Quantification of Permeability Heterogeneity for Reservoir Uncertainty Quantification

Bilal Rashid¹, Ann Muggeridge¹, Glyn Williams²

¹Imperial College London, ²BP
Outline

• Introduction – Impact of heterogeneity on recovery

• Overview of theory
  – Vorticity and shear-strain rate

• Applications
  – Shear-strain rate as a measure of heterogeneity
    • Test against previous measures (Dykstra & Parsons, Schmalz & Rahme, Shook et al.)

• Conclusions
Impact of Heterogeneity on Recovery

Styles of reservoir heterogeneity

Why quantify heterogeneity?

Heterogeneity Measures

Existing heterogeneity indices:

**Static**
- Dykstra-Parsons coefficient
- Lorenz coefficient

**Dynamic**
- Streamline measures
- Koval’s H factor

- Rule of thumb
- Relative
- Tested with geostatistical distributions

Quantify the *impact* of heterogeneity on flow

- Breakthrough time
- Sweep efficiency

*an absolute measure*
Overview of Theory

\[ \mathbf{v}(x, y) = \mathbf{v}(x_{IJ}, y_{IJ}) + \mathbf{J} \cdot dx + \cdots \]

\[ \mathbf{v}(x_{IJ}, y_{IJ}) = \frac{1}{2} \begin{pmatrix} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & 2 \frac{\partial v}{\partial y} & \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} & 0 & \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} & \frac{\partial v}{\partial x} & 0 \end{pmatrix} \]

Rate of Strain Tensor

Vorticity

Two measures?

Vorticity

Shear Rate
Overview of Theory

\[
\mathbf{v}(x, y) = \mathbf{v}(x_{IJ}, y_{IJ}) + \frac{1}{2} \left( \begin{array}{c} 2 \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} + 2 \frac{\partial v}{\partial y} \end{array} \right) + \frac{1}{2} \left( \begin{array}{cc} 0 & \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} & 0 \end{array} \right) \]

Rate of Strain Tensor

\[\propto\text{ Vorticity}\]

Cauchy Stokes Decomposition Theorem: From Mahani et al. (2009)
Overview of Theory

Shear-rate
\[ \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta y} \]

Vorticity
\[ \frac{\Delta v}{\Delta x} - \frac{\Delta u}{\Delta y} \]
Application: Heterogeneity Index

\[ J = \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \]

Propose: The variation in shear is a robust measure of heterogeneity.

\[ c_v(\text{Shear}) = \frac{\text{Standard Deviation}}{\text{Mean}} \]
Application: Heterogeneity Index

Test heterogeneity indices using all 85 layers from SPE 10 Model 2
Miscible – Immiscible
M=1/10/100
Diff. Well Patterns

SPE Model 2 Permeability Map
Heterogeneity Indices

**Static**

Dykstra-Parsons coefficient
(Jensen & Currie 1990)

**Dynamic**

Streamline simulations
Lorenz Coefficient (Shook et al. 2009)
Variation of time of flight distribution
Methodology

Single phase displacement simulation (Finite volume)

Calculate shear-rate field

Calculate $C_V$ of shear-rate

Generate streamlines (tracer flow) 3DSL

Use streamline data to calculate:
- Dynamic Lorenz coefficient $C_V$ of TOF

Compare with normalised breakthrough time for:
- Miscible
- Immiscible
- Line Drive
- Q5 Spot
Results – Base Line

Dykstra-Parsons coefficient

- Layer from SPE Model 2

Breakthrough time (PV1)

Dykstra-Parsons Coefficient

Tarbert

Upper-Ness
Results – Base Line

- Dykstra-Parsons coefficient
  - Dynamic Lorenz coefficient

- Miscible
  - Poor Sensitivity

- Immiscible
Results - TOF

\( C_v(\text{TOF}) \)

**Miscible**

**Immiscible**

\( \frac{\text{Max(\text{TOF})} \text{ - } \text{Min(\text{TOF})}}{\text{Mean(\text{TOF})}} \)
Results - Shear

Shear Heterogeneity Index

**Miscible**

Breakthrough Time (PVI)

\[ C_v(\text{Shear}) \]

Line Drive

**Immiscible**

Breakthrough Time (PVI)

\[ y = 0.65x \]

Recovery at 1 PVI

Shear Heterogeneity Index
Results - Shear

C_v(Shear) Line Drive

Quarter 5 Spot

Miscible

Immiscible
Conclusions

• $C_v(\text{Shear-Strain rate})$ is proportional to breakthrough time & recovery

• Allows:
  – Rapid evaluation of the impact of heterogeneity on breakthrough time
  – Reliable for:
    • Realistic geological models
    • Range of mobility ratios
    • Different well patterns

• May be used to both rank realisations & estimate recovery