Capital requirements, monetary policy and the fundamental problem of bank risk taking

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September 2015

Abstract

We ask what should be the role of limits on bank capital structure and monetary policy, the latter aimed at influencing the required return on bank liabilities, in countering a tendency for banks to take excessive risks because of asymmetric information and limited liability. We find the two policy instruments operate as imperfect substitutes. A tightening of either instrument can improve 'prudence', by disincentivising banks against undertaking lending projects with low probability of success; but only at the cost of decreased 'participation', whereby more banks will forego the opportunity to lend. We show conditions when each instrument should be used.

Keywords: capital requirements, macroprudential policy, monetary policy

JEL Classification: E52, E58, G21, G28

The views in this paper are those of the authors, not of the Bank of England or its Monetary Policy Committee. We would like to thank Douglas Gale, Franklin Allen, and John Moore for helpful discussions.

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1 Introduction

A major policy question for governments, financial regulators and central banks is how best to reduce the probability of a repeat of the financial crisis which hit most developed economies in 2007-2008. Central to this issue is the relative role of macro prudential policy and monetary policy in reducing financial instability, particularly in the banking sector.

In this paper we ask what should be the role of limits on bank capital structure and monetary policy, the latter aimed at influencing the required return on bank liabilities, in countering a tendency for banks to take excessive risks because of asymmetric information and limited liability.We show that banks take too many risks and typically lend too much when they know more about the probability of success of loans than do those who provide debt funding. There are only imperfect tools available to bank regulators and central banks to counter the tendency for banks to take too many risks. One tool is to force banks to use more equity funding than they would choose themselves. Another tool is to raise the cost of debt to banks and counter the tendency to excessive lending by using monetary policy – setting a slightly higher policy rate than would otherwise be appropriate. Both are rather blunt instruments and we show both have costs. But they can help and we show how they might be optimally used.

There is a growing literature on the impact of capital requirements on bank lending; there is also a much larger (and older) literature on the effects of monetary policy on banks. But there is less research on the interaction of monetary policy with bank capital requirements in offsetting inefficient lending - which is the focus of this paper.

The broad consensus in the existing literature is that stricter capital requirements can curb excessive risk-taking and lending (see for instance Furlong and Keeley (1989); and Gersbach and Rochet (2013)); and a tightening of monetary policy could achieve similar effects, albeit potentially at a cost to other macroeconomic considerations (Farhi and Tirole (2009) and De Nicolò et al (2010)). A separate body of work exists on the role of bank capital in the transmission of monetary policy (such as Kashyap and Stein (1994), and Bolton and Freixas (2006))[1].

More recently, Angeloni and Faia (2013) compared the performance of different Taylor Rules under four banking regimes. They concluded that the optimal monetary policy rule should incorporate some response to financial conditions. Angelini, Neri and Panetta (2014) analysed the impact of monetary policy and capital requirements on macroeconomic performance and stability, and discussed the need for cooperative arrangements between the authorities responsible for each instrument.

This paper presents a simple theoretical framework to model the trade-offs involved in using monetary policy and capital requirements to influence bank lending decisions. We analyse what an optimal combination of capital requirements and monetary policy might look like. The model we use is an extension of the framework developed in Bernanke and Gertler's (1990) paper on financial fragility and economic performance. Their model described an economy with two types of risk-neutral agents: entrepreneurs that have access to risky investment projects; and

[1] See Borio and Zhu 2012 for a survey of this literature

households from whom the entrepreneurs must borrow in order to fund their investment project. Entrepreneurs must undertake costly screening before they can find out the probability of success of their project, so some with low endowments (and thus high funding requirements) may choose to forego the lending opportunity even before the screening stage. Those that do undertake screening are incentivised to take excessive risks. This is because entrepreneurs enjoy limited liability and cannot credibly reveal their probability of success to their creditors. The extent to which an individual entrepreneur is prone to excessive risk-taking is also exacerbated by lower initial endowments. Bernanke and Gertler concluded that the dependence of aggregate investment on the initial distribution of endowment introduces 'financial fragility'. A 'financially fragile' economy, defined as one with a sizeable proportion of its entrepreneurs operating around the threshold level of endowment between screening and not screening, may experience a dramatic collapse in investment if subjected to a negative shock to endowments.

