invasion percolation model while there was a range of saturations in which two-phase could flow in a three-dimensional network. In addition, the residual non-wetting phase saturation was much higher in a two-dimensional network than in a three-dimensional network. For a uniform distribution of pore-sizes, the residual non-wetting phase saturation might be overestimated by as much as 1/3 by using a two-dimensional network relative to a three-dimensional network of the same coordination number [17]. However, Mohanty et al. [84] simulation results retained some qualitative features of the experimental displacements – the wide pore size distribution led to a steady fall in the residual oil saturation while the narrower pore distribution introduced a change in the slope of residual oil saturation with increasing capillary number.

![Figure 2.11: Comparison between two-dimensional (2-D) model simulations results (residual oil saturation vs. porosity divided capillary number) with three-dimensional coreflood experimental results [84].](image)
Using a regular three-dimensional cubic lattice with spherical pores and cylindrical throats, Mani and Mohanty [49] developed a pore network model to simulate capillarity-controlled gas invasion into a water-wet medium containing oil and water. The primary drainage and secondary imbibition were simulated using the quasi-static assumption and the displacement mechanisms described by Mohanty and Salter [86]. The fluid distributions at different stages of the imbibition process provided the initial conditions for the three phase simulations. To account for rate effects, they assumed a fixed conductance in wetting layers. Their results showed that water relative permeability for the strongly water-wet system modeled, was a function of its saturation alone while gas and oil relative permeability curves depended on the gas and oil saturation, respectively, the saturation history and the spreading coefficient.

Mogensen and Stenby [87] presented a two-phase dynamic model of imbibition using a three-dimensional network with throats having either circular, rectangular or triangular cross-sections. Both pores and throats contributed to the total void space and to the permeability of the network. They included a snap-off criterion through an expression based on the aspect ratio of a throat and its adjacent pores and solved for pore pressures using a conjugate gradient method preconditioned by symmetric successive overrelaxation. They performed a sensitivity study to evaluate the effects of contact angle, aspect ratio, capillary number and coordination number on the residual oil distribution after a waterflood. For computational efficiency, wetting layer flow rates were assumed to be constant throughout the simulation and the fluid viscosities were also assumed to be the same. Despite these assumptions, the model was computationally intensive resulting in the largest lattice size of 15×15×15 because it required solving the pressure field once or several times for each element filled.

The effects of contact angle, capillary number and initial wetting phase saturation on flow patterns, residual non-wetting phase saturations and relative permeabilities in two- and three-dimensional networks were studied by Hughes and Blunt [88]. They used a perturbative model to simulate imbibition by assuming a fixed conductance for wetting layers and using the pressure drop across them, in addition to the local capillary pressure, to rank and fill elements. They also assumed a fixed flow rate in the wetting phase throughout the medium when the pressure field was computed and
thereby prevented swelling of the wetting fluid at the displacement front. They validated their model by comparing the simulated results with experimental results of Lenormand and Zarcone [89] using the same throat size distribution and fluid parameters. The simulated results showed qualitative similarities to the experimental results. Their model was much faster than fully dynamic models and Figure 2.12 shows the simulated results of the effects of flow rate on the residual non-wetting phase saturation for different contact angles at different initial water saturations using a 20×20×20 network.

Singh and Mohanty [90] developed a rule-based dynamic network model for two-phase flow through a 30×8×8 network with cubic pores and square cross-section throats. They assumed, among others, that throats did not contribute to the saturation in the network and the radius of curvature of the wetting/non-wetting interface in a pore was constant with respect to time and saturation. The model was used to study primary drainage with constant inlet flow rate and constant inlet pressure boundary conditions. They studied the effects of capillary number, viscosity ratio and pore-throat size distribution on wetting phase residuals and relative permeability. Their simulation results identified three distinct flow patterns – stable, viscous fingering and capillary fingering which was consistent with the two-dimensional results of Lenormand et al. [74]. Figure 2.13 shows the simulated non-wetting phase saturation and velocity profiles for viscous fingering, stable displacement and capillary fingering. The velocity profiles show that the non-wetting phase fractional flow is a function of saturation only and hence, the drainage process could be described with a Buckley-Leverett theory / displacement profile.

Singh and Mohanty [90] developed a heuristic scheme to implement wetting layer flow in their network model. The procedure involved computation of the capillary pressure drop at each interface location. Then an amount of wetting phase was removed from the interface in proportion to the computed capillary pressure drop at that location. The total amount of wetting phase that could be removed through films was set at 1% of the volume flowing through the bulk. This implied that the wetting layer conductance was approximately two orders of magnitude smaller than the bulk conductance.
Figure 2.12: Residual saturation curves for three-dimensional simulations with (a) $S_{wi} = 0$ (primary imbibition), (b) $S_{wi} = 8\%$ and (c) $S_{wi} = 15\%$ [88].
Figure 2.13: Simulation results of non-wetting phase saturation and velocity profiles for (A) viscous fingering with mobility ratio, $M = 10$, (B) stable displacement with $M = 1$ and (C) capillary fingering with $M = 1$. $Q_o$ is the non-wetting phase flow rate and the legends show the time elapsed in seconds [90].
The effects of displacement rate on imbibition patterns, relative permeabilities and residual saturations were investigated by Nguyen et al. [7] using a regular 20×20×20 cubic network with pores and throats having square cross-sections. Their dynamic network model showed that swelling of wetting films was a capillary pressure driven, non-linear diffusive process. They concluded from their simulation results that film swelling and snap-off were the dominant displacement mechanisms at low rates (capillary number, $N_{\text{cap}} < 10^{-8}$) and the displacement pattern was similar to bond percolation while for very high rates ($N_{\text{cap}} > 10^{-2}$) the displacement time was insufficient for films to swell significantly and snap-off might be completely suppressed. For intermediate rates ($10^{-6} < N_{\text{cap}} < 10^{-3}$), frontal displacements, film swelling, snap-off and snap-off nucleated displacements were all important in determining the displacement patterns.

Al-Gharbi and Blunt [91] developed a dynamic pore network model for simulating two-phase flow in porous media that accounted for flow in wetting layers, meniscus oscillation and the dynamics of snap-off. The model was used to investigate the effects of capillary number and viscosity ratio on displacement patterns and fractional flow in primary drainage. The porous medium was represented as a network of pores and throats with triangular cross-sections whose inscribed radii varied sinusoidally. Both fluids were assumed to have the same viscosity and they used an equivalent electrical resistors network to calculate the fluid resistance. The model was computationally intensive resulting in the largest network size of 30×30.

Using a network geometry and topology that was representative of Berea-type sandstone, Nguyen et al. [92] investigated the effects of displacement rate and wettability on imbibition relative permeability. Their results showed that contact angles and displacement rates had a similar effect on computed relative permeability curves – high displacement rates and large contact angles suppressed snap-off while low displacement rates and small contact angles favored snap-off. Figure 2.14 shows comparisons between predicted and experimental relative permeability curves for water-wet Berea for both the dynamic model of Nguyen et al. [92] and quasi-static models of Blunt et al. [2] and Valvatne and Blunt [4]. The contact angle required for a good match for the dynamic model ($30^\circ$) was smaller than for the quasi-static model.
(30° – 90°) because displacement rate played an important role in suppressing snap-off.

Figure 2.14: Comparison between predicted and experimental relative permeability curves (a) quasi-static model of Blunt et al [2]; and Valvatne and Blunt, [4] and (b) dynamic model of Nguyen et al [7].
2.6 Project Overview

A new method to generate stochastic random networks representing the pore space of different rocks with given input pore and throat size distributions and connectivity is developed. A new rule-based, rate-dependent imbibition model is also developed by extending the Hughes and Blunt [88] pertubative model. The model is used to study the effects of capillary number, $N_{cap}$, mobility ratio, $M$ and network size on displacement patterns, saturation profiles and residual non-wetting phase saturation. Buckley-Leverett profiles are reproduced directly from the model by using large networks generated from the stochastic method. The model does not assume any of the simplified assumptions made by the previous dynamic models discussed above and many others. Some of the assumptions and their implications are summarized below.

2.6.1 Network Type and Pore Geometry

Most of the previous models were based on either a simple, regular two- or three-dimensional network or a perturbed/distorted network that did not capture any of the statistics associated with a real porous medium. The assumption of cylindrical and spherical pore geometry implies that pores and throats can only contain one fluid at a time and hence wetting layer flow could not be considered in the models that made this assumption. However, micromodel experiments have shown that in imbibition, the displacement occurred first via thin film spreading [73, 89].

In this work, a stochastic network generation algorithm presented in chapter 3 is used to generate topologically equivalent stochastic networks with triangular, square and circular pore geometries. While networks extracted from pore-space images have fixed dimensions and impractical to scale up to core scale and beyond, the stochastic network generation algorithm can be used to generate a network of arbitrary size with given input pore and throat size distributions and connectivity. The distributions and connectivity, used for the generation of the stochastic networks, are obtained from a topologically equivalent network extracted from the reconstructed microstructure of Berea sandstone [93].
2.6.2 Fluid Volume and Viscosity

Both pores and throats in topologically equivalent networks extracted from reconstructed images of process-based method contain fluid and contribute to the network porosity. Hence, assuming that all the fluids are contained in the pores and that the throats do not contribute to the porosity does not reflect the reality in a porous medium. Many previous models also assumed fluids of equal viscosities for computational simplicity and thereby could not model the effects of mobility ratio on displacement patterns and residual non-wetting phase saturation. In waterflooding projects, the oil being displaced, in most cases, always has higher viscosity than the displacing water.

In this model, both pores and throats have volume and contribute to the network porosity and fluids of different viscosities are used. The mobility ratio (ratio of non-wetting phase viscosity to wetting phase viscosity) is varied from 1 to 20 and the effects of the variation on saturation profiles, fractional flow and residual non-wetting phase saturation are investigated and discussed.

2.6.3 Wetting Layer Conductance and Flow Rate

As stated earlier, the key feature that distinguishes dynamic model from quasi-static models is the explicit computation of pressure field after every time step. The wetting layer conductance changes with pressure. However, most of the previous dynamic models either did not consider flow in wetting layers or assumed fixed wetting layer conductance throughout the simulation. In addition, they assumed constant flow rate in the wetting phase and wetting layers.

Flow in the wetting layers is considered in this work and the conductivity is allowed to change with pressure at every time step. The model does not assume constant flow rate in the wetting phase and layers and thereby allows swelling of the wetting phase near an advancing imbibition front.
2.6.4 Non-Wetting Phase Pressure Gradient

Most of the previous models are only suitable for water displacing light oil or water displacing gas because pressure drops in the non-wetting phase are neglected and constant non-wetting phase pressure is assumed throughout the network. However, there is non-wetting phase viscous pressure drop for the connected and flowing region of the non-wetting phase. In this work, the pressure drops are considered and hence, the model is applicable for cases where the oil viscosity is higher than the water viscosity.

2.6.5 Computational Efficiency and Network Size

Despite all the simplified assumptions made by the previous models, most of them are still computationally intensive and the largest network sizes considered range from few hundred pores to less than 10,000 pores. However, the new model is rule based and much faster. Hence, networks containing up to 100,000 pores are considered. Using a large network facilitates the reproduction of Buckley-Leverett profiles directly from pore-scale modeling thereby providing a bridge between pore-scale and macro-scale transport.

The new method of generating stochastic random networks is presented in the next chapter. The stochastic networks can be arbitrarily large and hence are not limited by the size of the original image or network. The generated large networks are validated by comparing the predicted drainage and imbibition relative permeability data from the original network and large stochastic network with experimental data.
Chapter 3

Stochastic Network Generation Algorithm

In this chapter, the current methods of generating stochastic networks are briefly reviewed and a new algorithm is presented. The stochastic random networks, generated from the new algorithm, represent the pore space of different rocks with given input pore and throat size distributions and connectivity – these distributions can be obtained from an analysis of pore-space images or from an original reconstructed network. The generated networks can be arbitrarily large and hence are not limited by the size of the original image or network. Validation of the generated networks and consistency checks are also discussed.

Ioannidis et al. [94] extracted pore and throat size distributions and pore connectivity information for stochastically simulated porous media using measurements of porosity and autocorrelation from binary images of thin-sections. Starting with regular cubic lattices, they removed nodes and bonds according to a site percolation process until the remaining percolating components had the desired average coordination numbers corresponding to the simulated porous media. Cubic pores and rectangular throats were then distributed on the remaining nodes and bonds. The generated network was used to predict absolute permeability and formation resistivity factor based on simulations of primary drainage.

Comparison of their predicted and experimental measurements showed that predicted permeability was reduced by more than a factor of four while the formation factor was increased by less than 50%. Comparisons of relative permeability curves computed on networks derived from the images of Fontainebleau sandstone (with average coordination number, \( \bar{z} \approx 3.3 - 3.8 \)) with those computed on a regular cubic lattice (\( \bar{z} = 6 \)), regular lattices and diluted lattices with average coordination number similar to that of the image rock network and with identical geometric characteristics also showed poor agreement [9]. Differences between model predictions and experimental
Sok et al. and Arns et al. [9, 10] proposed a better method of generating stochastic networks that did not only match the average coordination number but also matched the coordination number distribution completely and found reasonable agreement between relative permeability curves computed on these networks with those for the imaged rock networks. Their algorithm, termed “Current Algorithm” is briefly summarized below.

### 3.1 Current Algorithm

The following four-part algorithm summarizes Sok et al. [10] and Arns et al. [9] method for generation of stochastic networks. Details of this method can be found in their papers.

1. Construct a highly connected network; for example, a network based on the cubic close packing of spheres (with coordination number, \( z = 12 \)).
2. Randomly assign a coordination number of between 3 and \( z -1 \) to each site. The coordination number can be drawn from an experimental data set or from a specified distribution.
3. Mark bonds in the network one by one until, with the exception of inconsistencies, all desired coordination have been obtained.
4. Build the diluted network such that it contains only the bonds declared to be needed during step 3.

### 3.2 Proposed New Algorithm

The major limitation of the current method however, is that the stochastic network model is a subset of a regular lattice as shown in Figure 3.1 and randomization of the network is achieved through perturbation of the network skeleton as shown in Figure 3.2. Image-based networks in general and networks extracted from process-based reconstruction in particular also have some limitations – computationally very intensive; fixed dimensions; and impractical to scale up to core scale and beyond. To overcome these constraints, a stochastic network generation algorithm that can be
Figure 3.1: A regular network of size 20 x 10 x 2 [8].

Figure 3.2: Randomized network with variable coordination number [8].

used to generate a network of arbitrary size with given input pore and throat size distributions and connectivity is presented below. These distributions can be obtained from an analysis of pore-space images or from original networks.
• Assign the desired dimensions (x, y, z) to the random network.

• Place same or equivalent number of pores as in the original distribution/network randomly within the network.

• As the pores are being placed, assign pore index (PIdx) of 1 to the first pore, PIdx of 2 to the second pore and so on. The last pore placed in the network is assigned PIdx of n, where n is the total number of pores placed in the random network.

• The coordination number and geometrical information (radius, volume and length) for each pore from the original network are randomly assigned to the pores in the stochastic network.

• Using equation 3.1, assign a weight to each pore.

• Impose a maximum distance between two connected pores.

• Using the PIdx as the selection criterion, take the first pore with PIdx of 1 and coordination number of j and connect it with j nearest pores that satisfy the imposed condition. The remaining allowed number of connections for each of the connected nearest pores are decreased by one. For each pair of connected pores, calculate the branch weight using equation 3.2.

• Then, take the next pore with PIdx of 2 and coordination number of j and connect it with k nearest pores that satisfy the imposed condition. k = j if the selected pore has not been connected to any pore before or k = 0 if the selected pore has been connected with j nearest pores previously or k equals the remaining number of connections if the number of previously connected pores is less than j. The remaining number of connections for each of the connected nearest pores are decreased by one. For each pair of connected pores, calculate the branch weight using equation 3.2.

• Repeat the above step for the next pore with PIdx of 3, 4, ……(n-1), until the last pore with PIdx of n has been selected.

• Once all the pores in the network have been connected, sort the branch weights for all connected pores.

• Sort the throat geometrical information (radius, length and volume) from the original network based on radius.

• To obtain a correlated network, the smallest throat along with other geometrical information is assigned to the branch with the smallest weight, the
second smallest throat to the branch with the second smallest weight and so on.

