The Impact of Porous Media Heterogeneity on Non-Darcy Flow Behaviour

Bagus P. Muljadi ¹  Martin J. Blunt ¹  Ali Q. Raeini ¹  Branko Bijeljic ¹

January 10, 2015

Imperial College Consortium on Pore-Scale Modelling, Imperial College, London, United Kingdom.
Motivation

1. Non-Darcy flow can reduce the gas-well productivity index by 50%.\(^1\)

2. Proppant selection study: Low stress oil well in Lake Maracaibo, Venezuela.\(^2\)

<table>
<thead>
<tr>
<th>Analytical Predictions:</th>
<th>Disregarding Multiphase and Non-Darcy Effects</th>
<th>Includes non-Darcy Effects, Disregards multiphase Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Production Rate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(bopd) 20/40 Sand</td>
<td>280</td>
<td>97</td>
</tr>
<tr>
<td>20/40 RCS</td>
<td>283</td>
<td>104</td>
</tr>
<tr>
<td>20/40 LWC</td>
<td>293</td>
<td>266</td>
</tr>
<tr>
<td><strong>Effective Conductivity</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(md-ft) 20/40 Sand</td>
<td>3193</td>
<td>261</td>
</tr>
<tr>
<td>20/40 RCS</td>
<td>4089</td>
<td>300</td>
</tr>
<tr>
<td>20/40 LWC</td>
<td>8392</td>
<td>931</td>
</tr>
<tr>
<td><strong>Dimensionless Fracture</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conductivity (Fcd)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20/40 Sand</td>
<td>0.53</td>
<td>0.04</td>
</tr>
<tr>
<td>20/40 RCS</td>
<td>0.68</td>
<td>0.05</td>
</tr>
<tr>
<td>20/40 LWC</td>
<td>1.40</td>
<td>0.16</td>
</tr>
<tr>
<td><strong>Cash Flow after 3 years of</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>production ($000, net of proppant cost)</td>
<td>2043</td>
<td>1728</td>
</tr>
<tr>
<td>20/40 Sand</td>
<td>2031</td>
<td>753</td>
</tr>
<tr>
<td>20/40 RCS</td>
<td>2067</td>
<td>1813</td>
</tr>
<tr>
<td>Most Economic Proppant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>for assumed frac geometry</td>
<td>20/40 LWC</td>
<td>20/40 LWC</td>
</tr>
<tr>
<td>Time required to payout</td>
<td></td>
<td></td>
</tr>
<tr>
<td>incremental investment for</td>
<td></td>
<td></td>
</tr>
<tr>
<td>premium proppant</td>
<td>20/40 RCS</td>
<td>never</td>
</tr>
<tr>
<td></td>
<td>20/40 LWC</td>
<td>1.6 years</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; 1 month</td>
</tr>
<tr>
<td>% return on incremental</td>
<td></td>
<td></td>
</tr>
<tr>
<td>investment after three years</td>
<td>20/40 RCS</td>
<td>-40%</td>
</tr>
<tr>
<td></td>
<td>20/40 LWC</td>
<td>59%</td>
</tr>
</tbody>
</table>


\(^2\) Vincent et al., *SPE Western Regional Meeting*, SPE-54630-MS, 1999
Introduction

Darcy equation:

\[-\nabla p = \frac{\mu}{K_D} U,\]  \hfill (1)

Forchheimer equation \(^1\):

\[-\nabla p = \frac{\mu}{K_F} U + \beta \rho U^2,\]  \hfill (2)

\[\rho = \text{pressure}\]  \[U = \text{volumetric velocity}\]

\[K_D = \text{Darcy permeability}\]  \[K_F = \text{Forchheimer permeability}\]

\[\mu = \text{fluid viscosity}\]  \[\beta = \text{non-Darcy coefficient}\]

\[\rho = \text{fluid density}\]

---

\(^1\)Forchheimer, P., Zeit Ver Deutsch Ing, 1901; (45):1781-1788.
3-D images

<table>
<thead>
<tr>
<th>Sample</th>
<th>Resolution $\mu m$</th>
<th>Porosity $\phi$</th>
<th>$L$ $\mu m$</th>
<th>$N_{\text{vox}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>beadpack</td>
<td>2.0</td>
<td>0.359</td>
<td>100</td>
<td>$300 \times 300 \times 300$</td>
</tr>
<tr>
<td>Bentheimer</td>
<td>3.0035</td>
<td>0.211</td>
<td>139.9</td>
<td>$500 \times 500 \times 500$</td>
</tr>
<tr>
<td>Estaillades</td>
<td>3.3113</td>
<td>0.108</td>
<td>253.2</td>
<td>$500 \times 500 \times 500$</td>
</tr>
</tbody>
</table>

Table: Description of the images of beadpack, Bentheimer and Estaillades.