We make a number of changes to the Bernanke Gentler (1990) model:

1. *Banks*: First we re-interpret 'entrepreneurs' as 'banks'; 'risky investments' as 'risky bank lending projects'; and 'initial endowment' as 'initial bank capital'. Banks need to screen potential loans and to raise funds from households in order to engage in bank lending. Adapting the Bernanke Gentler model in this way provides us with the basis of a model where aggregate economic outcomes depend on the extent of leverage in banks.

2. *Debt and Equity*: We distinguish between two types of funding contracts: banks in our model can raise funds from households either in the form of debt or in the form of equity. Debt contracts promise to pay the creditor a fixed sum; whereas equity contracts promise a share of the return on the risky lending project (net of any debt obligations). There may be a non-zero pay-off for the risky project in the event it fails. Providers of debt have priority claim over this 'liquidation value' of the risky project.

3. *Policy tools*: The policy maker has two levers with which to ameliorate the effects of the asymmetry of information that combine with limited liability to make market outcomes inefficient: monetary policy and regulatory capital requirements. Monetary policy sets the safe rate paid on the outside option (i.e. a risk-free deposit facility) that is available to both banks and households. So a tightening of monetary policy can be seen as an increase in the risk-free rate, or a decline in the premium offered by risky bank lending. This safe rate is paid by the central bank, and recouped through taxes set by the fiscal authority and required to keep the central bank solvent should it set a higher interest rate. So what we call monetary policy (the setting of a rate of remuneration on deposits at the central bank) has clear fiscal implications. (In fact what we call the monetary policy instrument is equivalent to a decision by the fiscal authority to offer savings accounts to households with a guaranteed interest rate the cost of which are financed out of taxation). Regulatory capital requirements prohibit banks with initial endowments less than the regulatory minimum from proceeding with the risky lending project, unless they raise the additional equity from households. Banks for which the capital regulation is binding will be financed by a mixture of debt and external equity.

4. *Intertemporal optimisation*: For given starting endowment households optimise their consumption profile across periods, in light of their expected rate of return from depositing at the central bank and from providing funding to banks.

We find that in the context of this model monetary policy and prudential capital requirements operate as imperfect substitutes. A tightening of either instruments can improve 'prudence' (by disincentivising the banks against undertaking lending projects with sub-optimally low probability of success); but only at the cost of decreased 'participation' (where decreased 'participation' means that more banks will choose to forego the lending opportunity even before they discover its probability of success through costly screening). The substitutability between the policy instruments, and this trade-off between 'prudence' and 'participation', implies that the optimal level of the central bank policy rate falls as prudential capital requirements are tightened. We find that, across a broad range of calibrations, using capital requirements is more efficient as a way to get the level of risk-taking right in the economy than using monetary policy (interest rates). But there are situations where monetary policy is the more effective instrument. We show how the optimal setting of each policy instrument depends on the probability distribution of returns on bank loans, parameters that determine the desire of households to save, and the initial amount of equity in the banking sector.

2 The Model

The analytical framework we use is a two-period general equilibrium model, at the centre of which is the lending decision of banks. Banks are intermediaries that allow households to finance risky projects. Banks can use external financing to leverage up their funding of risky lending projects. They have private information on the likelihood of success on the projects they undertake, and limited liability in the event of failure. This information asymmetry give rise to moral hazard and the Modigliani-Miller theorem does not hold.

The specific assumptions are as follows:

2.1 Agents

There are two types of risk-neutral agents in the economy: households that consume in both periods; and banks (or rather their owner-managers) which engage in risky lending in the first period and consume only in the second period. We normalise the size of the aggregate population of agents (banks and households) to 1 and denote the proportion of banks and households in the population by μ and $(1-\mu)$ respectively.