- For an uncorrelated network, assign throat radius along with other geometrical information from the specified distribution or the original network randomly to the branch.

A flowchart for the stochastic algorithm is shown in Figure 3.3.

\[
P_i = \left( \frac{D_i - D_{\text{min}}}{D_{\text{max}} - D_{\text{min}}} \right)^n \tag{3.1}
\]

where the \(i\)-th pore is assigned a weight of value \(P_i\), which depends on its diameter \(D_i\); \(D_{\text{min}}\) and \(D_{\text{max}}\) are the minimum and maximum pores diameter and \(n\) is the correlation parameter. For \(n = 0\), there is no correlation between sizes of contiguous pores and throats; for \(n < 0\), small pores prefer to be connected with large throats; for \(n > 0\), large pores prefer to be connected with large throats. Whether \(n < 0\) or \(n > 0\), the pore sizes determine the throat sizes that are connected to them.

The average of the two adjacent pore weights is taken as the branch weight and the smallest throat is placed at the branch with the smallest weight, the second smallest of the throats at the branch with the second smallest weight and so on. With this method, the throat size distribution in the specified distribution or the original network is the same as the desired one and is independent of the pore size distribution. Throat weight \(T_i\) is given as:

\[
T_i = \frac{P_1 + P_2}{2} \tag{3.2}
\]

where \(P_1\) and \(P_2\) are the weights of pore 1 and pore 2 connected by throat \(i\) respectively. The following three conditions must be satisfied to ensure a geologically realistic network.
Figure 3.3: A flowchart for the stochastic algorithm

1 = Pore Index (PIdx); 2 = Coordination Number (CN); 3 = Throat Index (TIdx)
1. The maximum distance between two connected pores should be specified.
2. Each throat should be smaller than the two pores it connects.
3. Ensure that two adjacent pores do not overlap.

Consider a small stochastic 2D network consisting of 5 pores and 6 throats. The pore geometric data is tabulated in Table 3.1 and the pore weights are calculated from the pore radii using equation 3.1. Similarly, the unsorted and sorted throat sizes are tabulated in Table 3.2. The pores are placed randomly in a 2D network as shown in Figure 3.4. The coordination number and pore radius are taken together and assigned to the pores as they are being placed in the network. The first pore to be placed in the network is assigned pore index of 1, the second pore is assigned pore index of 2 and so on until the last pore is placed and assigned a pore index of 5.

Table 3.1: Pore geometric data for a stochastic 2D network.

<table>
<thead>
<tr>
<th>Pore Index</th>
<th>Coordination Number</th>
<th>Pore Radius (µm)</th>
<th>Pore Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>8</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>10</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>12</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 3.2: Unsorted and sorted throat radius for a stochastic 2D network.

<table>
<thead>
<tr>
<th>Unsorted Throat Data</th>
<th>Sorted Throat Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Throat Index</td>
<td>Throat Radius (µm)</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>
We impose 60% of the total network length in $x$ direction as the maximum connection distance between two pores. Based on the pore index, we select the first pore and find the nearest pores that satisfy the imposed condition. For simplicity a pore with pore index $j$ is called pore $j$. Pores 2, 3 and 5 satisfy the imposed condition but since the coordination number of the selected pore (pore 1) is 2, it is connected to only pores 2 and 5. They are nearer to pore 1 than pore 3 and the numbers of connection for these two pores (pores 2 and 5) are decreased by one accordingly. Hence, there is a connection (branch/throat) between pore 1 and pore 2 as well as pore 1 and pore 5. The branch weights are calculated using equation 3.2.

Then we pick the next pore (pore 2, based on the pore index) and find the nearest pores to it. Pores 1, 3, 4, and 5 satisfy the imposed condition but pore 1 has previously been connected to two pores (including pore 2) based on its coordination number and cannot be reconnected to pore 2 or any other pore again. Reconnection of pore 1 and 2 if pore 1 has not been connected to the maximum number of pores based on its coordination number is possible and this will result in parallel connections. However, since parallel connections are absent in the original Berea network, we do not allow parallel connections in our stochastic networks.
The remaining allowed number of connections for pore 2 is three and we connect it with pores 3, 4 and 5 and decrease the number of connections for these pores by one accordingly. We calculate the branch weights for these connections using equation 3.2. The next pore based on the selection criterion is pore 3. It has a coordination number of 2 and has been previously connected to pore 2. Hence, the remaining number of connection for pore 3 is one and the nearest pores that satisfy the imposed conditions are pores 1, 2, 4 and 5. However, pores 1, 2 and 4 have all been connected to the maximum possible number of pores based on their coordination numbers and the only pore left is pore 5. Pore 3 is thus connected to pore 5 and the branch weight for this new connection is also calculated using equation 3.2.

We select the next pore (pore 4) with coordination number 1 and it has previously been connected to pore 2. It can not be connected with any other pore again. Similarly, the last pore – pore 5 – has a coordination number of 3 and has been previously connected with three pores (pores 1, 2 and 3) and can not be connected with any other pore again.

All the pores have now been connected based on their coordination numbers and we sort the branch weight from the smallest to the largest. We then assigned the sorted throat radius to the sorted branch weight as follows. The branch with the smallest weight is assigned the smallest throat radius, the second smallest weight to the second smallest radius and so on until the last branch with the largest weight is assigned the largest throat radius. Table 3.3 shows the sorted branch weight and the assigned throat radius while Figure 3.5 shows the stochastic 2D network.
Table 3.3: Sorted branch weight and the assigned throat radius for a stochastic 2D network.

<table>
<thead>
<tr>
<th>S/No</th>
<th>Connected Pores Indexes</th>
<th>Sorted Branch Weight</th>
<th>Assigned (Sorted) Throat Radius(µm)</th>
<th>Throat Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 and 4</td>
<td>0.375</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>2 and 3</td>
<td>0.5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1 and 2</td>
<td>0.625</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3 and 5</td>
<td>0.625</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>1 and 5</td>
<td>0.75</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2 and 5</td>
<td>0.825</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 3.5: A 2D stochastic network showing the randomly placed pores and the connected throats.

Based on the random placement of the pores and the imposition of the maximum distance between two connected pores, we might have slightly fewer throats in the stochastic network compared to the original one. Pore and throat size distributions as well as coordination distribution from a topologically equivalent Berea network generated from the reconstructed Berea sandstone [93] are used as the bases in this work.
3.3 Description of the Berea Network

The Berea network has a dimension of (3×3×3) mm$^3$ and consists of 12,349 pores and 26,146 throats. It has a porosity of 0.24, an absolute permeability of 2.5 Darcy, 92.3% of the elements have triangular cross sections, 6.5% have square cross sections and the rest are cylindrical elements. The network connectivity varies between 1 and 19, with an average of 4.19. Figure 3.6 shows a plot of average pore volume against coordination number for the original Berea network and confirms that there is a correlation between coordination number and pore size.

Figure 3.7 shows the pore and throat radius distributions for the Berea network, the overlap of the pore and throat radii implies that there are some throats with larger radii than pores. Hence, the need for the imposed condition that throat radius should always be smaller than the radii of the two pores it connects. Correlation between radii of pores and connecting throats is expected for consolidated sandstone. The correlation coefficient for the reconstructed Berea sandstone is 0.72, as shown in Figure 3.8, and we use $n = 1.0$ for the generation of the stochastic networks. Using a Rhino 3-D visualization software package, a ball and stick representation of the original Berea network is as shown in Figure 3.9.
Figure 3.6: Correlation between pore coordination number and average pore volume

### 3.4 Generated Equivalent Stochastic Berea Networks

Four equivalent Berea networks of dimension (3×3×3) mm$^3$ consisting of 12,349 pores and 26,018 throats; (6×6×6) mm$^3$ consisting of 98,792 pores and 207,995 throats; (9×6×6) mm$^3$ consisting of 148,188 pores and 311,489 throats; and (12×6×6) mm$^3$ consisting of 197,584 pores and 414,980 throats were generated and used in this
Figure 3.7: Pore and throat radius distributions for the Berea network.

Figure 3.8: Correlation between radii of pores and connecting throats for the reconstructed Berea [95].
Figure 3.9: A ball and stick representation of the original Berea network.

Figure 3.10: A ball and stick representation of an equivalent Berea network.
study. The average coordination number is 4.17 for (3×3×3)mm$^3$ and 4.19 for other stochastic networks compared with 4.19 of the original network. Figure 3.10 shows a ball and stick representation of the equivalent Berea network of dimension (3×3×3)mm$^3$ using a Rhino 3-D visualization software package.

3.5 Comparison Between the Original and an Equivalent Berea Network

We compare the geometrical, topological, correlational as well as predicted transport properties (drainage and imbibition relative permeability curves) using the quasi-static network model of Valvatne and Blunt [4] for the original and an equivalent Berea network in the subsection below.

3.5.1 Geometrical and Topological Data

Table 3.4 shows the geometric and topological data comparison for the original and stochastic Berea networks shown in Figures 3.9 and 3.10 respectively. The percentage difference in the total number of connected throats and average coordination number is 0.49% and 0.48% respectively. However, the percentage difference in the maximum throat radius is 38.56%. The reason for this huge difference is explained below.

Due to the imposition of the maximum distance between two connected pores (condition 1), the stochastic network has a total of 26,018 throats and they are all connected while the original network has a total of 26,146 connected throats. This implies that the equivalent network has 128 fewer throats and this translates to 0.49% difference. The assignment of the throat radii to the throats started from the throat with the smallest weight assigned with the smallest radius, the second smallest weight assigned the second smallest radius and so on. Thus, the stochastic network has the same smallest throat radius of 0.9µm but a lower largest throat radius of 34.9µm compared with 56.8µm in the original network. The 128 throats that are removed from the stochastic network have radii in the range of (35.0µm ≤ radius ≤ 56.8µm) as shown in Figure 3.7.
The length between two connected pore centers is the throat total length and this can be calculated from the pore center coordinates. The throat total length minus the radii of the two pores it connects is the throat length. One of the three conditions imposed on the stochastic network is that the maximum distance between two connected pores should be specified. This ensures that the calculated throat total lengths and throat lengths are reasonable and comparable to those in the original distribution.

The volume, shape factor and length, for each throat in the original network data, are taken together with the radius and assigned randomly to throats in the stochastic network. This is to ensure that the predicted permeability for the stochastic network remains as close as possible to that from the original network and partly that the porosity for the two networks remains the same value. While volumes and shape factors can be assigned to throats in the stochastic network, assignment of throat lengths is not physically justifiable. This is due to the fact that the calculated lengths differ from the original lengths for most of the throats.

While it is possible to assign coordination number and other geometrical information for each pore in the original network randomly to pores in the stochastic networks in order to preserve the correlation between coordination number and pore size, it is difficult to ensure that all pores are connected to the desired number of throats based on the randomly assigned coordination numbers. The stochastic networks always have fewer (less than 0.5%) throats than the original network due to the imposition of the maximum distance between two connected pores. Thus, the actual coordination numbers of very few pores in the stochastic network will be less than the randomly assigned values. Hence, the randomly assigned values are changed to the actual values once the network generation is completed.

Consequently, it may be difficult to preserve the topological data (coordination number distribution) in the stochastic network completely. However, we preserve the pore geometric data – pore radius, shape factor and length completely. This is essential since the pore sizes determine the throat sizes that are connected to them as a result of the imposed pore-throat correlation.
Table 3.4: Comparison between the original and stochastic Berea network data.

A. Geometrical Information

<table>
<thead>
<tr>
<th></th>
<th>Original Data</th>
<th>Code Data</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of pores</td>
<td>12,349</td>
<td>12,349</td>
<td>0.00</td>
</tr>
<tr>
<td>No of throats</td>
<td>26,146</td>
<td>26,018</td>
<td>0.49</td>
</tr>
<tr>
<td>porosity</td>
<td>0.24</td>
<td>0.24</td>
<td>0.00</td>
</tr>
<tr>
<td>Abs. permeability</td>
<td>3230</td>
<td>2907</td>
<td>10.00</td>
</tr>
</tbody>
</table>

**Throat (\(\mu m\))**

<table>
<thead>
<tr>
<th></th>
<th>Original Data</th>
<th>Code Data</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum radius</td>
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<td>0.90</td>
<td>0.00</td>
</tr>
<tr>
<td>Maximum radius</td>
<td>56.80</td>
<td>34.90</td>
<td>38.56</td>
</tr>
<tr>
<td>Average radius</td>
<td>11.00</td>
<td>10.50</td>
<td>4.55</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>7.03</td>
<td>6.32</td>
<td>10.10</td>
</tr>
</tbody>
</table>

**Pore (\(\mu m\))**

<table>
<thead>
<tr>
<th></th>
<th>Original Data</th>
<th>Code Data</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum radius</td>
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<td>3.62</td>
<td>0.00</td>
</tr>
<tr>
<td>Maximum radius</td>
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<td>73.50</td>
<td>0.00</td>
</tr>
<tr>
<td>Average radius</td>
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<td>19.20</td>
<td>0.00</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>8.47</td>
<td>8.47</td>
<td>0.00</td>
</tr>
</tbody>
</table>

B. Topological Information

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<th>% Diff</th>
</tr>
</thead>
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<td>1</td>
<td>0.00</td>
</tr>
<tr>
<td>Maximum coordination no</td>
<td>19</td>
<td>19</td>
<td>0.00</td>
</tr>
<tr>
<td>Average coordination no</td>
<td>4.192</td>
<td>4.172</td>
<td>0.48</td>
</tr>
</tbody>
</table>

3.5.2 Correlational Data

The aspect ratio between the pore body radius and average connecting throat radii of the stochastic network compared with the original network is shown in Figure 3.11 and Table 3.5 compares the key aspect ratio data. Figure 3.11 and Table 3.5 show that 1.2% of the pores in the original network have aspect ratios that range from 7 to 49.04 while the maximum aspect ratio in the stochastic network is 16.89 and this explains why there is a huge percentage difference of 66% between the maximum aspect ratios. The stochastic network mean aspect ratio (2.55) is greater than that of the original network (2.06) with a negative percentage difference of 24%.
It is difficult to implement correlation without impacting some other parts of pore morphological description. Implementation of the pore-throat correlation in the stochastic network requires keeping the pore and throat sizes from the original network constant with the neighboring pore sizes determining the size of the throats. This generally results in different aspect ratio for the stochastic network irrespective of the correlation factor, $n$ used. The relative permeabilities obtained from stochastic network generated with $n = 1.0$ approximate those from the original network much better than the relative permeabilities obtained from network generated with $n = 0.72$. Hence, we use $n = 1.0$ for the generation of all the stochastic networks used in this study.

However, it is possible to impose the throat size distribution on the stochastic network and the original aspect ratio distribution on the pores and determine the pore sizes from the connected throat sizes and the imposed aspect ratio. This approach will generate stochastic networks with exactly the same aspect ratio and throat size distributions as the original network but the pore size distribution will be completely different from the original one. In order to preserve the pore and throat sizes as well as other geometric information from the original network, we choose the former approach in this study.
Figure 3.11: Comparison between the original and stochastic Berea network aspect ratios.

Table 3.5: Comparison between the original and stochastic Berea network aspect ratio.

<table>
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<th></th>
<th>Berea</th>
<th>Code</th>
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<td>1.00</td>
<td>0.96</td>
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<tr>
<td>max</td>
<td>49.04</td>
<td>16.89</td>
<td>65.56</td>
</tr>
<tr>
<td>mean</td>
<td>2.06</td>
<td>2.55</td>
<td>-23.70</td>
</tr>
<tr>
<td>std. deviation</td>
<td>1.61</td>
<td>1.58</td>
<td>1.50</td>
</tr>
<tr>
<td>var</td>
<td>2.58</td>
<td>2.50</td>
<td>2.98</td>
</tr>
</tbody>
</table>

3.5.3 Relative Permeability Curves

Though visualization of the original and stochastic Berea networks show that they are similar and comparison of the networks geometric data also show reasonable agreement, these do not confirm that the stochastic network replicates closely the microstructure of the original Berea network. We compare the predicted transport properties, such as relative permeability curves, on the original and stochastic...
networks to establish to what extent the stochastic network replicates the original network. Arns et al. [9] used this approach and compared drainage relative permeability curves as shown in Figure 3.12, for a diluted stochastic network generated from a cubic star network (with coordination number, \( z = 26 \)) with a coordination number distribution that closely matched those of the Fontainebleau networks of different porosity.

