Generalized network modelling of two-phase flow

Ali Raeini, Branko Bijeljic and Martin Blunt

Imperial College consortium on pore-scale modelling,
Annual meeting,
November 2016
Outline

- **Background**
  - Experimental relative permeability as an outcome of interactions between forces across scales

- **Generalized network modelling**
  - Network extraction
  - Flow simulation
  - Validation

- **Conclusion and future work**
Micro-/nano-scale variations in the void space

Variations of pore space corners and grain-wall roughness and corner angles

Pore-size variations in mm-scale void-space images

$\Delta x \sim 0.1 \ \mu m$

Beadpack

Sandstone

$\Delta x \sim 1 \ \mu m$

Carbonate

$\Delta x \sim 5 \ \mu m$

Andrew et al (2013)
Capillary heterogeneity at core scale

Krevor et al (2012)
Experimental core-scale relative permeability measurements

Reynolds and Krevor (2015)
Direct simulations are computationally expensive

- Simulations on 1 million cells took ~ 1 week on 24 processors, for a relatively high capillary number of $10^{-5}$
- Simulation time increases as we decrease the capillary number
- Two-phase flow REV-size is about 1 billion grid-blocks
Background:
upscaling from pore-scale to Darcy-scale using network modelling

Network extraction → Flow simulation

Rock image

Conventional network elements

Reservoir simulation


Conventional network elements

Water-wet

Oil-wet
Igor Bondino et al. (Total) (SCA2012):
  • different network extraction algorithms give significantly different results for the same fluid/rock system

Source of the problem:
  Oversimplification of the pore-space

Summary: conductances and volumes may not be assigned correctly to capillary radii and hence causing uncertainty in the flow modelling predictions
There is a one-to-one relationship between any 3D geometry and its medial surface → no information loss → no additional uncertainty due to oversimplification of the pore space.

Generalized pore network can be viewed as a coarse medial-surface representation of the pore space.

Picture from: http://www.agg.ethz.ch/research/medial_axis
Background – generalised network model

Half-throat corners

Medial-axis transformation of the half-throat corners:
Flow simulation: interface tracking + post-processing

There is a one-to-on relationship between capillary radius and interface location for each half throat

→ allowing the interface location to be recorded and tracked as a single scalar variable
Generalized network modelling - summary

- The necessary parameters are extracted directly from the micro-CT image as tabulated functions and used in a generalized network flow simulator:
  - MS-P theory is used to relate $Pc$ to Volume ($Sw$) for each pore/throat
    - The necessary parameters: pore/throat radius and corner angles.
      - extracted directly from the micro-CT image
  - Direct simulation is used to compute throat conductivities
  - Effect of contact angle and other fluid properties is added during flow simulation.
Generalised pore-network extraction
Parametrisation of pore space

Pore-space converted into voxelated (micro-CT) format, on which Navier-Stokes equations are solved: and converted into generalised network format.
Parametrisation of pore space

Step 2.
2.1. Extract ball hierarchy (medial surface),
2.2. find local maxima of the radius and
2.3. Assign a pore index to each of the local maxima
Parametrisation of pore space

Step 3.0. Find pore-to-pore connections (throats)
Step 3.1. Identify and label individual corners on the throat medial axis
Parametrisation of pore space

Step 3.1. Map corner labels to the original image and analyse direct simulation results for each corner at Level 0
Parametrisation of pore space

Step 3.3. More balls from throat centres are deleted from corners to analyse direct simulations at Level 2
Corner conductivities are calculated from tabulated functions extracted from direct single-phase flow simulations.
Comparison - conventional network extraction

Corner conductivities are calculated using correlations, from the shape of the corner.

Conventional network extraction,
Only throat corners are visualised.
Network extraction validation using synthetic geometries

<table>
<thead>
<tr>
<th>cross-sectional shape</th>
<th>Input geometry corner angles, $2\gamma$ (degrees)</th>
<th>Predicted corner angles, $2\gamma$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CNM</td>
<td>GNM</td>
</tr>
<tr>
<td>Star</td>
<td>60</td>
<td>22-33</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>35-43</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>99-119</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>59-62</td>
<td>59-62</td>
</tr>
<tr>
<td></td>
<td>51-65</td>
<td>51-65</td>
</tr>
<tr>
<td>Star</td>
<td>45</td>
<td>23-34</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>34-43</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>97-118</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>42-52</td>
<td>48-53</td>
</tr>
<tr>
<td></td>
<td>48-53</td>
<td>48-53</td>
</tr>
<tr>
<td></td>
<td>48-54</td>
<td>48-54</td>
</tr>
<tr>
<td>Triangle</td>
<td>60</td>
<td>21-25</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>33-42</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>112-122</td>
</tr>
<tr>
<td></td>
<td>53-60</td>
<td>55-57</td>
</tr>
<tr>
<td></td>
<td>45-49</td>
<td>45-49</td>
</tr>
<tr>
<td>Triangle</td>
<td>40</td>
<td>20-30</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>33-45</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>103-116</td>
</tr>
<tr>
<td></td>
<td>66-98</td>
<td>66-98</td>
</tr>
</tbody>
</table>
Network extraction demonstration – Berea sandstone