Both households and banks have access to a risk-free deposit facility at the central bank. This risk-free deposit facility remunerates any amount deposited at the risk-free rate $(1+r)$. This is a purely real model so we should think of r as a safe real rate offered by the central bank and potentially requiring fiscal backing to make good on the promise. One could equally think of the policy rate as being directly chosen by the fiscal authorities who offer real government debt to households and finance the real interest costs by levying taxes. In this sense our labelling the use of the policy rate r as monetary policy, rather than debt management or fiscal policy, is somewhat semantic.

The risky lending technology that is unique to banks always requires 1 unit of input, and pays out $y_h > 1 + r$ when it succeeds (with probability p), and $y_l < 1 + r$ when it fails (with probability $(1 - p)$). We do not think of the risky lending as literally being on one project. Rather we interpret lending as being on a type of similar loans which have an overall probability of success or failure. Different banks have access to different types of loans. Furthermore, banks have initial endowment $\omega < 1$, so all need to obtain external financing from households in order to lend[2]. In the event that the risky lending fails, banks and any external equity providers are protected by limited liability. Debt providers have priority claim on y_l, the liquidation value of the project.

2.1.1 Banks

A bank is characterised by **two random variables**:

1. The probability of success on its lending projects, p, which is uniformly distributed between $[0, 1]$ with probability density function $h(p)$ and cumulative density function $H(p)$. Each bank draws a realisation of p when it undertakes loans. But a bank can find out its own realisation, p_i, only by incurring a fixed screening cost of $C < 1$. While the distribution of p is common knowledge, we assume there is no way for any bank to credibly convey its realisation of p_i to other agents. By construction, lending is not worthwhile in the absence of screening: $y_l + E[p](y_h - y_l) < 1 + r$. This implies that banks which do not screen will not want to lend.

2. The level of initial endowment for banks, ω, is uniformly distributed between $[\omega_{lb}, \omega_{ub}]$, where $0 \leq \omega_{lb} < \omega_{ub} < 1$. The p.d.f. and c.d.f. of ω are denoted by $f(\omega)$ and $F(\omega)$ respectively. Both the distribution of ω and each bank's realisation of ω_i are publically observed. p and ω are assumed to be independently distributed.

2.1.2 Households

The representative household has an (labour) endowment of value W, with which it needs to finance consumption across two periods in order to maximise the utility function[3]:

$$U = \left[c_t^\rho + \beta \left(E_t\left(c_{t+1}\right)\right)^\rho\right]^{\frac{1}{\rho}} \tag{1}$$

where

[2] One way to see the distinction between banks and households is that only banks have the ability to screen risky projects. This screening technology allows banks to discover the probability of success on the risky project for a fixed cost C<1, before having to commit a full unit of input into the project. By construction, the risky project is not worthwhile in the absence of screening. This ensures households would not be interested in conducting these risky lending activities directly.

[3] This is effectively an Epstein-Zin utility function of the form $U_t = \left[(1-\beta) c_t^\rho + \beta \left(\left[E_t U_{t+1}^\alpha\right]^{1/\alpha}\right)^\rho\right]^{1/\rho}$, where α, the risk aversion parameter, is set to 1 (for risk-neutrality).

- c_s denotes consumption in period s;
- ρ determines the elasticity of intertemporal substitution, $\frac{1}{1-\rho}$; and
- β denotes the rate of time preference.

Any endowment that is not consumed by households in the first period is saved. Savings are transferred to the second period through either the safe deposit facility or through providing funding to banks. We assume that $W \geq \frac{\mu}{1-\mu}(1 - E[\omega])$, such that every bank's lending could be feasibly funded if the household saving rate was high enough.

All parameters of the model, with the only exception of p_i (the probability of success for an individual bank), are assumed to be common knowledge across all agents.