Figure: 2-D cross sections of 3-D gray-scale images of (a) beadpack, (b) Bentheimer, and (c)Estaillades
3-D images

(a) the beadpack

(b) Bentheimer

(c) Estaillades

Figure: Voxelised pore spaces through which the flows are simulated.

---

Flow equation

\[
\frac{\partial \vec{u}}{\partial t} + \nabla \cdot (\vec{u} \vec{u}) - \nabla \cdot (\mu \nabla \vec{u}) = -\nabla p, \quad (3)
\]

\[
\nabla \cdot \vec{u} = 0,
\]

\[
\vec{u} = 0, \text{ on grain boundaries.}
\]

Numerical & physical parameters:

- OpenFOAM solver based on Finite volume method.\(^1\)
- Constant pressure boundary condition at the inlet and the outlet faces.
- No-slip (zero normal and tangential velocity) boundary condition at pore-grain interface.
- Water with viscosity \(\mu = 0.001 \frac{kg}{ms}\) and density \(\rho = 1000 \frac{kg}{m^3}\).
- Intel Xeon E5-2695 2.40Ghz 30MB cache; Each simulation is run in parallel on 16 nodes.

Criteria for Non-Darcy Flow and the $\beta$ Factor

Critical Reynolds numbers:

1. Permeability-based Reynolds number:

$$Re_K = \frac{\rho U \sqrt{K_D}}{\mu},$$

(4)

2. Characteristic length-based Reynolds number:

$$Re_L = \frac{\rho U L}{\mu}.$$  

(5)

Dimensionless permeability

$$K^* = \frac{K_{app}}{K_D}$$

(6)

The onset of non-Darcy flow is the point $K^* = 0.99$.\textsuperscript{1}

Numerical results; departure from Darcy flow

Figure: Pressure gradient (○); Pressure gradient assuming $-\nabla p = \frac{\mu U}{K_D}$ (black line).

(a) the beadpack

(b) Bentheimer

(c) Estaillades
The Onset of Non-Darcy Flow

Figure: The dimensionless permeability $K^*$ as a function of (a) $Re_K$ and (b) $Re_L$ for Estaillades (□), Bentheimer (◇), and the beadpack (○).
Numerical results

Figure: Plots of tortuosities $T = \frac{\sum_i u_i}{\sum_i \{u_f\}_i}$ and apparent permeabilities $K_{app}$ as functions of $Re_L$. □ denotes $K_{app}$; ♦ denotes $T$. 

(a) the beadpack  
(b) Bentheimer  
(c) Estaillades
Flow features - Estaillades

(a) Darcy regime, $Re_K = 3.17 \times 10^{-7}$, $K^* = 1$

(b) transition regime, $Re_K = 3.154 \times 10^{-5}$, $K^* = 0.0995$

(c) Forchheimer regime, $Re_K = 3.275 \times 10^{-4}$, $K^* = 0.94$

Figure: Plots of flow streamlines within Estaillades pores (grey) at selected locations.
Flow features - Bentheimer

(a) Darcy regime

\[(Re_K = 4.45 \times 10^{-6})\]

(b) Forchheimer regime

\[(Re_K = 8.65 \times 10^{-3})\]

Figure: Plots of flow streamlines within Bentheimer pores (grey) at selected locations.
Flow features - the beadpack

(a) Darcy regime

\( (Re_K = 2.23 \times 10^{-3}) \)

(b) Forchheimer regime

\( (Re_K = 2.06 \times 10^{-1}) \)

Figure: Plots of flow streamlines within beadpack pores (grey) at selected locations.
Estimated $\beta$ factors

(a) the beadpack, $\beta = 2.57 \times 10^5$

(b) Bentheimer, $\beta = 2.07 \times 10^6$

(c) Estaillades, $\beta = 6.15 \times 10^8$
The beadpack $\beta$ factor

comparison to experimental data

Ergun equation\(^1\)

\[
\beta_{\text{Ergun}} = \frac{0.142887}{K^{0.5} \phi^{1.5}} = 2.795 \times 10^5 \frac{1}{m}.
\]  

(7)

$K$ is expressed in Darcy, and $\beta$ in $\frac{1}{cm}$.
Our estimated $\beta$ factor for beadpack is $2.57 \times 10^5 \frac{1}{m}$.