We simulated primary drainage and secondary imbibition and generated relative permeability curves for both the original and stochastic networks using the Valvatne and Blunt quasi-static two-phase model. Figures 3.13 and 3.14 compare the predicted relative permeability curves for the original Berea, stochastic Berea and a distorted \( 23^3 \) cubic network with experimental data from Oak [68]. The cubic network consists of 12,167 pores and 26,076 throats with an average coordination number of 4.2 and similar geometric properties to the original Berea network. The primary drainage relative permeability curves (Figure 3.13) were simulated at contact angle of \( 0^0 \) while the secondary imbibition displacements were simulated at contact angles of \( 50^0 \) – \( 60^0 \) (Figure 3.14).
Both the drainage and imbibition relative permeability curves for the stochastic network are similar to the original Berea network curves. However, the cubic network drainage and imbibition relative permeability curves are far off from the original network curves. These confirm that the stochastic network replicates the microstructure of the original Berea network much better than the distorted cubic network.
Figure 3.13: Comparison of predicted primary drainage relative permeability curves for the original Berea network, a stochastic Berea network of size $(3\times 3\times 3)\text{mm}^3$ and a distorted cubic network with experimental data.

Figure 3.14: Comparison of predicted secondary imbibition relative permeability curves for the original Berea network, a stochastic Berea network of size $(3\times 3\times 3)\text{mm}^3$ and a distorted cubic network with experimental data.
3.5.4 Pore-space correlations

Correlations in pore structures, whether pore-throat correlation, throat-throat correlation or pore-pore correlation, have an impact on the absolute and relative permeabilities. While Jerauld and Salter [17] observed larger absolute permeabilities for their correlated networks compared with uncorrelated networks, Bryant et al. [30] observed the opposite effect on their correlated networks constructed from random packing of equal spheres. That is their uncorrelated network had a permeability that is 78% larger than that of the correlated network, representing a real sphere packing. It is also interesting to note that while Jerauld and Salter [17] observed that correlations between neighboring pore-throats increases the non-wetting phase and slightly decreases the wetting phase relative permeabilities, Mani and Mohanty [18] observed that both the wetting and non-wetting relative permeabilities increase with correlation for primary drainage. These apparently contradictory results can be explained by the exact nature of the correlation. For instance, for sphere packs Bryant et al. [30] showed that there was a negative correlation between neighboring pore sizes: large pores tended to be next to smaller ones – this explained why the permeability was lower than for an uncorrelated network. Blunt [62] studied the impact of spatial correlations in pore and throat size on relative permeability and concluded that some degree of correlation was necessary to explain the hysteresis trends observed in consolidated rocks.

As stated earlier, we use a correlation parameter, \( n = 1.0 \) and the assignment of the throats for a correlated stochastic network is determined by the average size of the two adjacent pores. This implies that both throat-throat and pore-throat correlations are present in the stochastic networks, while we assume that there is no spatial correlation in pore size. We observe higher non-wetting phase and slightly lower wetting phase relative permeabilities for the stochastic network compared with the original network as shown in Figures 3.13 and 3.14. While these are consistent with the observations of Jerauld and Salter [17] it is difficult to say whether they are due completely to the imposed pore-throat correlation only or partly to the impose correlation and other factors.
3.6 Consistency Check

Figures 3.15 and 3.16 show comparisons between drainage and imbibition relative permeability curves for stochastic Berea networks of dimension $(3 \times 3 \times 3)\, mm^3$ and $(12 \times 6 \times 6)\, mm^3$. The results are similar and demonstrate the consistency of the stochastic network generation algorithm. It also shows that the original Berea network of dimension $(3 \times 3 \times 3)\, mm^3$ is sufficiently large to give representative results.

![Figure 3.15: Comparison of primary drainage relative permeability curves for $(3\times3\times3) mm^3$ and $(12\times6\times6) mm^3$ stochastic networks.](image)

Figure 3.15: Comparison of primary drainage relative permeability curves for $(3\times3\times3) mm^3$ and $(12\times6\times6) mm^3$ stochastic networks.
Figure 3.16: Comparison of secondary imbibition relative permeability curves for (3×3×3)mm$^3$ and (12×6×6)mm$^3$ stochastic networks.

The new rule-based, rate-dependent imbibition model is presented in the next chapter and the model uses the large networks generated in this section for different simulations. The simulation results are presented in chapters 5 and 6.
Chapter 4

Dynamic Pore-Network Model

The main forces responsible for the displacements in pore network models and their computation; and the new rule-based, rate-dependent imbibition model are presented in this chapter. The model accounts for viscous forces by solving for the wetting and non-wetting phase pressure and allows wetting layer swelling near an advancing flood front. A new time-dependent algorithm that accounts for partial filling of elements is incorporated into the model and the algorithm is also presented in this section.

4.1 Displacement Forces – Computation of Capillary Pressure

The computation of capillary pressure for piston-like displacement, pore body filling and snap-off are based on the quasi-static model of Valvatne and Blunt [4] and details can be found in [95]. However, for completeness, we present the basic mathematical details in this sub-section. There are three main displacement mechanisms by which water can displace oil: piston-like, pore body filling and snap-off [16]. Piston-like mechanism is the displacement of oil from a throat by an invading interface in the center of an adjoining water filled pore as shown in Figure 4.1. The threshold capillary entry pressure is given by the expressions below.

For a cylindrical throat of radius $r$ with no contact angle hysteresis, $\theta_a = \theta_r$.

$$P_c = \frac{2 \sigma \cos \theta_r}{r}$$  \hspace{1cm} (4.1)

where $\sigma$ is the interfacial tension and $\theta_a$ and $\theta_r$ are the advancing and receding contact angles respectively. For a polygonal shaped element with no contact angle hysteresis [1]

$$P_c = \frac{\sigma \cos \theta_r \left(1 + 2 \sqrt{\pi G}\right)}{r} F_d(\theta_r, G)$$  \hspace{1cm} (4.2)
If there is contact angle hysteresis, $\theta_a > \theta_r$, the computation of the threshold capillary pressure is more involved and relevant equations for spontaneous imbibition and forced water invasion are given in [1, 95] and [3].

Figure 4.1: Piston-like displacement mechanism; this can only occur if there is an adjacent element whose center is filled with oil.

The threshold capillary pressure for filling a pore is limited by the largest radius of curvature required to invade the pore. This radius depends on the size of the pore body and the number of adjacent oil-filled throats [16]. For a pore with a coordination number $z$, there are $z-1$ possible events referred to as $I_1$ to $I_{z-1}$. Figure 4.2 shows $I_1$ and $I_2$ events.

Figure 4.2: Pore-body displacement mechanism; (a) $I_1$ – only one throat is filled with oil and (b) $I_2$ – two throats are filled with oil.

We use the parametric model presented by Blunt [47] to compute the threshold capillary pressure for an $I_n$ mechanism

$$P_c = \frac{2\sigma \cos \theta_a}{r} - \sigma \sum_{i=1}^{z} A_i x_i$$

(4.3)
where \( A_i \) are input parameters and \( x_i \) are random numbers between 0 to 1. If only one of the connecting throats contains oil, \( I_i \), the filling of the pore is similar to that of a piston-like displacement.

As the capillary pressure decreases during the course of an imbibition displacement, the water layers in the corners of pores and throats swell. Figure 4.3 shows the evolution of water layer swelling and snap-off mechanism in a throat. This will only occur if there is no adjacent element whose center is filled with water, and corner water layers swell so much that the water/oil interface becomes unstable. For a strongly water wet system, the threshold capillary pressure is given by \( P_c = \sigma/r \). If there is contact angle hysteresis, the computation of the threshold capillary pressure is given in [1, 95] and [3]. The capillary pressure for snap-off is always lower than that of piston-like displacement; hence, snap-off only occurs if piston-like displacement is not topologically feasible. Snap-off can, however, still be an important displacement process and can occur ahead of the connected front where the oil is connected to the outlet.

![Snap-off displacement mechanism](image)

Figure 4.3: Snap-off displacement mechanism; it can only occur if there is no adjacent element whose center is filled with water.

### 4.2 Displacement Forces – Computation of Viscous Pressure Drop

To compute the viscous pressure drop in water and oil, the conductance of each phase in each element needs to be specified. The conductance of a phase \( p \), \( g_p \), in an element using Poiseuille’s law, can be defined as:

\[
q_p = g_p \Delta p_p
\]  

(4.4)
where $\Delta p_p$ is the pressure drop across the element and $q_p$ is the flow rate of phase $p$.

For single phase flow with viscosity $\mu_p$ in a circular tube with radius $r$, an analytic expression for $g_p$ is given by:

$$g_p = k \frac{A^2 G}{\mu_p} = \frac{1}{2} \frac{A^2 G}{\mu_p}$$  \hspace{1cm} (4.5)

where $A$ is the cross-sectional area, $G$ is the shape factor, $k$ is 0.6 and 0.5623 for equilateral triangles and squares respectively [96].

For polygonal elements containing water and oil, water resides in corners while oil occupies the centers of the elements. Determination of the conductance, $g_p$ for water in corners and oil in the centers involves determination of the area occupied by water; and the area occupied by oil is then the total area minus the water area. For flow of oil in the centers, we use equation (4.5) but with $A$ replaced by the cross-sectional area occupied by oil and the appropriate value of $k$. For flow of water in corners, exact analytic results are not possible hence; we use empirical correlations derived from solutions of the Stoke’s equation [1, 3].

The flow rate $q_{p,ij}$ between two pores $i$ and $j$ is given as:

$$q_{p,ij} = \frac{g_{p,ij}}{l_{ij}} (P_{p,j} - P_{p,i})$$ \hspace{1cm} (4.6)

where $q_{p,ij}$ is the volumetric flow rate of phase $p$ flowing from pore $i$ to pore $j$, $l_{ij}$ is the distance from center of pore $i$ to center of pore $j$, $P_{p,i}$ and $P_{p,j}$ are the phase $p$ pore pressures in pores $i$ and $j$ respectively and the total bulk fluid conductance $g_{p,ij}$ is the harmonic conductance between pores $i$ and $j$:

$$\frac{1}{g_{p,ij}} = \frac{1}{g_{p,t}} + \frac{1}{2 \left( \frac{1}{g_{p,i}} + \frac{1}{g_{p,j}} \right)}$$ \hspace{1cm} (4.7)

$g_{p,t}$ is the conductivity of the throat connecting pore $i$ to pore $j$. We calculate the pore pressures by applying volume conservation in each pore:

$$\sum_{j=1}^{z} q_{p,ij} = 0$$ \hspace{1cm} (4.8)
where $z$ is the coordination number of pore $i$.

A linear set of equations can be defined by imposing equation (4.8) at every pore using equation (4.6) for each flow rate. An algebraic multigrid solver developed by Ruge and Stueben [97] is used to solve for the pressure field. We use ‘no-flow’ boundary conditions for the side faces and assume a zero pressure at the outlet and a constant pressure at the inlet. For each throat, we take the average of the connecting pore pressures as the throat pressure.

### 4.3 Computation of Global Pressure for Imbibition

For each element in the network, depending on the shape and the connection number, we compute the capillary pressure for all possible displacement events: piston-like displacement in throats, pore body filling and snap-off. We obtain both water $\left(P_w^{\text{local}}\right)$ and oil $\left(P_o^{\text{local}}\right)$ local pressures for each element from the pressure solution and compute the pressure drops as stated below:

\[
\Delta P_w = P_{w}^{\text{inlet}} - P_{w}^{\text{local}} \tag{4.9}
\]

\[
\Delta P_o = P_{o}^{\text{local}} - P_{o}^{\text{outlet}} \tag{4.10}
\]

where $\Delta P_w$ is the water pressure drop between the inlet and the element, $\Delta P_o$ is the oil pressure drop from the element to the outlet, $P_{w}^{\text{inlet}}$ is the constant water pressure at the inlet and $P_{o}^{\text{outlet}}$ is the constant oil pressure at the outlet.

\[
P_c^{\text{local}} = P_{o}^{\text{local}} - P_{w}^{\text{local}} \tag{4.11}
\]

where $P_c^{\text{local}}$ is the local capillary pressure for each element. By rearrangement and substitution of equations (4.9), (4.10) and (4.11), we obtain

\[
P_{w}^{\text{inlet}} = P_{o}^{\text{local}} - P_c^{\text{local}} + \Delta P_w \tag{4.12}
\]

\[
P_{\text{global}} = -P_{w}^{\text{inlet}} = P_{c}^{\text{local}} - P_{o}^{\text{local}} - \Delta P_w \tag{4.13}
\]

where $P_{\text{global}}$ is the global pressure for each element and is used for ranking of all elements for displacement events. Rearranging equation (4.13) gives:

\[
P_c^{\text{local}} = P_{\text{global}} + P_{o}^{\text{local}} + \Delta P_w \tag{4.14}
\]
In quasi-static displacement, we assume that both the water and oil pressure gradients are negligible and equation (4.13) reduces to:

\[ P_{\text{global}} = P_{\text{local}} \]  
(4.15)

Thus, in quasi-static modeling, the local capillary pressure is the global capillary pressure over the entire network and this is used to define the fluid configuration in each element and the corresponding volume of each phase.

The perturbative approach of the Hughes and Blunt model [88] includes the viscous pressure drop in water in the expression for \( P_{\text{global}} \) but assumes that the oil pressure gradient is negligible and equation (4.13) reduces to:

\[ P_{\text{global}} = P_{\text{local}} - \Delta P_w \]  
(4.16)

Neglecting the pressure gradient in the oil phase implies that their model is suitable for only water/gas or water/light oil flows. They used equation (4.17) to calculate and scale-up \( \Delta P_w \) for different \( N_{\text{cap}} \) corresponding to the desired flow rates, \( (Q_w)_{\text{desired}} \).

\[ (\Delta P_w)_{\text{desired}} = \frac{(Q_w)_{\text{desired}}}{(Q_w)_{\text{multiphase}}} \Delta P_w \]  
(4.17)

For each \( N_{\text{cap}} \) value, they calculated \( (\Delta P_w)_{\text{desired}} \) for each element and determined \( P_{\text{global}} \) using equation (4.16). All possible displacement events are ranked and sorted in terms of global pressure and the event with the highest global pressure is executed first.

4.4 Proposed New Model

The assumption of fixed conductivity for corner flow in the Hughes and Blunt model [88] implies that water layers in the corners are prevented from swelling. In reality, the wetting layers swell near an advancing imbibition front. In this model, we allow the water layers to swell and include both the viscous pressure drops in water and oil in the expression for \( P_{\text{global}} \), equation (4.13). This allows us to model the effects of viscosity ratio \( M \), in addition to \( N_{\text{cap}} \), on the imbibition displacement patterns and saturation profiles. We define and give relevant mathematical expressions that are used in this model in the subsections below.
### 4.4.1 Maximum Local Capillary Pressure

We compute $P_{\text{global}}$ and $(\Delta P_w)_{\text{desired}}$ for each element for a given $N_{\text{cap}}$ using equations (4.13) and (4.17) respectively. The last element to be filled after each displacement event sets $P_{\text{global}}$ and the local capillary pressure for each element is calculated using equation (4.14). However, there is a maximum local capillary pressure for each element and this is the initial local capillary pressure, $P_{\text{local}}^{\text{initial}}$ before the commencement of the imbibition process. Allowing the local capillary pressure to be greater than this implies that the wetting layers will shrink on imbibition which is not physical.

\[ P_{\text{local}} = \min\left[ P_{\text{local}}^{\text{initial}}, P_{\text{local}}^{\text{local}} \right] \quad (4.18) \]

### 4.4.2 Minimum and Optimum Time Step

The water flow rate for each throat in the network is given by equation (4.6) while the pore flow rates are determined by summing all the connecting throat positive (inlet) flow rate into the pore assuming outlet flow is zero. For each element in the network, the filling time is determined from:

\[ t_i = \frac{V_{i,\text{lw}}}{q_{i,w}} \quad (4.19) \]

where $V_{i,\text{lw}}$ is the volume of oil in element $i$ and $q_{i,w}$ is the water flow rate in that element. The minimum time step, $t_{\text{min}}$ is the minimum filling time and is given by:

\[ t_{\text{min}} = \min\left[ t_i \right]_{i=1}^m \quad (4.20) \]

where $m$ is the total number of elements in the network. We determine the optimum time step, $t_{\text{opt}}$ by carrying out displacement processes starting from an arbitrary high time step and gradually decreasing the time step until there is a convergence of saturation profiles for the specific $N_{\text{cap}}$. The maximum time step that gives convergence is $t_{\text{opt}}$. For a fully dynamic model, the minimum filling time, $t_{\text{min}}$ is the model time step and this makes it computationally intensive.
### 4.4.3 Updating Water Layer Volume

After each displacement event, the water layers in all elements where the bulk fluid is oil will swell. The extent of swelling depends on the time step and (or) the local capillary pressure. We update the water layer volume due to time-step swelling for element $i$ with equation (4.21) and call this volume $v_{i,\text{tstep}}$.

$$v_{i,\text{tstep}} = v_{i}^{(n+1)} = v_{i}^{n} + I_{\text{opt}}q_i$$  \hspace{1cm} (4.21)

where $(n+1)$ is the current time step and $n$ is the previous time step.