2.2 Timing

The model has two periods. The timing in the first period is sub-divided into three stages (see Table 1):

- In the first stage of period 1, each individual bank draws a realisation of their initial endowments, ω_i, from the common distribution for endowments. The representative household receives it starting wealth W. Given the realisation of ω_i, each bank decides whether or not to undertake costly screening to discover the success rate of their risky lending, p_i, drawing a realisation of the success rate from the common distribution. Banks that do not screen can either provide funding to other banks or deposit the entirety of their endowments in the risk-free facility[4].
- In the second stage, those banks that did screen and discover their p_i, decide whether to proceed with lending. When banks proceed with lending, they commit the entirety of their initial endowment [5] and decide whether to fund the shortfall through debt, additional equity, or a combination of the two. Any external equity injections take place before the bank seeks debt. By assumption, households can observe the capital of banks (composed of a bank's initial endowment plus any subsequent equity injections) but not the probability of their success.
- An equity contract between a household and a bank takes the form $\left\{\frac{\tilde{\omega}}{\omega+\tilde{\omega}}, \tilde{\omega}\right\}$, where $\tilde{\omega}$ is the size of the equity injection, ω is the bank's own endowment, and $\frac{\tilde{\omega}}{\omega+\tilde{\omega}}$ is the share of net return on loans promised to the external equity providers. A debt contract takes the form $\{R(\omega + \tilde{\omega}, r), (1 - \omega - \tilde{\omega})\}$, where $\omega + \tilde{\omega}$ is the total capital of the bank and $(1 - \omega - \tilde{\omega})$ is the amount of debt sought. $R(\omega + \tilde{\omega}, r)$ denotes the gross amount (principal plus interest) that is promised on debt. A bank that funds its shortfall only through debt sets $\tilde{\omega} = 0$; whereas a bank that uses only equity sets $\tilde{\omega} = 1 - \omega$. All intermediate cases are permitted.

[4] For simplicity, we assume that banks choose to fund other banks when they are indifferent between that and using the safe deposit facility.

[5] Banks would only proceed with the project if its expected return exceeds the certain return from the deposit facility (recall that all banks are assumed to be risk neutral). Therefore, banks would prefer to commit the entirety of their endowment rather than to place a portion in the safe deposit facility.

Table 1: Timing Assumptions

Period.Stage	1.1	1.2	1.3	2
Household	Receives W	Discover $E_t\left(1+r_{hh}\right)$, decides how much to save/consume	Save/Consume	Consume
Bank	Receives ω_i, decides whether to screen	Discover p_i, decides whether to lend/invest	Lend/Invest	Consume

- The representative household works out the expected rate of return on its saving: $E_t\left(1+r_{hh}\right)$, and decides on how much to consume (and save) in the first period. All household saving/consumption and all bank lending/investment then takes place.

In the second period, the returns from bank lending and the safe-deposit facility are realised. The proceeds from bank lending are divided according to any contracts agreed in the first period (either equity or debt contracts). Both banks and households consume the entirety of their resources.

2.3 Mechanics

Two key thresholds drive the mechanics of the model:

1. The threshold level of a bank's initial endowment, $\hat{\omega}$, at which point it is indifferent between screening and simply depositing its endowments in stage 1 to earn the safe rate; and

2. The threshold level of the success probability of lending, $\hat{p}$, at which point a bank in stage 2 is indifferent between proceeding after screening and depositing at the safe-rate .

The level of bank endowment matters for the screening decision because a lower initial endowment implies a larger funding gap and higher cost of funding. A bank with initial endowment less than $\hat{\omega}$ would find that the funding cost, plus the fixed cost of screening, exceed the expected return from undertaking risky lending. The endowment threshold $\hat{\omega}$ is defined formally in Section 3.2**.**

The realisation of a bank's success probability p_i determines whether it proceeds with the project after screening. A bank with $p_i < \hat{p}$ finds that it can achieve a higher expected return by funding other banks or using the risk-free deposit facility, so will forego the lending opportunity. We define $\hat{p}$ more formally in Section 3.1.