Element Indices
- Mapped to the original image

A primitive visualization of the network model, colours show different discretization levels of the corner elements
Validation with direct simulations - **star-shaped pore-throat system**

### Star-shaped geometry parameters:
- Corner Angle: 45 degrees
- \( \frac{R_{pore}}{R_{throat}} = 2.25 \)
- \( \frac{R_{pore}}{L_{throat}} = 1.25 \)

### Direct simulation
- Simulations run on 12 processors
- Simulation time:
  - 3-4+ hours for \( \frac{R_{throat}}{\delta x} = 5 \)
  - 2-3+ days for \( \frac{R_{throat}}{\delta x} = 10 \)

### Network model
- Simulation time: **less than 1 second**

Note: network model visualization is primitive, the actual calculations are done based on a fixed 4-segment linear interpolation scheme for each half corner.
Improvements to direct simulations:

- Mesh quality improvements:

- Improved surface tension model using explicit interface reconstruction

Thanks to Mosayeb Shams for sharing his code, Arthur Moncorgé and Stephane Zaleski for their feedback and advice
Post-processing direct simulations

Corner Indices from network extraction are used directly as input control-volumes during direct simulations,

The code that is used for computing relative permeability curves is used to analyse individual corners extracted from rock images.

Each throat (pore-to-pore connection) is divided to 8 half throat corners during network extraction stage.
Layer saturation/conductivities depend on:

- Corner cross-sectional shape
- Variation of the cross-sectional area along the pore-to-pore connection
- Flow pattern / rates
Direct simulations, star-shaped, drainage, piston-like invasion followed by layer flow,
Parameters to be studied: (Compute their Pc-volume-conductivity relationship)

- Contact angle
- Corner angle
- Longitudinal interface curvature
- Different pore geometries

Triangular geometry:

- More Complex test cases to verify cooperate-pore filling algorithm
- Run direct simulations on small, but high resolution micro-CT images
Network extraction applied on the images from the IC pore-scale modelling website, and their statistics.

Corner-angle statistics

Pore/throat radius statistics
Network extraction validation

Distance map distribution:
- **Generalized network model (GNM)**
- **Conventional network model (CNM)**
- **Original image**

![Bentheimer](image)

![Doddington](image)

![Estaillades](image)

![Ketton](image)
Relative permeability predictions – water-wet Bentheimer sandstone (from IC-PSM website, 1000^3 @ 3um) compared to experimental data
Relative permeability predictions – oil-wet
Bentheimer sandstone (from IC-PSM website, 1000³ @ 3um)
Relative permeability predictions – **mix-wet**
Bentheimer sandstone (from IC-PSM website, 1000^3 @ 3um)
## Summary: Errors/uncertainties

### Description of the pore space:

<table>
<thead>
<tr>
<th>Method</th>
<th>Discretization</th>
<th>Resolution</th>
<th>Error</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segmentation / noise</td>
<td>stair-case</td>
<td>high</td>
<td>$O(\delta x)$ to $O(\delta x^2)$</td>
<td>Low to high</td>
</tr>
<tr>
<td>Conventional pore network</td>
<td>stair-case</td>
<td>low</td>
<td>$O(\delta x)$, Refinement not possible</td>
<td>High</td>
</tr>
<tr>
<td>Direct simulations</td>
<td>linear</td>
<td>high</td>
<td>$O(\delta x^2)$</td>
<td>Low to medium</td>
</tr>
<tr>
<td>Generalised networks</td>
<td>linear</td>
<td>medium</td>
<td>$O(\delta x^2)$, Refinement not implemented</td>
<td>Low to medium</td>
</tr>
</tbody>
</table>

### Fluid/rock properties:

- Contact angle distribution
- Clay / micro-porosity identification
Conclusion

➢ The generalised network model computations are comparable in accuracy with the direct simulations and in speed with the network models (a gain of about 6 orders of magnitude in performance)

✔ Corner extraction and connectivity-tracking provides richer computations and predictions and gives us the opportunity to have a pore-by-pore comparison on complex micro-CT images

➢ Further work is needed to quantify the range of uncertainties/variation in experimental measurements and compare them with network model predictions
Acknowledgement

Thanks to Total for providing the financial support

&

Thank you for your attention
Spare slides