The last element filled after each displacement event sets the $P_{\text{global}}$ and the $P_{\text{local}}$ for all elements are calculated based on this $P_{\text{global}}$ using equations (4.14) and (4.18). The water layers will adjust (swell or remain constant) to the new local capillary pressure equilibrium. This new volume is called equilibrium volume due to local capillary pressure, $v_{\text{lpce}}$. The value of $v_{\text{lpce}}$ when the local capillary pressure equals the snap-off pressure is the maximum water volume that can reside in the layers.

We compare $v_{i,\text{tstep}}$ and $v_{\text{lpce}}$ using equation (4.22) and set the minimum value to the current water layer volume, $V_{i,\text{curr}}$ for each element and update the fluid configuration and conductance accordingly as outlined below.

$$V_{i,\text{curr}} = \min(v_{i,\text{tstep}}, v_{\text{lpce}})$$  \hspace{1cm} (4.22)

### 4.4.4 Water Corner Conductance Approximation

Depending on the current volume of oil to be displaced in each element, the time step taken and the local capillary pressure; the oil can be completely or partially displaced. An algorithm on complete and partial filling is outlined under Time-dependent Partial Filling Algorithm section. After every displacement event, the water and oil volumes of all elements are updated using equation (4.22) and the area occupied by the water in corners and the oil at the center can be determined from the water and oil volume for each element respectively.

We take the following steps and approximate the water corner conductance of each element as follows:
Step 1
Determine the initial (minimum) water corner conductance, \( g_{c,\text{min}}^{w} \), for each element and the corresponding initial (minimum) water corner volume \( V_{w,\text{min}} \) and the area \( A_{w,\text{min}} \) before the commencement of the imbibition process.

Step 2
Determine the maximum water corner conductance, \( g_{c,\text{max}}^{w} \), for each element and the corresponding maximum water corner volume \( V_{w,\text{max}} \) and the area \( A_{w,\text{max}} \). The \( g_{c,\text{max}}^{w} \) is the conductance at snap-off pressure.

Step 3
Determine the local \( P_{\text{local}} \) for each element, allow the water layers to swell, calculate the corner conductance, \( g_{c,\text{lpc}}^{w} \), current volume, \( V_{w,\text{lpc}} \) and area, \( A_{w,\text{lpc}} \).

Step 4
Update the volume of each element after every displacement event using equation (4.22) and determine the current volume \( V_{w,\text{cur}} \) of water in corners. Calculate the wetting phase area, \( A_{w,\text{cur}} \) from the volume.

Step 5
Compare \( V_{w,\text{cur}} \), \( V_{w,\text{lpc}} \) and \( V_{w,\text{max}} \) for each element and compute, assign or approximate the current conductance \( g_{c}^{w} \) as stated below:

(a) If \( V_{w,\text{cur}} = V_{w,\text{lpc}} \) and \( V_{w,\text{lpc}} \leq V_{w,\text{max}} \), then \( g_{c}^{w} = g_{c,\text{lpc}}^{w} \).

(b) If \( V_{w,\text{cur}} \geq V_{w,\text{lpc}} \) and \( V_{w,\text{cur}} > V_{w,\text{max}} \), we fill the element completely and then compute the conductance using equation (4.5) as usual for a completely filled element.

(c) If \( V_{w,\text{cur}} \neq V_{w,\text{lpc}} \) and \( V_{w,\text{cur}} \leq V_{w,\text{max}} \), then we interpolate between the \( g_{c,\text{min}}^{w} \) and \( g_{c,\text{max}}^{w} \) and approximate the \( g_{c}^{w} \) as follows...
\[ g_c^w = g_{c,\min}^w + \frac{(g_{c,\max}^w - g_{c,\min}^w)}{(w_{c,\max}^2 - w_{c,\min}^2)} (A_{w,cur}^2 - A_{w,\min}^2) \]  

(4.23)

\[ g_{c,\min}^w, \ g_{c,\max}^w \text{ and } g_{c,\lpc}^w \text{ are calculated using empirical correlations derived from} \]

solutions of the Stoke’s equation [1, 3].

### 4.5 Filling Sequence

Depending on the specified initial water saturation and (or) the capillary pressure desired at the end of the primary drainage process, some elements (small throats and pores) in the network will remain water filled and un-invaded by oil. In this model, we set the irreducible clay volume to zero but we do not allow the drainage process to reach zero water saturation. This is to ensure that there is water layer flow at the beginning of the imbibition process. The ratio of the bulk oil flow to the water layer flow depends on the specified initial water saturation and we use 20% initial water saturation in most of the results presented here.

At the beginning of the imbibition process we compute the \( P_{\text{global}} \) for all possible displacement events and rank all possible elements using the \( P_{\text{global}} \). We fill one element at a time and the last element to be filled sets the \( P_{\text{global}} \) for the network. An element can be filled completely or partially during a displacement event depending on whether the filling mechanism is piston-like or snap-off. Piston-like displacement for each element occurs at a high flow rate and hence, we allow complete displacement of oil and complete filling by water. Filling by snap-off (layer swelling) for each element however, occurs at a low flow rate and we use the time-dependent partial filling algorithm (below) to check whether an element can be partially or completely filled within the specified time step. We also allow complete filling of snap-off elements once the water layer volume reaches the maximum layer volume.

Once an element is displaced, whether partially or completely from the sorted list, we allow the water in layers/corners of all other elements to swell within the specified time step and the new local capillary pressure. The layer volume is updated using equation (4.22).
We note the initial volume of water in each element at the end of the drainage process and before the commencement of the displacement for every time step and set this as the previous volume, \( v^n \). We update the volume of all elements (due to displacement and layer swelling) after each time step and set this as the current volume, \( v^{(n+1)} \). We compute the summation of the difference between the current and previous volumes for all elements except the currently displaced element and set this as the volume of water injected due to layer swelling, \( v_{\text{low}} \). The difference between the current and previous volumes for the currently displaced element is \( v_{\text{dsp}} \). The total actual volume of water injected, \( v_{\text{actual}} \) within a time step is the summation of \( v_{\text{low}} \) and \( v_{\text{dsp}} \).

\[
 v_{\text{low}} = \sum_{i=1}^{m} \left( v^{(n+1)} - v^n \right) 
\]

\[
 v_{\text{actual}} = v_{\text{low}} + v_{\text{dsp}} 
\]

where \( m \) is the total number of network elements minus one.

### 4.6 Time-dependent Partial Filling Algorithm

The algorithm can be summarized as follows:

- **Injected (Water) Volume Computation**: for a given \( N_{\text{cap}} \), the volume of water available for injection at every time step, \( v_{\text{step}} \) is given by equation (4.26) while the total volume, \( V \) available for injection throughout the imbibition process is given by equation (4.27).

\[
v_{\text{step}} = Q \Delta t
\]

\[
V = QT
\]

where \( \Delta t \) is the time step, \( T \) is the total flow duration and \( Q \) is the desired flow rate. The total number of time step requires for the injection of \( V \) is given by \( V / v_{\text{step}} = T / \Delta t \).

- **Water Layer Volume Distribution**: for secondary imbibition, the primary drainage displacement leaves water layers in pores and throats throughout the network. We calculate the initial volume of water \( v_{\text{ew}} \), and determine the initial volume of the non-wetting phase, \( v_{\text{enw}} \) from the known total volume for each element \( v_{et} \). We update \( v_{ew} \) and \( v_{enw} \) after each time step.
\[ v_{enw} = v_{et} - v_{ew} \] (4.28)

- **Pressure Solution:** we set up the pressure solution using equations (4.6) and (4.8), assign a constant pressure across the inlet, assign zero pressure at the outlet and obtain the water and oil pressures for each element as well as the water pressure drop between the inlet and the element using equation (4.9).

- **Scaling of Pressure:** we scale up the water pressure drop for each element using equation (4.17) to obtain the volumetric inlet flow rate \( Q \) for the wetting phase that corresponds to the desired \( N_{cap} \). We follow the procedure outlined below (Scaling and Adjusting Oil Pressure) to scale and adjust the oil pressure for each element.

- **Sorting Pressure and Ranking:** we calculate the global (sorting) pressure using equation (4.13) and rank the elements based on this pressure.

- **Filling of Elements:** we take the following steps to fill the elements.
  
  **Step 1:** we take the first or next element on the sorted list, if the displacement is piston-like, we fill the element completely and set:
  
  \[ V = V - v_{enw} \] (4.29)

  and go to step 3. If the displacement is snap-off, go to step 2.

  **Step 2:** we calculate the volume of water, \( v_e \) that can enter the element in a time step \( \Delta t \).
  
  \[ v_e = q_e \Delta t \] (4.30)

  where \( q_e \) is the total inlet water flow rate into the element assuming that outlet flow rate is zero. We also calculate the change in volume of water in layers and corners \( v_{lpce} \), due to swelling as a result of reduction in the local capillary pressure and set:
  
  \[ v_e = \min \left[ v_e, v_{lpce} \right] \] (4.31)

  - **Complete Filling:** if either \( v_e \geq v_{enw} \) or \( v_e \geq V_{mlv} \) (the maximum layer volume) and \( V > v_{enw} \), fill the element completely with water and set
    
    \[ v_{ew} = v_{et}, \quad V = V - v_{enw} \quad \text{and} \quad v_{dsp} = v_{enw}. \]

    Then go to Step 3.
Partial Filling: if \( v_e < v_{ew} \), and \( V > v_e \), fill the element partially with the available \( v_e \) and set \( v_{ew} = v_{ew} + v_e \), \( V = V - v_e \) and \( v_{dsp} = v_e \). Then go to Step 3.

Final Filling: if neither of the above criteria apply, this implies that either \( V < v_{ew} \) or \( V < v_e \). In either case, fill the element with the available volume of water and set \( v_{ew} = v_{ew} + V \), \( v_{dsp} = V \) and \( V = 0 \). Then go to Step 7.

**Step 3**: we allow the water in layers/corners of all other partially filled or yet to be filled elements to swell using equations (4.30) and (4.31), update their configurations, compute \( v_{lsw} \) (the summation of the layer swelling, equation (4.24)) and update the total volume of water injected accordingly.

\[
V = V - v_{lsw}
\]  
(4.32)

**Step 4**: we compute \( v_{\text{actual}} \) using equation (4.25). If \( v_{\text{actual}} < v_{\text{tstep}} \), the next available element on the sorted list is displaced (by taking steps 1 and 2 only) and we update its volume and \( v_{\text{actual}} \) accordingly.

**Step 5**: We then compare \( v_{\text{actual}} \) and \( v_{\text{tstep}} \) again and repeat step 4 until both volumes are either equal or \( v_{\text{actual}} \) is greater than \( v_{\text{tstep}} \). In most cases \( v_{\text{actual}} \) will be slightly greater than \( v_{\text{tstep}} \). This slight increase can be accounted for by adjusting the current time step and thereby reduce the total available time remaining. Alternatively, the remaining total water volume available for injection can be reduced accordingly. We choose both options in this study.

**Step 6**: we recompute the water, oil and global pressures for all elements and resort the sorted list and go to step 7.

**Step 7**: repeat steps 1 to 6 until all the available water has been injected and \( V = 0 \) or all the remaining non-wetting phase is trapped. Then go to step 8.

**Step 8**: end of the imbibition process.

A flowchart for the partial filling algorithm is shown in Figure 4.
Figure 4.4: A flow chart for the partial filling algorithm

1. **Start**
   - Calc. total vol. of water available: \( V = Q_w T \)
   - Determine the optimum time step: \( \Delta t \)
   - Calc. time step volume: \( V_{\text{step}} = Q_w \Delta t \)

2. **Next time step**
   - Calculate: \( P_{\text{sort}} = P_c^{\text{local}} - \Delta P_w - \Delta P_{nw} \)
   - Rank all elements on the sorted list
   - Take the first/next element on the list

3. **Fill the element completely**
   - Remove the element from the list. Set: \( v_{\text{dsp}} = v_o \)
   - \( V = V - v_o \)

4. **Calc. \( v_{we} = f_n (q_e, \Delta t, P_c^{\text{local}}) \)**
   - for all other partially filled or yet to be filled elements. Set: \( v_{lsw} = \sum v_{we} \)
   - \( V = V - v_{lsw} \)

5. **\( V_{\text{actual}} = v_{lsw} + v_{dsp} \)**

6. **Piston-like displacement?**
   - yes
   - \( v_{we} \geq v_o \) & \( V \geq v_{we} \)?
   - yes
   - \( v_{we} < v_o \) & \( V \geq v_{we} \)?
   - yes
   - \( v_{we} \geq v_{lsw} \) & \( V \geq v_{we} \)?
   - yes
   - \( V_{\text{actual}} \geq V_{\text{step}} \)?
     - yes
     - Update saturation and conductance
     - Check for trapping
     - no
     - Element still available on the list? & \( V \geq 0 \)?
       - yes
       - Displace the next element on the list
       - \( V_{\text{actual}} = V_{\text{actual}} + v_{dsp} \)
       - no
       - Stop
4.7 Trapping and Updating the Sorted List

Water in corners and layers ensures global connectivity of water phase during primary oil flooding and trapping, though possible, is unimportant. However, trapping of oil becomes important during secondary oil drainage when completely oil-filled elements or oil layers surround water filling pore centers. By assigning the water phase in each element to clusters, the water within the isolated water clusters is trapped and cannot be displaced.

During water flooding oil can be completely surrounded by water and cannot be displaced. Isolated oil clusters are trapped and the oil in them is no longer in pressure communication with the outlets. Hence, we do not allow either trapped oil or water to be displaced during simulations of waterflooding. Before the oil in an element is displaced either partially or completely therefore, we check whether there is a connected path for the oil to the outlet. Once the oil in an element is displaced completely, either through piston-like or snap-off, the element does not contain oil any longer. Hence, we need to check the neighboring / surrounding elements for trapping and remove those elements with trapped oil from the list. We also need to update the capillary entry pressure, global and local capillary pressure for the surrounding elements and resort the list before the next displacement. For partially-filled elements, we assume that the wetting phase resides in layers and we update the water conductance accordingly. A partially-filled element already displaced from the sorted list cannot be re-inserted.

4.8 Oil Sub-Network

From equation (4.13), it is obvious that both the water and oil pressures are required for the computation of $P_{global}$. Water maintains hydraulic continuity from the inlet to the outlet through the water layers in corners of pores and throats. Hence, the water pressure can be calculated for all pores in the network at any time during the displacement processes. Connectivity of the oil phase in the network however, changes with time depending on $N_{cap}$. 

88
At low capillary number ($N_{cap} \leq 10^{-6}$), oil remains connected from the inlet to the outlet at any time during the displacement processes. Oil also maintains hydraulic continuity from the inlet to the outlet at the commencement of the displacement processes for intermediate to high $N_{cap} > 10^{-6}$. As the displacement progresses however, oil is displaced from the network, its saturation decreases and the oil connected region start-point progressively moves towards the outlet. We call this changing oil connected region “Oil Sub-Network” and the pores and throats in this region are determined using a burning algorithm. No pressure drop occurs outside the oil sub-network. Within the sub-network however, the oil pressure is calculated for all pores using the pressure solver. The disconnected region may contain trapped/bypassed oil.

### 4.9 Scaling and Adjusting of Oil Pressure

We use equation (4.33) to scale the oil pressure to the flow rate corresponding to the desired $N_{cap}$.

\[
\left( P_{o, \text{local}} \right)_{\text{desired}} = \left( \frac{Q_{w}}{Q_{o}} \right)_{\text{multiphase}} P_{o, \text{local}}
\]  

(4.33)

However, there is a need to adjust the oil pressure due to the changing oil sub-network as outlined below.