For the rest of the paper, we will refer to $\hat{\omega}$ as the '**participation threshold**' and $\hat{p}$ as the '**prudence threshold**'. $\hat{\omega}$ is described as the 'participation threshold' because banks with initial endowment below this level do not 'participate' in bank lending. $\hat{p}$ is described as the 'prudence threshold' because the success rate banks are willing to accept is an indicator of how prudent they are in handling funds from households. Imprudent banks will tend to take excessive risks to take advantage of their private information and limited liability.

Note that not all banks will lend, and not all household endowment will be saved. So there can be a mismatch in the aggregate demand and aggregate supply of funding for banks. When the supply of funding exceeds banks' demand for them, any surplus is placed in the safe deposit facility in the central bank. When banks' demand for funds exceed the supply from households, banks bid up the expected return on deposits until demand equals supply. We denote the excess of the expected return on deposits over the safe rate by δ_r. If $\delta_r > 0$, no households use the safe saving facility offered by the central bank and the expected rate of return on deposits (debt contracts) with commercial banks is given by $r + \delta_r$. In general we say that the **market for funds** clears when there are no excess demand for funds.

3 The First-Best - the model under perfect information

Under the first-best, households can observe a bank's probability of success, p_i, as well as its level of capitalisation $(\omega_i + \tilde{\omega}_i)$.

3.1 The Prudence Threshold - First-Best Contracts in the Banking Sector

Working backwards, we will start by examining the prudence threshold (at Stage 2 of the game) for those banks that have discovered their individual realisation of p_i through screening. The prudence threshold for banks, $\hat{p}$, is defined as the success rate required to make a bank indifferent between undertaking the risky lending project and simply depositing its endowments with other banks.

Specifically, for a bank with initial endowment ω_i, and which is looking for $\tilde{\omega}_i$ in additional equity and $1-(\omega_i + \tilde{\omega}_i)$ in debt from households, its prudence threshold $\hat{p}_i = \hat{p}(\omega_i + \tilde{\omega}_i)$ and the terms of its funding contract are jointly determined by the following three equations[6]:

1. The condition where the bank is indifferent between lending and depositing:

$$\begin{aligned} \frac{\omega}{\omega + \tilde{\omega}} \left(\hat{p}\left(y_h - R\right) + (1-\hat{p}) \max\left[0, y_l - R\right]\right) &= \omega(1 + r + \delta_r);\ \text{or} \\ \hat{p}\left(y_h - R\right) + (1-\hat{p}) \max\left[0, y_l - R\right] &= \left(\omega + \tilde{\omega}\right)\left(1 + r + \delta_r\right) \end{aligned} \tag{2}$$

   where $\tilde{\omega}$ is the size of the equity injection from households; $\omega + \tilde{\omega}$ is the total capital of the bank; and $\frac{\omega}{\omega+\tilde{\omega}}$ is the share of the return retained by the bank. A bank with capital of $\omega + \tilde{\omega}$ needs to finance the remainder of the project $(1 - \omega - \tilde{\omega})$ through debt. $R(\omega + \tilde{\omega}, p)$ denotes the gross amount (principal plus interest) that is promised on that debt. $\delta_r \geq 0$ adjusts to ensure there can never be an excess demand for funds (more details in section 3.3).

[6] In the interest of brevity, we suppress the i subscript henceforth where possible.