---

\(^1\)Ergun, S., *Chemical Engineering Progress*, 1952; 48(2):89-94
Table: Estimated $\beta_{Bentheimer} = 2.07 \times 10^6 \, \text{m}^{-1}$; $\beta_{Estaillades} = 6.15 \times 10^8 \, \text{m}^{-1}$

<table>
<thead>
<tr>
<th>Source</th>
<th>Empirical Model</th>
<th>Units of $\beta$ and $K$</th>
<th>Bentheimer $\beta \times 10^6 (\text{m}^{-1})$</th>
<th>Estaillades $\beta \times 10^8 (\text{m}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Janicek and Katz 1</td>
<td>$\frac{1.82 \times 10^8}{K^{5/4} \phi^{3/4}}$</td>
<td>$\frac{1}{\text{cm}}$, mD</td>
<td>2.09</td>
<td>1.53</td>
</tr>
<tr>
<td>Jones 2</td>
<td>$\frac{6.15 \times 10^{10}}{K^{1.55}}$</td>
<td>$\frac{1}{\text{ft}}$, mD</td>
<td>0.62</td>
<td>0.68</td>
</tr>
<tr>
<td>Cooper et al 3</td>
<td>$\frac{10^{-3.25} T^{1.943}}{K^{1.023}}$</td>
<td>$\frac{1}{\text{cm}}$, mD</td>
<td>5.32</td>
<td>1.83</td>
</tr>
<tr>
<td>Geertsma 4</td>
<td>$\frac{0.005}{K^{0.5} \phi^{5.5}}$</td>
<td>$\frac{1}{\text{cm}}$, cm$^2$</td>
<td>13.71</td>
<td>24.77</td>
</tr>
<tr>
<td>Liu et al 5</td>
<td>$\frac{8.91 \times 10^8 T}{K \phi}$</td>
<td>$\frac{1}{\text{ft}}$, mD</td>
<td>5.85</td>
<td>2.97</td>
</tr>
<tr>
<td>Thäuvin &amp; Mohanty 6</td>
<td>$\frac{1.55 \times 10^4 T^{3.35} \phi^{0.29}}{K^{0.98}}$</td>
<td>$\frac{1}{\text{cm}}$, D</td>
<td>2.84</td>
<td>1.43</td>
</tr>
<tr>
<td>Coles &amp; Hartman 7</td>
<td>$\frac{1.07 \times 10^12 \phi^{0.449}}{K^{1.88}}$</td>
<td>$\frac{1}{\text{ft}}$, mD</td>
<td>0.36</td>
<td>0.79</td>
</tr>
<tr>
<td>Li et al 8</td>
<td>$\frac{11500}{K \phi}$</td>
<td>$\frac{1}{\text{cm}}$, D</td>
<td>1.52</td>
<td>0.61</td>
</tr>
</tbody>
</table>

The onset of non-Darcy flow
comparison to experimental data

<table>
<thead>
<tr>
<th>Source</th>
<th>Criterion</th>
<th>Reported onset</th>
<th>Our predicted onset for:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Re = \frac{\rho D_p U}{\mu}$</td>
<td></td>
<td>beadpack</td>
</tr>
<tr>
<td>Chilton &amp; Colburn¹</td>
<td>$10 - 80$ (packed particles)</td>
<td>2.79</td>
<td>0.196</td>
</tr>
<tr>
<td>Fancher &amp; Lewis²</td>
<td>$0.4 - 3$ (sandstones)</td>
<td>2.79</td>
<td>0.196</td>
</tr>
<tr>
<td>Ergun³</td>
<td>$\frac{\rho D_p U}{\mu(1-\phi)}$</td>
<td>$3 - 10$</td>
<td>4.35</td>
</tr>
<tr>
<td>Bear⁴</td>
<td>$\frac{\rho D_p U}{\mu}$</td>
<td>$3 - 10$</td>
<td>2.79</td>
</tr>
<tr>
<td>Scheidegger⁵</td>
<td>$\frac{\rho D_p U}{\mu}$</td>
<td>$0.1 - 75$</td>
<td>2.79</td>
</tr>
<tr>
<td>Hassanizadeh &amp; Gray⁶</td>
<td>$\frac{\rho D_p U}{\mu}$</td>
<td>$1 - 15$</td>
<td>2.79</td>
</tr>
</tbody>
</table>

³ *Chemical Engineering Progress*, 1952; 48(2):89-94.
Conclusions

1. The critical Reynolds number ReK for Estaillades is two orders of magnitude smaller than for Bentheimer and three orders of magnitude smaller than for the beadpack.

2. Steady eddies in heterogenous Estaillades as a precursor of the non-Darcy flow.

3. In a homogeneous beadpack, the fast flow path characterises the Forchheimer regime, and at much higher Reynolds numbers.

4. The estimated $\beta$ factors and the onset of non-Darcy flow agree with experimental data.

5. X-ray imaging along with direct pore-scale simulation is a viable alternative to experiments and empirical correlations.