**Step 1**: we compute a rate, $r_i$ at which each element is filling.

\[
r_i = \frac{\Delta V_{wi}}{\Delta t}
\]  

\[
\sum_{i=1}^{m} r_i = Q_{\text{desired}}
\]  

where $\Delta V_{wi}$ is the change in water volume for element $i$ and this corresponds to the volume of oil displaced within the time step $\Delta t$, $Q$ is the total injection rate, $m$ is the total number of elements in the network.

**Step 2**: we define $r_i(x)$ where $x$ is the coordinate of the element center and compute $Q(x_i)$ as follows:

\[
Q(x_i) = \sum_{i, x \leq x_i} r_i
\]  

(4.36)
Step 3: we use equation (4.37) to adjust the oil pressure.

\[
(p_{o}^{local})_{adj} = \frac{Q(x_t)}{Q} (p_{o}^{local})_{desired}
\]

(4.37)

4.10 Model Assumptions / Approximations

4.10.1 Outlet Water Flow Rate and Maximum Injected Water Volume

The outlet water flow rate from the network is assumed to be zero and this implies that only oil is allowed to flow out of the network. Consequently, the maximum water volume that can be injected per unit pore volume, theoretically, is limited to \((1 - S_{wi})\), where \(S_{wi}\) is the initial water saturation in the system. However, the actual water volume that can be injected is limited to movable oil volume \((1 - S_{wi} - S_{or})\), where \(S_{or}\) is the residual oil saturation. However, much more (several pore volumes) of water is injected to displace oil in laboratory displacement experiments when measuring fluid production and pressure drop data for relative permeability calculations. We accommodate this by simulating injection in a long network, so near the inlet \(S_{or}\) is reached before breakthrough.

4.10.2 Complete Filling of Elements

The wetting layer volume in partially filled elements as a result of snap-off mechanism increases with time due to wetting layer swelling. Complete filling is assumed when the wetting layer volume in these partially filled elements reaches the maximum layer volume as stated under section 4.6 (Time-dependent Partial Filling Algorithm). In reality however, these elements should still contain oil and complete filling should have been a gradual process. Also, as stated under section 4.5 (Filling Sequence), piston-like displacement for each element occurs at a high flow rate and hence, we assume complete displacement of oil and complete filling by water. However, due to insufficient time, many of the elements filled by piston-like mechanism will be partially filled in reality.

Consequently, oil production as a result of displacement by water will be over-predicted by the model when compared with experimental data and the water pressure
in the completely filled elements will be higher than what it should have been if the elements had been partially filled. Hence, the model will also predict a lower water pressure drop between elements when compared with experimental data. These limitations can be overcome by refining the model as suggested in section 7.1 – Prediction of Relative Permeability from Displacement Experiments.

The simulation results for large networks generated from the reconstructed microstructure of Berea sandstone for strongly water-wet, mixed-wet and oil-wet are presented in the next chapter.
Chapter 5

Water-wet Berea Sandstone Results

Starting from a large stochastic Berea network of dimension $(6\times6\times6)\,mm^3$ fully saturated with water and water-wet, oil displaces water and the throats and pores are filled in order of increasing capillary pressure. The filling continues until the network water saturation, $S_w = 0.2$. This primary drainage displacement process can only occur through piston-like displacement whereby the center of an element can only be filled if it has an adjacent element containing oil and is the same as the quasi-static model of Valvatne and Blunt [4] and details can be found in [95].

After primary drainage, the wettability of the oil-contacted part of the rock will be altered and the network might become weakly water-wet or mixed-wet. In addition to wettability alteration, contact angles are affected by the direction of flow due to surface roughness. Contact angle hysteresis between the flooding cycles is thus modeled using the Class III model suggested by Morrow [98]. By defining a distribution of intrinsic contact angles, the receding and advancing value can be found using the relationship illustrated in Figure 5.1.

![Figure 5.1: Relationship between receding and advancing contact angles on a rough surface as a function of intrinsic contact angle measured at rest on a smooth surface [98].](image-url)

Figure 5.1: Relationship between receding and advancing contact angles on a rough surface as a function of intrinsic contact angle measured at rest on a smooth surface [98].
For a (strongly) water-wet system without contact angle hysteresis however, a thick wetting film coats the surface of the pore space and the network will still maintain its wettability and hence both receding and advancing contact angles are set to zero. Waterflooding commences with injection of water at a constant rate corresponding to the desired capillary number and the process continues until all the elements on the sorted list have been displaced completely or the remaining oil in the network is completely trapped or a specified water saturation is reached. We validate the model by comparing the displacement patterns and relative permeability curves with the Hughes and Blunt model and quasi-static model. The validation results and other simulation results are presented below.

### 5.1 Model Validation

Figures 5.2 to 5.5 show comparisons of the displacement patterns for the dynamic model the Hughes and Blunt model and quasi-static model. The irreducible clay volume is set to zero, initial water saturation at the end of the drainage process is 20\% and the initial water distribution is as depicted in Figure 5.2. After injecting 30\% pore volume of water ($S_w = 0.5$), Figures 5.3, 5.4 and 5.5 show the displacement patterns for the quasi-static, Hughes and Blunt, and the dynamic model respectively. The mobility ratio $M = 1$, contact angle, $\theta = 0$ and $N_{cap} = 3.0 \times 10^{-8}$ for the Hughes and Blunt model and the dynamic model. These Figures show that the displacement patterns for the dynamic model at low capillary number are the same as those of the quasi-static model and the Hughes and Blunt model.

To avoid boundary effects, a section of the network (25\% to 75\% along the model) is used for calculating relative permeability and water saturation using the average pressure drops at the section boundaries.

$$k_p = \frac{q_{mp} \Delta P_{sp}}{q_{tp} \Delta P_{mp}}$$

(5.1)

where $q_{mp}$ and $q_{tp}$ are the total flow rates of phase $p$ in multiphase and single-phase conditions; and $\Delta P_{mp}$ and $\Delta P_{sp}$ are the multiphase and single-phase pressure drops for phase $p$ across the selected section. The relative permeability curves for the three
models are as shown in Figure 5.6. The three relative permeabilities are the same: the water is always well connected and capillary forces control the displacement.

Figure 5.2: Initial water distribution at $S_{wi} = 0.2$; (a) a 2D projection and (b) a 3D pattern.

Figure 5.3: Displacement patterns for the quasi-static model of Valvatne and Blunt with average $S_w = 0.5$; (a) a 2D projection and (b) a 3D pattern.
Figure 5.4: Displacement patterns for the Hughes and Blunt model at $N_{cap} = 3.0 \times 10^{-8}$ with average $S_w = 0.5$; (a) a 2D projection and (b) a 3D pattern.

Figure 5.5: Displacement patterns for the dynamic model at $N_{cap} = 3.0 \times 10^{-8}$ with average $S_w = 0.5$; (a) a 2D projection and (b) a 3D pattern.
Figure 5.6: Comparison of predicted relative permeability curves for the dynamic and Hughes and Blunt (H&B) models at $N_{cap} = 3.0 \times 10^{-8}$ with a quasi-static model. The irreducible clay volume is zero and initial water saturation $S_{wi} = 0.2$.

For $N_{cap} > 3.0 \times 10^{-8}$, Figures 5.7 to 5.9 compare the displacement patterns for the dynamic model with the Hughes and Blunt model. For $N_{cap} = 3.0 \times 10^{-5}$ the dynamic model predicts a less diffuse displacement front, since it suppresses filling through layers in comparison with the Hughes and Blunt model.

Figure 5.10 shows the predicted residual oil saturation, $S_{or}$ as a function of $N_{cap}$ for the two models. For $N_{cap} \leq 10^{-5}$ the results are similar, but for higher values of $N_{cap}$ we see a continued decrease in $S_{or}$ for the dynamic model, as the displacement front becomes more piston-like. The Hughes and Blunt model fails to see any further decrease in $S_{or}$ – the model is no longer valid and is qualitatively dissimilar to experimental results [84, page 42].
Figure 5.7: Comparison of 2D displacement pattern projections at $N_{cap} = 3.0 \times 10^{-7}$ for (a) the Hughes and Blunt model with (b) the dynamic model. The total number of elements filled at $S_w = 0.5$ in (a) = 10,607 and (b) = 10,616.

Figure 5.8: Comparison of 2D displacement pattern projections at $N_{cap} = 3.0 \times 10^{-6}$ for (a) the Hughes and Blunt model with (b) the dynamic model. The total number of elements filled at $S_w = 0.5$ in (a) = 10,540 and (b) = 10,219.

Figure 5.9: Comparison of 2D displacement pattern projections at $N_{cap} = 3.0 \times 10^{-5}$ for (a) the Hughes and Blunt model with (b) the dynamic model. The total number of elements filled at $S_w = 0.5$ in (a) = 10,058 and (b) = 8,655.
Figure 5.10: Comparison of the effects of capillary number, $N_{\text{cap}}$, on residual oil saturation, $S_{\text{or}}$, for the dynamic model with the Hughes and Blunt model.

### 5.2 Effects of Rate on Displacement Patterns and Saturation Profiles.

Figure 5.11 shows 2D projections of displacement patterns for a water-wet system with receding contact angle equal to advancing contact angle, $\theta_r = \theta_a = 0^\circ$ for (a) initial condition, $S_{wi} = 0.2$; and after 0.3 pore volume ($PV$) of water has been injected ($S_w = 0.5$) for (b) $N_{\text{cap}} = 3.0 \times 10^{-8}$ and (c) $N_{\text{cap}} = 3.0 \times 10^{-5}$ while Figure 5.12 shows the saturation profiles for mobility ratio, $M = 1$ and $N_{\text{cap}}$ of (a) $3.0 \times 10^{-8}$; (b) $3.0 \times 10^{-6}$; and (c) $3.0 \times 10^{-5}$ for a water-wet system with initial water saturation, $S_{wi} = 0.2$ and average water saturations, (from) $S_w = 0.3$ to $S_w = 0.6$.

The flow duration and the optimum time step for the different values of $N_{\text{cap}}$ are tabulated in Table 5.1. At low $N_{\text{cap}}$ ($3.0 \times 10^{-8}$), the capillary pressures dominate the displacement events and the viscous pressure drops are insignificant. The flow duration (3,600 seconds) and the optimum time step (9 seconds) are large and the wetting layers have sufficient time to swell. Hence, snap-off mechanisms are prevalent and piston-like displacements are suppressed. Consequently, the
displacement events as shown in Figure 5.11(b), are random and can happen anywhere in the network and there is no visible displacement front. This is equivalent to a quasi-static displacement and the saturation is uniformly distributed within the network as the wetting fluid invades the smallest pores and throats throughout the system as shown in Figure 5.12(a).

Figure 5.11: 2D projection of displacement patterns for a water-wet system with \( \theta_r = \theta_a = 0^\circ \) for (a) initial condition, \( S_{wi} = 0.2 \); (b) \( N_{cap} = 3.0 \times 10^{-8} \) and \( S_w = 0.5 \); and (c) \( N_{cap} = 3.0 \times 10^{-5} \) and \( S_w = 0.5 \).

As the capillary number, \( N_{cap} \) increases, the viscous pressure drop increases and become significant compared with the capillary pressure, the flow duration and time step decrease; and the wetting layers do not have sufficient time to swell. At \( N_{cap} \) of \( 3.0 \times 10^{-6} \), the flow duration is 36 seconds with a time step of 0.09 seconds; both the capillary and viscous forces contribute to the displacement events, snap-off mechanisms are reduced and piston-like mechanisms increase. Figure 5.12(b) shows the transition saturation profiles from a capillary dominated to a viscous dominated displacement.

The viscous pressure drop completely dominate displacement events at \( N_{cap} \geq 3.0 \times 10^{-5} \) and for a mobility ratio of 1.0, the displacement is piston-like as shown in Figure 5.11(c). The flow duration (3.6 seconds) and optimum time step (0.009 seconds) are small due to the high flow rate and the wetting layers have very little or no time to swell and snap-off mechanisms are completely suppressed. Hence, there is a distinct and clear displacement front with only water flowing behind the front in the presence
Figure 5.12: Effects of rate on average saturation profiles for mobility ratio, $M = 1$ and $N_{cap} = (a) 3.0 \times 10^{-8}$; (b) $3.0 \times 10^{-6}$; and (c) $3.0 \times 10^{-5}$ for a water-wet Berea sandstone.
of residual oil saturation and only oil flowing ahead of the front in the presence of connate water saturation as shown in Figure 5.12(c).

Table 5.1: Flow duration for injecting 0.4 pore volume, $PV$ and optimum time step for different flow rates.

<table>
<thead>
<tr>
<th>Flow Rate (m/sec)</th>
<th>Capillary Number, $N_{cap}$</th>
<th>Flow Duration (seconds)</th>
<th>Optimum Time Step (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.2\times10^{-7}$</td>
<td>$3.0\times10^{-8}$</td>
<td>3600</td>
<td>9</td>
</tr>
<tr>
<td>$1.2\times10^{-5}$</td>
<td>$3.0\times10^{-6}$</td>
<td>36</td>
<td>0.09</td>
</tr>
<tr>
<td>$1.2\times10^{-4}$</td>
<td>$3.0\times10^{-5}$</td>
<td>3.6</td>
<td>.009</td>
</tr>
</tbody>
</table>

5.3 Reproduction of Buckley-Leverett Profile - Effects of Mobility Ratio, $M$ on Saturation and Velocity Profiles

In 1942 Buckley and Leverett [99] presented one-dimensional frontal advance equation (equation 5.2) which states that for a constant volumetric rate of water injection ($q_t$), the velocity, $v_{S_w}$ of a plane of constant water saturation is directly proportional to the derivative of the fractional flow equation, $df_w/dS_w$ evaluated for that saturation [100].

\[ v_{S_w} = \left. \frac{dx}{dt} \right|_{S_w} = \left. \frac{q_t}{A} \right|_{S_w} \frac{df_w}{dS_w} \tag{5.2} \]

where $x$ is the distance along path of flow, $t$ is the time, $\phi$ is the porosity and $A$ is the cross-sectional area. A derivation of this equation is presented in Appendix A. Rearranging equation 5.2 gives the dimensionless velocity, $v_D$ as

\[ v_D = \left. \frac{\phi A}{q_t} \frac{dx}{dt} \right|_{S_w} = \left. \frac{df_w}{dS_w} \right|_{S_w} \tag{5.3} \]

The effects of mobility ratio, $M$ on saturation and velocity profiles are investigated at constant injection rates corresponding to different $N_{cap}$ values. The values of mobility
ratio, $M$ used in the simulations are 1, 5, 10 and 20. Figures 5.13 to 5.16 show the effects of mobility ratio, $M$ on the saturation and velocity profiles for $N_{cap}$ of $3.0 \times 10^{-8}$ for a water-wet Berea sandstone. These figures confirm that for capillary dominated displacements and laminar flow; $M$ does not have any significant effect on the saturation and velocity profiles. Figure 5.17 compares the saturation and velocity profiles for different mobility ratios after injecting 0.3 pore volume of water. Thus, at an average water saturation of 0.5 or any other value, the effect of mobility ratio, $M$ on saturation and velocity profiles is indistinguishable for $N_{cap}$ of $3.0 \times 10^{-8}$ for a water-wet system.

Figure 5.13: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-8}$ and $M = 1$ for a water-wet system.

Figure 5.14: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-8}$ and $M = 5$ for a water-wet system.
Figure 5.15: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-8}$ and $M = 10$ for a water-wet system.

Figure 5.16: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-8}$ and $M = 20$ for a water-wet system.

Figure 5.17: Comparison of (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-8}$ at an average $S_w = 0.5$ for different mobility ratios for a water-wet system.
Similarly, Figures 5.18 to 5.21 show that the effects of mobility ratio, $M$ on saturation and velocity profile for $N_{cap}$ of $3.0 \times 10^{-7}$ for a water-wet system are insignificant. Figure 5.22 compares the saturation and velocity profiles for different mobility ratios at an average water saturation of 0.5.

Figure 5.18: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-7}$ and $M = 1$ for a water-wet system.

Figure 5.19: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-7}$ and $M = 5$ for a water-wet system.
Figure 5.20: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-7}$ and $M = 10$ for a water-wet system.

Figure 5.21: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-7}$ and $M = 20$ for a water-wet system.