2. The condition where the households are receiving, in expectation, their required return from providing debt funding to a bank:
$$pR + (1-p)\min[y_l, R] = (1-(\omega+\tilde{\omega}))(1+r+\delta_r) \tag{3}$$

3. The condition where for the households providing additional equity to the bank, their expected return from the capital injection is at least equal to the expected return on debt/deposits:

$$\begin{aligned}\frac{\tilde{\omega}}{\omega+\tilde{\omega}}\left(p\left(y_h-R\right)+(1-p)\max\left[0,y_l-R\right]\right) &\geq \tilde{\omega}(1+r+\delta_r)\\ \text{or } p\left(y_h-R\right)+(1-p)\max\left[0,y_l-R\right] &\geq (\omega+\tilde{\omega})(1+r+\delta_r)\end{aligned} \tag{4}$$

This last condition is always satisfied given equation 2, and the fact that banks will only proceed with $p \geq \hat{p}_{fb}$, where $\hat{p}_{fb}$ is the first-best level of prudence that reflects risk neutrality of all agents.

Proposition 1. *When p is common knowledge between all agents, banks will not take excessive risks, regardless of the level of their capitalisation.*

$$\hat{p}(\omega_i+\tilde{\omega}_i) = \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)} \equiv \hat{p}_{fb} \text{ for } \forall(\omega_i+\tilde{\omega}_i) \tag{5}$$

Proof. The intuition of the proof is simple. Creditors to the bank can perfectly observe the risks the bank is taking, so they can structure their contracts such that there are no incentives for the bank to engage in excessive risk-taking. More formally:

Let $\omega^*(p)$ denote the level of capital $(\omega+\tilde{\omega})$ such that $y_l = R(p,\omega^*)$, so for a bank with $\{p,\omega^*(p)\}$, we can re-arrange equation 3 and 2 to give:

- $\omega^*(p) = \omega^* = 1 - \frac{y_l}{(1+r+\delta_r)}$ [so ω^* is independent of p]; and
- $R(p,\omega^*) = R(\omega^*) = (1-\omega^*)(1+r+\delta_r)$ [so $R(\omega^*)$ is independent of p]; and
- $\hat{p}(\omega^*) = \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)} = \hat{p}_{fb}$

We will refer to any bank with $\omega+\tilde{\omega} \geq \omega^*$ as a 'fully capitalised' bank, because such a bank can pay its debt in full - even in the bad state[7].

Consider the case where $\min[y_l, R(p,\omega+\tilde{\omega})] = R(p,\omega+\tilde{\omega})$ (the debt obligation is less than the liquidation value), re-arranging equations 2 and 3 gives:

$$\begin{aligned} R(p,\omega+\tilde{\omega} \mid y_l > R) &= (1-(\omega+\tilde{\omega}))(1+r+\delta_r)\\ \hat{p}(\omega_i+\tilde{\omega}_i \mid y_l > R) &= \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}\end{aligned}$$

[7] These banks can still 'fail' in the sense that their equity holders may be wiped out in the bad state.

Consider the case where $\min\left[y_l, R\left(p, \omega+\tilde{\omega}\right)\right] = y_l$ (the debt obligation is greater than the liquidation value):

So from equation 3 we have $pR + (1-p)\, y_l = (1-(\omega+\tilde{\omega}))\,(1+r+\delta_r)$, or

$$R\left(p, \omega+\tilde{\omega} \mid y_l < R\right) = \frac{(1-(\omega+\tilde{\omega}))\,(1+r+\delta_r) - (1-p)\, y_l}{p}$$

Substitute $R\left(\hat{p}, \omega+\tilde{\omega} \mid y_l < R\right)$ into equation 2 to give $\hat{p}\left(y_h - \frac{(1-(\omega+\tilde{\omega}))(1+r+\delta_r)-(1-\hat{p})y_l}{\hat{p}}\right) = (\omega+\tilde{\omega})\,(1+r+\delta_r)$, which simplifies to

$$\hat{p}\left(\omega+\tilde{\omega} \mid y_l < R\right) = \frac{(1+r+\delta_r) - y_l}{(y_h - y_l)}$$

Since it is easy to see that R is a decreasing function of $(\omega+\tilde{\omega})$, from the definition of ω^* we can conclude that $y_l < R$ whenever $\omega+\tilde{\omega} < \omega^*$, and $y_l > R$ whenever $\omega+\tilde{\omega} > \omega^*$.