Figure 5.22: Comparison of (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-7}$ at an average $S_w = 0.5$ for different mobility ratios for a water-wet system.
At intermediate $N_{cap}$ of $3.0 \times 10^{-6}$ however, the effects of mobility ratio, $M$ on the saturation and velocity profiles become obvious and increases with increasing value of $M$ as shown in Figures 5.23 to 5.26. Comparison of the saturation and velocity profiles for different mobility ratios at an average water saturation of 0.5 is depicted in Figure 5.27.

Figure 5.23: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-6}$ and $M = 1$ for a water-wet system.

Figure 5.24: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-6}$ and $M = 5$ for a water-wet system.
Figure 5.25: Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-6}$ and $M = 10$ for a water-wet system.

Figure 5.26: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-6}$ and $M = 20$ for a water-wet system.

Figure 5.27: Comparison of (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-6}$ at an average $S_w = 0.5$ for different mobility ratios for a water-wet system.
For $N_{cap} \geq 3.0 \times 10^{-5}$, Figures 5.28 to 5.31 show the saturation and velocity profiles for different values of mobility ratio, $M$ for a water-wet system. We observe that at late time, the saturation profiles collapse onto a single curve when plotted as a function of dimensionless velocity. These profiles, for $N_{cap}$ of $3.0 \times 10^{-5}$, show that the velocity of a plane of constant water saturation is directly proportional to the derivative of the fractional flow equation evaluated for that saturation. Hence, Buckley-Leverett profiles have been reproduced directly from pore-scale modeling.

We note however, the two-step saturation profiles in Figures 5.29 to 5.31. At the macro-scale, whether the displacing fluid is gas or water, both the oil and the displacing fluid flow together and simultaneously through the same pores. The actual amount of oil displaced during the process depends upon the relative ease with which the two fluids can flow – mobility ratio. At the pore-scale, both oil and water flow simultaneously through polygonal network elements and the partial filling algorithm allows gradual and continuous displacement of oil by water as the displacement progresses. We assume complete displacements of oil in elements filled by piston-like mechanism because piston-like displacement occur at high flow rates. However, due to the small time step at $N_{cap} \geq 3.0 \times 10^{-5}$ many of the elements filled by piston-like mechanism are only partially filled. This assumption might partly contribute to the two-step profiles noted in these figures.

![Figure 5.28: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 1$ for a water-wet system.](image-url)
Figure 5.29: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 5$ for a water-wet system.

Figure 5.30: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 10.0$ for a water-wet system.

Figure 5.31: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 20.0$ for a water-wet system.
Figure 5.32: Comparison of (a) saturation profiles and (b) dimensionless velocity profiles for \( N_{cap} = 3.0 \times 10^{-5} \) at an average \( S_w = 0.5 \) for different mobility ratios for a water-wet system.

Comparison of the saturation and velocity profiles for different mobility ratios at an average water saturation of 0.5 for \( N_{cap} \) of \( 3.0 \times 10^{-5} \) is depicted in Figure 5.32. This Figure shows that the higher the mobility ratio the less mobile the oil phase and the higher the saturation of the oil phase left behind the displacement front. Thus, the higher the mobility ratio, the higher the maximum speed of the injected water, the earlier the water breakthrough and the higher the residual oil saturation at water breakthrough. However, the apparent residual saturation (the oil saturation near the inlet) is approximately independent of mobility ratio.

The next section below computes the cumulative oil production as well as residual oil saturation at water breakthrough for different mobility ratios, \( M \).

### 5.4 Effects of Mobility Ratio, \( M \) on Fractional Water Flow Curve and Cumulative Oil Production

The fractional flow of water, \( f_w \), at any point in the network, is defined as the water flow rate, \( q_w \), divided by the total flow rate.

\[
f_w = \frac{q_w}{q_w + q_o}
\]

\( (5.4) \)

Dividing both the numerator and denominator by \( q_w \), equation 5.4 becomes.
However, the oil-water ratio \( q_o/q_w \), can be expressed in terms of the water and oil relative permeabilities as

\[
\frac{q_o}{q_w} = \frac{k_o \mu_m}{\mu_w k_m} \quad (5.6)
\]

Substituting equation 5.6 into equation 5.5 gives equation 5.7 below which states that for a given rock – that is, a given set of water-oil relative permeability characteristics – the value of \( f_w \) depends upon the magnitude of oil and water viscosities [101].

\[
f_w = \frac{1}{1 + \frac{\mu_o}{\mu_w} \frac{k_o}{k_w}} \quad (5.7)
\]

The water fractional flow, \( f_w \) is obtained by integrating equation 5.3 and Figure 5.33 shows the real part of the fractional flow curves with water saturation increasing abruptly from initial water saturation to flood front saturation for different mobility ratios for \( N_{cap} \) of \( 3.0 \times 10^{-5} \) for a water-wet Berea sandstone.

\[
f_w = \nu_o \int dS_w = \frac{\phi A}{q_i} \int dx dS_w \quad (5.8)
\]
Figure 5.33: Fractional flow curves for $N_{cap} = 3.0 \times 10^{-5}$ for different mobility ratios $M$, for water-wet Berea sandstone.

The cumulative oil production at breakthrough, $N_{pdw}$, is given by equation (5.9) and the average water saturation at breakthrough, $S_{w}^{avg}$, is obtained by extending the tangent to the fractional flow curves to $f_w = 1.0$. The corresponding $S_w$ to $f_w = 1.0$ is the average water saturation at breakthrough.

$$N_{pdw} = W_{idw} = S_{w}^{avg} - S_{wi}$$  \hspace{1cm} (5.9)$$

where $S_{wi}$ is the initial water saturation and $W_{idw}$ is the dimensionless cumulative water injected at breakthrough. The average water saturation at breakthrough and cumulative production at breakthrough for different mobility ratios, $M$ for $N_{cap} = 3.0 \times 10^{-5}$ are tabulated in Table 5.2. The cumulative oil production at breakthrough decreases with increasing mobility ratio.
Table 5.2: Cumulative oil production for different mobility ratios for $N_{cap} = 3.0 \times 10^{-5}$. for a water-wet Berea sandstone.

<table>
<thead>
<tr>
<th>Mobility Ratio, $M$</th>
<th>Average Water Saturation at Breakthrough, $S_n^{avg}$</th>
<th>Initial Water Saturation, $S_{wi}$</th>
<th>Cumulative Production (PV)</th>
<th>Oil Saturation, $S_o$ at Breakthrough</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.830</td>
<td>0.200</td>
<td>0.630</td>
<td>0.17</td>
</tr>
<tr>
<td>5.0</td>
<td>0.775</td>
<td>0.200</td>
<td>0.575</td>
<td>0.23</td>
</tr>
<tr>
<td>10.0</td>
<td>0.720</td>
<td>0.200</td>
<td>0.520</td>
<td>0.28</td>
</tr>
<tr>
<td>20.0</td>
<td>0.640</td>
<td>0.200</td>
<td>0.440</td>
<td>0.36</td>
</tr>
</tbody>
</table>

5.5 Sensitivity Analysis

The model is used to investigate the effects of network size, contact angle and initial water saturation on the saturation profile for a water-wet system.

5.5.1 Influence of Network Size on Saturation Profile

We show the effect of network size for three different network sizes for low capillary number only. Figure 5.34 compares the saturation profiles, for a water-wet system, for stochastic Berea networks of (3×3×3)mm³, (6×6×6)mm³ and (9×6×6)mm³ generated with the stochastic algorithm. While Figure 5.34(a) shows a lot of noise in the saturation profiles for the (3×3×3)mm³ network due to its small size, there are no noise effects in a (6×6×6)mm³ stochastic network, Figure 5.34(b). Comparison of Figures 5.34(b) and 5.34(c) shows that there is no noticeable difference in the saturation profiles of (6×6×6)mm³ and (9×6×6)mm³ networks. This confirms that the (6×6×6)mm³ network is sufficiently large enough for multiphase flow simulations.
Figure 5.34: Effects of network size on saturation profiles for $N_{cap} = 3.0 \times 10^8$; mobility ratio, $M = 1$; for (a) (3×3×3)mm$^3$, (b) (6×6×6)mm$^3$ and (c) (9×6×6)mm$^3$ networks.
5.5.2 Influence of Contact Angle on Saturation Profiles

Figure 5.35 shows the effects of contact angle on saturation profiles for a water-wet system with mobility ratio, $M = 10$ and capillary number, $N_{cap} = 3.0 \times 10^{-8}$. Figure 5.35(a) shows the saturation profiles for a strongly water-wet with contact angle of $0^\circ$. The saturation profiles for weakly water-wet systems with intrinsic contact angles uniformly distributed between $30^\circ$ and $50^\circ$ is shown in Figure 5.35(b); and $60^\circ$ and $80^\circ$ in Figure 5.35(c). The saturation profiles in Figures 5.35(a) and 5.35(b) are the same and show that at low capillary number and for any value of intrinsic contact angle below $50^\circ$, the contact angle does not have any significant effect on the saturation profiles.

Figure 5.35: Effects of contact angle on saturation profiles for a water-wet system with mobility ratio, $M = 10$, $N_{cap} = 3.0 \times 10^{-8}$; for contact angle = (a) $0^\circ$; (b) $30^\circ$ – $50^\circ$; (c) $60^\circ$ – $80^\circ$. 
and (c) 60° – 80°; and (d) comparison of saturation profiles for the different contact angles at an average water saturation of 40%.

However, the saturation profiles in Figure 5.35(c) shows noticeable differences compared with those in Figures 5.36(a) and 5.35(b). These differences are due to the fact that as the intrinsic contact angle increases (above 50° in this case), the network becomes less water-wet, snap-off mechanism reduces and piston-like filling increases. At an average water saturation of 40%, Figure 5.35(d) compares the saturation profiles for the different contact angles.

While the saturation profiles for intrinsic contact angle ≤ 50° are the same at low flow rate, there are noticeable differences between the saturation profiles for a water-wet with advancing contact angle of 0°; weakly water-wet systems with intrinsic contact angles uniformly distributed between 30° and 50°; and 60° and 80° at high flow rates corresponding to capillary numbers, $N_{cap} \geq 3.0 \times 10^{-5}$ as shown in Figures 5.36(a), 5.36(b) and 5.36(c) respectively. As the intrinsic contact angle increases, the network becomes less water-wet, snap-off mechanism reduces and piston-like filling increases. At an average water saturation of 40%, Figure 5.36(d) compares the saturation profiles for the different contact angles.

### 5.5.3 Influence of Initial Water Saturation on Saturation Profiles

As stated under section 4.5 (Filling Sequence) the irreducible clay volume is set to zero in this model but the drainage process is not allowed to reach zero saturation to ensure that there is water layer flow at the beginning of the imbibition process. The effects of initial water saturation on saturation profiles at low and high capillary numbers are depicted in Figures 5.37 and 5.38 respectively for a water-wet system. At zero initial water saturation, there is no water layer flow at the beginning of the imbibition process and there is no water layer swelling both at low and high capillary numbers. Hence, the displacement is piston-like.
Figure 5.36: Effects of contact angle on saturation profiles for a water-wet system with mobility ratio, $M = 10$, $N_{cap} = 3.0 \times 10^{-5}$; for contact angle = (a) $0^\circ$; (b) $30^\circ$ – $50^\circ$; (c) $60^\circ$ – $80^\circ$; and (d) comparison of saturation profiles for the different contact angles at an average water saturation of 40%.

At low capillary number, $N_{cap} = 3.0 \times 10^{-8}$, the capillary pressures dominate the displacement events and the viscous pressure drops are insignificant. The saturation profiles for initial water saturation of 10% as shown in Figure 5.37(b) and 20% as shown in Figure 5.37(c) are the same. This implies that as long as there is water layer flow and water layer swelling, the ratio of the bulk oil flow to the water layer flow is insignificant and the displacement events are random and are due mainly to snap-off mechanisms. For initial water saturation of zero however, the displacement is piston-like and snap-off mechanism is completely suppressed as shown in Figure 5.37(a) because the ratio of the bulk oil flow to water layer flow is infinity. Figure 5.37(d) compares the saturation profiles for the different initial water saturations when the average water saturation is 40%. The profile for initial water saturation of zero shows
a clear and distinct saturation front compared with the uniform distribution in the other two cases.

Figure 5.37: Effects of initial water saturation on saturation profiles for a water-wet system with mobility ratio, $M = 10$, $N_{cap} = 3.0 \times 10^{-8}$; for initial water saturation of, $S_{wi}$ = (a) 0%; (b) 10%, (c) 20%; and (d) comparison of saturation profiles for the different initial water saturations at an average water saturation of 40%.
Figure 5.38: Effects of initial water saturation on saturation profiles for a water-wet system with mobility ratio, $M = 10$, $N_{cap} = 3.0 \times 10^{-5}$; for initial water saturation of $S_{wi} = (a) 0\%; (b) 10\%; (c) 20\%$ and (d) comparison of saturation profiles for the different initial water saturation of 40\%.

Figure 5.38 depicts the saturation profiles for capillary number, $N_{cap} = 3.0 \times 10^{-5}$ and mobility ratio, $M = 10$ for different initial water saturations. The displacement is completely piston-like as shown in Figure 5.38(a) and 5.38(b) for initial water saturations of zero and 10\% respectively. This is due to the fact that the wetting layer flow is too low and hence, layer swelling and snap-off mechanisms are completely suppressed. At initial water saturation of 20\% however, the saturation profile is Buckley-Leverett displacement profile as shown in Figure 5.38(c). This confirms that the water layer flow is high enough for wetting layer swelling near an advancing flood front. Hence, we use 20\% initial water saturation in most of the simulations in this model. Figure 5.38(d) compares the saturation profiles for the different initial water saturations when the average water saturation is 40\%.
5.6 Prediction of Relative Permeability from Steady-State Experiments

By tuning the intrinsic contact angle distributions, the quasi-static network model of Valvatne and Blunt [4] and several other quasi-static models [1, 3, 5, 6] have been used to predict steady-state experimental relative permeability data for water-wet as well as mixed-wet sandstones successfully. In all of these models, relative permeability and water saturation are calculated using the pressure drops either across the whole network or a section of the network to avoid boundary effects, equation (5.1).

This approach of calculating relative permeability as a function of water saturation is valid since the basic assumption of quasi-static models is that the flow is laminar. It is also valid for the dynamic model at $N_{cap} \leq 10^{-8}$ and with the same initial flowing water saturation the dynamic model reproduces the quasi-static model relative permeability curves successfully as shown in Figure 5.6.

Valvatne and Blunt [4] successfully predicted Oak’s steady-state experimental relative permeability data [68] with intrinsic contact angles uniformly distributed between 50 and 60 degrees. The experimental irreducible water saturation, $S_{wc} = 0.25$ due to micro and clay bound porosity. A constant clay volume that remains water filled is associated with each element to account for the irreducible water saturation.

We assume that the experimental irreducible water saturation, $S_{wc} = 0.25$ is made up of the initial flowing water saturation $S_{wi}$ and irreducible clay volume $C_{vol}$. $S_{wi} + C_{vol} = S_{wc} = 0.25$. By adjusting the associated clay volume for each element by a constant factor, the micro and clay bound porosity can be varied while keeping the net porosity constant. Thus, the network total porosity $\phi_t$, made up of net porosity $\phi_n$; and micro and clay bound porosity $\phi_c$, ($\phi_c = \phi_n + \phi_c$) will change once the micro and clay bound porosity is varied. However, these changes will not affect the absolute permeability of the network. To ensure that the irreducible water saturation in the simulation is the same as the experimental value, $S_{wi} + C_{vol}$ should always be equal to 0.25.
We have shown the effects of initial water saturation on saturation profiles in subsection 5.5.3. Using the same intrinsic contact angles distribution (50 – 60 degrees), we vary the initial flowing water saturation, $S_{wi}$ and the irreducible clay volume $C_{vol}$ in the network. $S_{wi} + C_{vol} = S_{wc} = 0.25$. Figure 5.39 compares the predicted relative permeability curves by the dynamic model at $N_{cap} \leq 10^{-8}$ and different values of $S_{wi}$ and $C_{vol}$ with the quasi-static and experimental data. This Figure further shows the sensitivity of the dynamic model to the initial flowing water saturation and that successful prediction of experimental data depends not only on the distribution of the contact angles but also on the value of the initial flowing water saturation. Figure 5.40(a) compares the predicted relative permeability curves at $S_{wi} = 0.03$ and $C_{vol} = 0.22$ with the quasi-static and experimental data. The corresponding saturation profiles, Figure 5.40(b), are uniform and show that the calculation of relative permeability as a function of water saturation using pressure drops across the network is valid.

![Figure 5.39](image-url)

**Figure 5.39**: Predicted relative permeability curves at different values of initial flowing water saturation, $S_{wi}$ and clay volume, $C_{vol}$ compared with the quasi-static prediction and experimental data by Oak [68].
Figure 5.40: (a) Predicted relative permeability curves compared with the quasi-static prediction and experimental data by Oak [68] and (b) corresponding water saturation profiles. The intrinsic contact angles are uniformly distributed between 50 and 60 degrees.
To avoid boundary effects, a section of the network (0.25 to 0.75 along the model) is used for calculating the relative permeability curves shown in Figure 5.39 and 5.40(a). The effects of using different fraction of the network are shown in Figure 5.41. This Figure shows that for a good prediction of experimental relative permeability data at $N_{cap} \leq 10^{-8}$ the fraction of the network used in the computation of the relative permeability curves should be greater than or equal to 0.5. The saturation profiles across the whole network are depicted in Figure 5.40(b) and they show that the saturation distributions may not be uniform if the fraction of the network used is less than 0.5.

The water flow rate in Oak’s steady-state experiment [68] varied from 0cm$^3$/min to 10.24cm$^3$/min and the oil and water viscosities were 1.39cp and 1.05cp respectively. The core was 7.6cm long with a diameter of 5.1cm. Thus, with a surface tension of 30mN/m the highest capillary number, $N_{cap} = 2.9 \times 10^{-6}$ and the mobility ratio, $M = 1.3$.

Nguyen et al. [92] successfully predicted Oak’s experimental data at a capillary number, $N_{cap} = 3.4 \times 10^{-6}$ using contact angle of 30 degrees, Figure 2.14. This implies a lower surface tension value of approximately 26mN/m. Figure 5.42 shows the...
saturation profiles across the whole network for the original Berea network of dimension (3×3×3)mm³ at \( N_{cap} = 3.4 \times 10^{-6} \), \( S_{wi} = 0.03 \), \( C_{vol} = 0.22 \) and intrinsic contact angles uniformly distributed between 25 and 35 degrees. The saturation profiles are piston-like and hence a very small fraction of the network, where the saturation distribution is approximately uniform, is required for computations of relatively permeabilities.

Figure 5.42: Saturation profiles at initial flowing water saturation \( S_{wi} = 0.03 \) and \( C_{vol} = 0.22 \) at \( N_{cap} = 3.4 \times 10^{-6} \).

Figure 5.43 compares the computed relative permeability curves for different fractions of the network with intrinsic contact angles uniformly distributed between 25 and 35 degrees at \( N_{cap} = 3.4 \times 10^{-6} \). This Figure shows that the smaller the fraction of the network used the smaller the end-points water relative permeability, \( K_{rw}^{\text{max}} \) and the higher the residual oil saturation, \( S_{or} \) as shown in table 5.3.

For intrinsic contact angles uniformly distributed between 25 and 35 degrees, the effects of \( N_{cap} \) on the residual oil saturation \( S_{or} \), and end-point water relative permeability \( K_{rw}^{\text{max}} \), are shown in Figure 5.44. At low \( N_{cap} \), snap-off mechanisms are prevalent and piston-like displacements are suppressed resulting in high \( S_{or} \). Table 5.4
tabulates the $S_{or}$ and $K_{rw}^{max}$ for different values of $N_{cap}$. The $S_{or}$ and $K_{rw}^{max}$ at $N_{cap}$ of $10^{-8}$ and $10^{-7}$ are almost equal and these values confirm that the capillary pressures dominate the displacement events and the viscous pressure drops are insignificant. At $N_{cap}$ of $10^{-6}$ however, the viscous press drops become significant, piston-like displacements become more prevalent and snap-off mechanisms are suppressed resulting in lower $S_{or}$ and higher $K_{rw}^{max}$.

Figure 5.43: Computed relative permeability curves for different fractions of the network size, $f$ at $N_{cap}$ of $3.4\times10^{-6}$ with intrinsic contact angles uniformly distributed between 25 and 35 degrees.

Table 5.3: Residual oil saturation, $S_{or}$ and end-point water relative permeability, $K_{rw}^{max}$ for different fractions of the network used in the computation of the relative permeability curves.

<table>
<thead>
<tr>
<th>Fraction of the network</th>
<th>$S_{or}$ (PV)</th>
<th>$K_{rw}^{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.206</td>
<td>0.38</td>
</tr>
<tr>
<td>0.2</td>
<td>0.229</td>
<td>0.29</td>
</tr>
<tr>
<td>0.1</td>
<td>0.262</td>
<td>0.22</td>
</tr>
</tbody>
</table>
The effects of contact angles on $S_{or}$ and $K_{rw}^{\text{max}}$ are similar to the effects of rate and are depicted in Figure 5.45. With relative permeability computed over a sub-network of 0.2 to 0.4 at $N_{\text{cap}}$ of $10^{-6}$, snap-off mechanisms are completely suppressed with intrinsic contact angles uniformly distributed between 50 and 60 degrees. The $S_{or}$ is very low and and $K_{rw}^{\text{max}}$ is very high as shown in table 5.5. At contact angle of 0 degree, piston-like displacements reduce and snap-off mechanisms increase resulting in higher $S_{or}$ and lower $K_{rw}^{\text{max}}$. Thus, as contact angle decreases, $S_{or}$ increases and $K_{rw}^{\text{max}}$ decreases.

Figure 5.44: Effects of $N_{\text{cap}}$ on the computed relative permeability curves with intrinsic contact angles uniformly distributed between 25 and 35 degrees. The fraction of the network used is 0.1 for $N_{\text{cap}}$ of $10^{-6}$ and 0.5 for $N_{\text{cap}}$ of $10^{-7}$ and $10^{-8}$.

Table 5.4: Residual oil saturation, $S_{or}$ and end-point water relative permeability, $K_{rw}^{\text{max}}$ for different $N_{\text{cap}}$.

<table>
<thead>
<tr>
<th>$N_{\text{cap}}$</th>
<th>$S_{or}$ (PV)</th>
<th>$K_{rw}^{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.4 \times 10^{-5}$</td>
<td>0.339</td>
<td>0.15</td>
</tr>
<tr>
<td>$3.4 \times 10^{-7}$</td>
<td>0.324</td>
<td>0.17</td>
</tr>
<tr>
<td>$3.4 \times 10^{-6}$</td>
<td>0.262</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Figure 5.45: Effects of contact angles on the computed relative permeability curves at \( N_{cap} \) of \( 3.4 \times 10^{-6} \) using a sub-network of 0.2 to 0.4.

Table 5.5: Residual oil saturation, \( S_{or} \) and end-point water relative permeability, \( K_{rw}^{max} \) for different contact angles at \( N_{cap} \) of \( 3.4 \times 10^{-6} \).

<table>
<thead>
<tr>
<th>Contact Angle (degree)</th>
<th>( S_{or} ) (PV)</th>
<th>( K_{rw}^{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 - 60</td>
<td>0.049</td>
<td>0.77</td>
</tr>
<tr>
<td>25 – 35</td>
<td>0.229</td>
<td>0.29</td>
</tr>
<tr>
<td>0</td>
<td>0.246</td>
<td>0.25</td>
</tr>
</tbody>
</table>

As shown in Figure 5.42, there is no representative elemental volume over which the saturation distribution varies smoothly and uniformly and Figure 5.43 shows the effects of using different fraction of the network on the computation of relative permeability curves. This makes the definition of the relative permeability problematic [88] and matching of experimental data by Oak [68] difficult. Figure 5.46 compares the predicted relative permeability curves computed over 0.2 and 0.1 fractions of the network with intrinsic contact angles equal to zero and uniformly distributed between 25 and 35 degrees respectively at \( N_{cap} \) of \( 3.4 \times 10^{-6} \) with the experimental data by Oak [68]. The predicted residual oil saturation, \( S_{or} \) and end-point water relative permeability, \( K_{rw}^{max} \) are fair in both cases.
Figure 5.46: Predicted relative permeability curves at $N_{cap}$ of $3.4 \times 10^{-6}$ compared with the quasi-static prediction and experimental data by Oak [68]. The fraction of the network used is (a) 0.2 with contact angle of 0 degree and (b) 0.1 with intrinsic contact angles uniformly distributed between 25 and 35 degrees.
5.7 Run Times

Figure 5.47 shows a plot of run times versus number of pores for a (3×3×3)mm$^3$, (6×6×6)mm$^3$ and (9×6×6)mm$^3$ stochastic networks consisting of 12,349, 98,792 and 148,188 pores respectively for $N_{cap} = 3.0 \times 10^5$ on a Pentium IV PC with 2.0GB RAM. A trend line added to the three data points gives a polynomial equation:

$$y = 5.0 \times 10^{-9}x^2 - 0.0002x + 2.9634$$

and $R^2 = 1.0$. Thus, the run time scales with the square of the number of pore $x$. $y(x) \in O(x^2)$. A stochastic network of dimension (50×6×6)mm$^3$ equivalent to a core length of 5cm will contain approximately 820,000 pores. This will require a run time of approximately 3360 hours (140 days). Hence, core-scale simulation is not possible using this model.

![Figure 5.47: A plot of run times against number of pores with a polynomial trend line.](image)

$y = 5E-09x^2 - 0.0002x + 2.9634$

$R^2 = 1$
Chapter 6

Mixed-wet Berea Sandstone Results

In a water-wet system, all network elements remain water-wet with a single distribution of contact angles. For a mixed-wet system however, some regions of the pore space are oil-wet with a distribution of contact angle greater than 90° while others remain water-wet with a distribution of contact angle less than 90°. It is assumed that the wettability of 50% of the oil contacted part of the pore space changed to oil-wet after primary drainage. The intrinsic contact angles are uniformly distributed between 50° - 60° and 90° – 130° for weakly water-wet and oil-wet regions respectively.

Piston-like advance in an intermediate-wet or a mixed-wet system leaves the center of the pore space filled with water. A layer of oil may be left sandwiched between the water in the corner(s) and the water in the center if $\theta_a > 90 + \beta_i$, where $\theta_a$ is the advancing contact angle in the element and $\beta_i$ is(are) the corner half angle(s) in that element. Oil may flow through these layers, which means that oil can be displaced as long as it has a pathway to the outlet of pores and throats that are either oil-filled or contain oil layers. We assume that the oil layers are stable and allow instantaneous flow through them during displacement events until the two oil/water interfaces on either side of the layer meet. These stable oil layers allow oil mobility down to very low saturations. Once the oil layers collapse they no longer allow flow.

After primary drainage, waterflooding commences with injection of water at a constant rate from an initial water saturation of 20% and continues until all the elements on the sorted list have been displaced completely or the remaining oil in the network is completely trapped or a specified water saturation is reached. The simulation results of the effects of injection rate on saturation profiles; mobility ratio on saturation and velocity profiles, fractional flow curves and cumulative oil production at breakthrough for mixed-wet systems are presented below.
6.1 Effects of Rate on Saturation Profiles

Figure 6.1 shows the saturation profiles for mobility ratio, \( M = 5 \) and \( N_{cap} \) of (a) \( 3.0 \times 10^{-8} \); (b) \( 3.0 \times 10^{-6} \); and (c) \( N_{cap} = 3.0 \times 10^{-5} \) for a mixed-wet system with initial water saturation, \( S_{wi} = 0.2 \) and average water saturations, \( S_w \) from 0.3 to 0.5. Due to the mixed-wet nature of the pore space, water re-enters the pore space as the non-wetting phase occupying the centers of the pores and throats thereby reducing wetting layer flow and snap-off mechanisms. At low flow rate corresponding to \( N_{cap} = 3.0 \times 10^{-8} \), the saturation profiles, Figure 6.1(a) shows that the saturation decreases towards the outlet of the network due to the reduction of snap-off mechanisms.

At \( N_{cap} \) of \( 3.0 \times 10^{-6} \) and \( 3.0 \times 10^{-5} \), the saturation profiles show that the displacement is piston-like and there is a distinct displacement imbibition front as shown in Figure 6.1(b) and 6.1(c) respectively. The higher the \( N_{cap} \) the shorter the distance moved by the imbibition front.

6.2 Reproduction of Buckley-Leverett Profile – Effects of Mobility Ratio on Saturation and Velocity Profiles

At \( N_{cap} \) of \( 3.0 \times 10^{-5} \), the effects of mobility ratio, \( M \) on the saturation and velocity profiles for a mixed-wet system are shown in Figures 6.2, 6.3 and 6.4 for mobility ratio of 5, 10 and 20 respectively. At \( M = 5 \), the saturation profiles, Figure 6.2(a), are piston-like and the velocity profiles, Figure 6.2(b), show that planes of constant water saturation move at the same velocity. At \( M = 20 \) however, the saturation and velocity profiles show Buckley-Leverett displacement profiles as shown in Figure 6.4.
Figure 6.1: Saturation profiles for mobility ratio, \( M = 5 \) and \( N_{cap} = \) (a) \( 3.0 \times 10^{-8} \); (b) \( 3.0 \times 10^{-6} \); and (c) \( 3.0 \times 10^{-5} \) for a mixed-wet Berea sandstone. The contact angles are distributed between 50° – 60° and 90° - 130° for the water-wet and oil-wet regions respectively and oil fraction = 0.5.
Figure 6.2: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 5$ for a mixed-wet system.

Figure 6.3: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 10$ for a mixed-wet system.

Figure 6.4: (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ and $M = 20$ for a mixed-wet system.
Figure 6.5: Comparison of (a) Saturation profiles and (b) dimensionless velocity profiles for $N_{cap} = 3.0 \times 10^{-5}$ at an average $S_w = 0.5$ for different mobility ratios for a mixed-wet system.

The displacement is still piston-like at $M = 10$ as shown in Figure 6.3 with the imbibition front tilting towards the outlet. Figure 6.5 compares the saturation and velocity profiles for the different values of $M$ at an average water saturation, $S_w = 0.4$. The higher the value of $M$, the greater the distance moved by the imbibition front and the earlier the water breakthrough.

While Figures 6.2 and 6.3 show piston-like displacement profiles, the saturation and velocity profiles for mobility ratios of 5 and 10 for a water-wet system show Buckley-Leverett displacement profiles as shown in Figure 5.29 and 5.30 respectively. This confirms that the presence of the oil-wet region in mixed-wet systems reduces the wetting layer flow thereby suppressing snap-off mechanisms and increases piston-like displacement.

### 6.3 Effects of Mobility Ratio on Fractional Water Flow Curve and Cumulative Oil Production

Figure 6.6 shows the fractional flow curves for different mobility ratios for $N_{cap}$ of $3.0 \times 10^{-5}$ for mixed-wet Berea sandstone. The average water saturation and cumulative oil production at breakthrough for different mobility ratios are tabulated in Table 6.1. The cumulative oil production at breakthrough decreases with increasing mobility ratio. While the fractional flow curves for different mobility ratios in water-wet system are widely and clearly separated as shown in Figure 5.33, those in Figure 6.6
are not due to the fact that water re-enters the pore space as the non-wetting space occupying the centers of the network elements thereby suppressing wetting layer flow and swelling.

Figure 6.6: Fractional flow curves for $N_{cap} = 3.0 \times 10^{-5}$ for different mobility ratios $M$, for a mixed-wet Berea sandstone.

The cumulative oil productions at breakthrough for mobility ratios of 5 and 10 are greater by 0.055PV and 0.025PV respectively in water-wet than mixed-wet system. This confirms that the displacement of a non-wetting fluid by a wetting fluid is more efficient than the displacement of a wetting fluid by a non-wetting fluid [101]. However, the cumulative oil production in water-wet system is less by 0.02PV than that of mixed-wet.
Table 6.1: Cumulative oil production at breakthrough for different mobility ratios for 
$N_{cap} = 3.0 \times 10^{-5}$ for mixed-wet Berea sandstone.

<table>
<thead>
<tr>
<th>Mobility Ratio, $M$</th>
<th>Average Water Saturation at Breakthrough, $S_{avg}^{w}$</th>
<th>Initial Water Saturation, $S_{wi}$</th>
<th>Cumulative Production (PV)</th>
<th>Oil Saturation, $S_{o}$ at breakthrough</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>0.720</td>
<td>0.200</td>
<td>0.520</td>
<td>0.28</td>
</tr>
<tr>
<td>10.0</td>
<td>0.695</td>
<td>0.200</td>
<td>0.495</td>
<td>0.305</td>
</tr>
<tr>
<td>20.0</td>
<td>0.660</td>
<td>0.200</td>
<td>0.460</td>
<td>0.340</td>
</tr>
</tbody>
</table>

6.4 Sensitivity Analysis

6.4.1 Influence of Contact Angle on Saturation Profiles

Figure 6.7 shows the effects of contact angle on the saturation profiles for mixed-wet Berea sandstone. Only the contact angle distribution in the oil-wet region of the pore space is changed and the distributions used in the simulations are: $90^\circ$ – $130^\circ$, $120^\circ$ – $160^\circ$ and $140^\circ$ – $180^\circ$. The Figure shows that variation in the contact angle distribution of the oil-wet region of the pore space does not have significant effects on the saturation profiles. Figure 6.7(d) compares the saturation profiles for the different contact angle distributions.

6.4.2 Influence of Initial Water Saturation on Saturation Profiles

At low flow rate corresponding to $N_{cap}$ of $3.0 \times 10^{-8}$, the effects of initial water saturation on the saturation profiles is depicted in Figure 6.8. For an initial water saturation $S_{wi} = 0$, the displacement is completely piston-like as shown in Figure 6.8(a). As the initial water saturation increases to $S_{wi} = 0.1$ and 0.2, the average water saturation profiles decreases towards the outlet as shown in Figures 6.8(b) and 6.8(c) respectively. This shows that as the initial water saturation increases, the wetting layer flow and swelling increase and piston-like mechanism is reduced. Figure 6.8(d) compares the saturation profiles for the different initial water saturations at an average network saturation $S_{w} = 0.4$. 

136
Figure 6.7: Saturation profiles for a mixed-wet system with mobility ratio $M = 10$, $N_{cap} = 3.0 \times 10^{-5}$; for oil-wet region contact angle = (a) $90^\circ - 130^\circ$; (b) $120^\circ - 160^\circ$; (c) $140^\circ - 180^\circ$ and (d) comparison of saturation profiles for the different contact angles at an average water saturation of 40%.

At high $N_{cap}$ of $3.0 \times 10^{-5}$, Figure 6.9 shows the saturation profiles for a mobility ratio, $M = 10$ for different initial water saturations for mixed-wet Berea sandstone. The displacement is completely piston-like as shown in Figure 6.9(a) and 6.9(b) for initial water saturations, $S_{wi} = 0$ and $S_{wi} = 0.1$ respectively and similar to the water-wet systems depicted in Figures 5.38(a) and 5.38(b). However, the profiles at $S_{wi} = 0.2$ are different for water-wet and mixed-wet system. While the profiles are Buckley-Leverett displacement profiles in water-wet system, Figure 5.38(c), they are piston-like, Figure 6.9(c), in mixed-wet system due to the presence of the oil-wet region. Figure 6.9(d) compares the saturation profiles for the different initial water saturations when the average water saturation, $S_w = 0.4$. 

137
Figure 6.8: Saturation profiles for a mixed-wet system with mobility ratio, $M = 10$, $N_{cap} = 3.0 \times 10^{-8}$; for initial water saturations, $S_{wi}$ = (a) 0%; (b) 10%, (c) 20%; and (d) comparison of saturation profiles for the different initial water saturations at an average water saturation of 40%.
Figure 6.9: Saturation profiles for a mixed-wet system with mobility ratio, $M = 10$, $N_{cap} = 3.0 \times 10^{-5}$; for initial water saturations, $S_{wi} = (a) 0\%; (b) 10\%, (c) 20\%$; and (d) comparison of saturation profiles for the different initial water saturations at an average water saturation of 40\%.
Chapter 7

Conclusions

A new method of generating stochastic random networks representing the pore space of different rocks with given input pore and throat size distributions and connectivity has been developed. While networks extracted from pore-space images have fixed dimensions and impractical to scale up to core scale and beyond, the stochastic network generation algorithm can be used to generate a network of arbitrary size with given input pore and throat size distributions and connectivity.

The drainage and imbibition relative permeability curves generated from the stochastic network are similar to those from the original Berea network and experimental data. The consistency of the random network generator is demonstrated by comparing drainage and imbibition relative permeability curves for different network sizes.

A new rule-based, rate-dependent imbibition model is also developed by extending the Hughes and Blunt [88] pertubative model. The model incorporates a new time-dependent algorithm that accounts for partial filling of elements, does not assume constant flow rate in the wetting phase and layers and thereby allows swelling of the wetting phase near an advancing imbibition front. Unlike previously developed models that are only suitable for water displacing light oil or water displacing gas, the model considered viscous pressure drops in the connected and flowing region of the non-wetting phase and hence, is applicable for cases where the oil viscosity is higher than the water viscosity.

The model is used to study the effects of capillary number, $N_{cap}$, and mobility ratio, $M$ on displacement patterns; saturation and velocity profiles; water fractional flow curves and residual non-wetting phase saturation at breakthrough. Networks containing up to 100,000 pores are considered because the model is rule-based and computationally much more efficient than previously developed models.
By using large networks generated from the stochastic method, we reproduce Buckley-Leverett displacement profiles from pore-scale modeling thereby providing a bridge between pore-scale and macro-scale transport. The effects of wettability variation on water fractional flow curve and cumulative oil production at breakthrough are examined and we show that the displacement of a non-wetting fluid by a wetting fluid is more efficient than the displacement of a wetting fluid by a non-wetting fluid [101]. A sensitivity study of the influence of network size, contact angle distribution and initial water saturation on saturation and velocity profiles is also performed. Further improvements and applications of the model are discussed below.

7.1 Prediction of Relative Permeability from Displacement Experiments

We have used the dynamic model to predict Oak’s steady-state experimental relative permeability data [68] at $N_{cap} \leq 10^{-8}$ successfully and fairly well at $N_{cap} = 3.4 \times 10^{-6}$. However, steady-state tests are tedious and time consuming. On the other hand, unsteady-state tests are much quicker and therefore more convenient for practical use. It involves displacement of oil from a sample of rock initially saturated by oil and water by injecting water at constant rates. The relative permeability curves are calculated from the pressure drop and fluid production data obtained from the experiments using Johnson-Bossler-Naumann (JBN) approach [102] or other approaches that are based on Buckley and Leverett theory.

Figure 7.1 shows low and high rate experimental relative permeability curves, computed using JBN method, compared with the steady state experimental data for a consolidated sandstone reservoir rock sample. The flow rates are $4\text{cm}^3/\text{hr}$ and $383.7\text{cm}^3/\text{hr}$ for the low and high rate tests respectively. The oil and water viscosities were 0.56cp and 0.3cp respectively. With interfacial tension of 25mN/m, the $N_{cap} = 1.2 \times 10^{-8}$ for the low rate test and $1.1 \times 10^{-6}$ for the high rate test.

We have shown the effects of rate ($N_{cap}$) on displacement patterns and saturation profiles in section 5.2 and on the relative permeability curves in section 5.6. At $N_{cap}$ of
1.2×10⁻⁸, displacement events will be dominated by snap-off and piston-like displacements will be suppressed. Hence, the residual oil saturation at breakthrough and at the end of the experiment will be very high. At $N_{cap} = 1.1\times10^{-6}$, snap-off mechanisms will be suppressed and piston-like displacements will increase due to increase in the viscous pressure drops. Consequently, the residual oil saturation at breakthrough and at the end of the experiment will be lower than at $N_{cap} = 1.2\times10^{-8}$.

However, while the final residual oil saturation is lower in the high rate test (0.16PV) than in the low rate test (0.24PV) as expected, the cumulative productions at breakthrough in the low and high rate tests are 0.621PV and 0.486PV respectively. Thus, the oil relative permeability curve at high rate test lies below the $K_{ro}$ at low rate. Due to these reasons, we will not be able to use the dynamic model to predict the relative permeability curves for these experimental data.

![Figure 7.1](image)

**Figure 7.1:** Comparison of low and high rates experimental relative permeability curves with experimental steady-state data for a consolidated sandstone reservoir sample.

Can the dynamic model be used to reproduce the unsteady-state experimental fluid production and pressure drop data and thereby predict relative permeability data? Fluid production and pressure drop data can be computed on the first quarter of either an original network or an enlarged network. However, the assumption that the outlet
water flow rate from the network is assumed to be zero would limit the total amount of water that can be injected in the simulation. Also, the assumptions of complete filling of partially filled network elements when the wetting layer volume reaches the maximum layer volume or the displacement is piston-like may affect the quantitative reproduction of the experimental production data as described in section 4.10 (Model Assumptions / Approximations).

For successful qualitative and quantitative reproduction of experimental fluid production and pressure drop data, the dynamic model needs to be refined as follows.

- All piston-like fillings will be based on the available volume of water in a time step, $\Delta t$ given by equation (4.30) and hence elements can either be partially or completely filled.
- Network elements that are available for displacement will be placed in two separate lists. The first list (List A) will be a sorted list containing all elements that are available for displacement but are yet to be displaced, while the second list (List B) will contain all elements that have been displaced but are partially filled.
- Once an element is displaced from List A and is partially filled, it will be added to the other partially filled elements in List B. Elements that have been displaced and filled completely will be removed from both lists.
- After each displacement event, the wetting layer volume of elements on both lists is allowed to swell as functions of time step and local capillary pressure.
- Filling of partially filled elements on List B continues even after all elements have been displaced from List A until desired water saturation is reached or all the partially filled elements are trapped.
- Complete filling of partially filled network elements when the wetting layer volume reaches the maximum layer volume will still be assumed due to the difficulty of calculating / approximating conductance when the water layer volume is greater than the maximum layer volume but less than the total element volume.
The above-suggested refinements will remove the assumptions of complete filling due to piston-like and the limitations on the maximum volume of water that can be injected into the network. It will also be possible to compute the fluid production and pressure drop data over the whole network and this will ensure that the scaling factor is based on the actual water flow rate in the network. However, the model will become computationally more intensive.

### 7.2 Gravity Fingering

The infiltration of water into an unsaturated soil or rock is an immiscible displacement process in which water seeps downward and displaces the resident air phase from the pore space. The displacement process is unstable with respect to gravity and can lead to the formation of gravity-driven fingers as shown in Figure 7.2 [103].

Figure 7.2: Transition from initially stable to unstable infiltration where gravity-driven fingers formed, [103].

The dynamic model can be used to model this unstable process by including gravity effect $\Delta \rho g z$ in the computation of the global pressure, where $\Delta \rho$ is the density difference between water and air, $g$ is the gravitational constant and $z$ is the height above datum of the element.
At the beginning of the displacement, viscous and capillary forces are of the same order of magnitude, and wetting layers form and swell in advance of an invading water front [2]. Behind the front however, drainage process could happen anywhere and air could displace water from the pore space. Hence, the imbibition and drainage processes would happen simultaneously. Two separate sorted lists: one for imbibition events and the other for drainage processes and the computation of the global pressure will be separate for each process.

While the gravity effects have already been included in the code, the drainage sorted list and computation of the global pressure for the drainage events have not been added to the code. Once these are included, the dynamic model could be used to study this fascinating problem.

### 7.3 Statistical Modeling - Upscaling

Rhodes et al. [104] have proposed a pore-to-field transport simulation approach and applied to single phase flow accounting for advection and diffusion using a particle-tracking approach in which the flux is linearly related to the solute concentration. In multiphase flow, however, the flux from node to node is nonlinearly dependent on saturation, making particle tracking and ensemble averaging problematic. The dynamic model could be used to set a framework for the node to node transport of saturation and upscaling from pore-scale to core- and field-scale for multiphase flow.

### 7.4 Core-Scale Simulations

As stated under secion 5.7, run times for the current simulations scale with the square of the number of pores. For core-scale simulations to be possible in a reasonable time there is need to improve the efficiency of the code.
References


Appendix A

A.1 Derivation of the Frontal Advance Equation

The only assumptions necessary for the derivation of the frontal advance equation are:

1. there is no mass transfer between the phases and
2. the phases are incompressible.

Consider an infinitesimal element of rock having a porosity $\phi$, an area $A$, and a length $\Delta L$, in the direction of flow. The mass rate of water at point $L$ is

$$ (q_w \rho_w)_L $$ \hspace{1cm} A.1

The mass rate of water leaving the element at point $L + \Delta L$ is

$$ (q_w \rho_w)_{L+\Delta L} $$ \hspace{1cm} A.2

The mass rate of water accumulation in the element is

$$ A \phi \Delta L \frac{\partial}{\partial t}(S_w \rho_w) $$ \hspace{1cm} A.3

The mass rate of water entering the element of rock minus mass rate of water leaving is equal to the rate of mass accumulation of water in the rock element. Thus

$$ (q_w \rho_w)_L - (q_w \rho_w)_{L+\Delta L} = A \phi \Delta L \frac{\partial}{\partial t}(S_w \rho_w) $$ \hspace{1cm} A.4

Equation A.4 can be rewritten

$$ \frac{\partial}{\partial L}(q_w \rho_w) + A \phi \frac{\partial}{\partial t}(S_w \rho_w) = 0 $$ \hspace{1cm} A.5

However the fluids are assumed to be incompressible hence, the water density $\rho_w$, is not a function of either time or distance. So

$$ \frac{\partial}{\partial L}(q_w) + A \phi \frac{\partial}{\partial t}(S_w) = 0 $$ \hspace{1cm} A.6

Equation A.6 can be rewritten

$$ \frac{\partial S_w}{\partial t} \bigg|_L = -\frac{1}{A \phi} \frac{\partial q_w}{\partial L} $$ \hspace{1cm} A.7

Since the water flow rate, $q_w$, is a function of both the water saturation and time,
\[ dq_w = \left( \frac{\partial q_w}{\partial S_w} \right)_t dS_w + \left( \frac{\partial q_w}{\partial t} \right)_{S_w} dt \]  
\text{A.8}

Taking the derivative with respect to length L at a fixed time, t, yields
\[ \left( \frac{\partial q_w}{\partial L} \right)_t = \left( \frac{\partial q_w}{\partial S_w} \right)_t \left( \frac{\partial S_w}{\partial L} \right)_L \]  
\text{A.9}

Similarly, the water saturation, \( S_w \), is a function of both distance and time, so at a constant saturation
\[ dS_w = \left( \frac{\partial S_w}{\partial L} \right)_L dL + \left( \frac{\partial S_w}{\partial t} \right)_L dt = 0 \]  
\text{A.10}

Thus
\[ \left( \frac{\partial S_w}{\partial t} \right)_L = - \left( \frac{\partial S_w}{\partial L} \right)_t \left( \frac{\partial L}{\partial t} \right)_{S_w} \]  
\text{A.11}

Substituting equations A.9 and A.11 in equation A.7 gives
\[ \left( \frac{\partial L}{\partial t} \right)_{S_w} = \frac{1}{A\phi} \left( \frac{\partial q_w}{\partial S_w} \right)_t \]  
\text{A.12}

The fraction of water in the total flowing stream is \( f_w \).
\[ q_w = q_t f_w \]  
\text{A.13}

where \( q_t \) is the total velocity. Differentiating with respect to water saturation, \( S_w \), at constant time, t
\[ \left( \frac{\partial q_w}{\partial S_w} \right)_t = q_t \left( \frac{\partial f_w}{\partial S_w} \right)_t + f_w \left( \frac{\partial q_t}{\partial S_w} \right)_t \]  
\text{A.14}

However, since the fluids are incompressible, the change in total velocity with saturation at any time is zero. Thus
\[ \left( \frac{\partial q_w}{\partial S_w} \right)_t = q_t \left( \frac{\partial f_w}{\partial S_w} \right)_t \]  
\text{A.15}

Substituting equation A.15 in equation A.12 gives
\[ \left( \frac{\partial L}{\partial t} \right)_{S_w} = \frac{q_t}{A\phi} \left( \frac{\partial f_w}{\partial S_w} \right)_t \]  
\text{A.16}

This equation states that for a constant volumetric rate of water injection (\( q_t \)), the velocity of a plane of constant water saturation is directly proportional to the derivative of the fractional flow equation, \( df_w / dS_w \), evaluated for that saturation [100, 101].