So $\hat{p}\left((\omega+\tilde{\omega}) > \omega^*\right) = \hat{p}\left((\omega+\tilde{\omega}) < \omega^*\right) = \hat{p}\left(\omega^*\right) = \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}$. We define this first-best level of prudence as $\hat{p}_{fb} \equiv \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}$. □

Proposition 2. *After banks screen in the first-best:*

1. banks invest in the risky lending project if and only if $p \geq \hat{p}(\omega+\tilde{\omega}, r)$; and

2. 'fully-capitalised' banks weakly prefer debt funding to equity funding. 'Not-fully-capitalised' banks are indifferent between using all debt, or using a mixture of debt and additional equity (provided that the additional equity injection does not make the bank 'fully-capitalised').

   For simplicity, we assume that when banks are indifferent between debt and equity, they will proceed with the project using debt finance only.

Proof. Part (1) follows from the definition of $\hat{p}(\omega+\tilde{\omega})$: banks with success probability $p < \hat{p}(\omega+\tilde{\omega}, r)$ would receive a higher expected return from depositing their endowment with other banks than from lending.

Part (2) For 'fully-capitalised' banks ($\omega+\tilde{\omega} > \omega^*$), debt can be obtained at the risk-free rate [see the proof for proposition 1], but equity comes at the cost of sharing the proportion of the expected return - and that expected return exceeds the expected return on debt. We know that a bank will proceed with the lending project if and only if the expected return is weakly greater than the safe-rate of return (from part (1) and equation 2). Thus equity is more expensive than debt for a 'fully-capitalised' bank.

For 'not-fully capitalised' banks, households with perfect information effectively set debt contracts as a function of p and ω. Additional equity injections lowers the cost on the bank's entire stock of debt, so equity becomes just as attractive as debt [the Modigliani-Miller Theorem holds]. We offer a more formal proof of this proposition in the Annex. □

Corollary 1. *In the absence of capital regulations, banks will not seek external equity injections when they undertake risky lending.*

3.2 The Participation Threshold - First Best Case

Having established the 'prudence threshold' of banks, we continue to work backwards to find banks' 'participation threshold'. Upon discovering its realisation of ω_i at Stage 1 of the game, a bank decides whether to engage in costly screening. The 'participation threshold' of banks is defined as the level of initial endowment that equates the expected value of screening (V) to the fixed cost of screening (C). The costly screening process can be interpreted as buying an option in the risky lending opportunity. The value of the option will depend on the funding structure of the bank, in particular how much additional equity it is trying to attract from households.

Formally, V is given by:

$$\begin{aligned} & V(\omega, \tilde{\omega}, r+\delta_r) \\ & \equiv E_p\left[\max\left\{0\ ,\ \frac{\omega}{\omega+\tilde{\omega}}\begin{pmatrix} p\left(y_h-R\right)+ \\ (1-p)\max\left[0, y_l-R\right]\end{pmatrix} -\ \omega(1+r+\delta_r)\right\}\right] \\ & = \frac{\omega}{\omega+\tilde{\omega}}\int_{\hat{p}}^{1}\left[p\left(y_h-y_l\right)+y_l-(1+r+\delta_r)\right]h(p)dp \end{aligned} \tag{6}$$

where the last line follows because the debt is provided by households at an expected return of $1+r+\delta_r$ (more detail can be found in the annex).

Definition. The participation threshold for banks, $\hat{\omega}$, is implicitly defined by:

$$V(\hat{\omega}, \tilde{\omega}, r+\delta_r) = \frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}}\int_{\hat{p}}^{1}\left[p\left(y_h-y_l\right)+y_l-(1+r+\delta_r)\right]h(p)dp = C \tag{7}$$

where C is the fixed cost of screening.

We know that in the absence of any capital requirements ($\omega_{reg}=0$), $\tilde{\omega}=0$; and in the first-best $\hat{p}(\omega+\tilde{\omega}) = \hat{p}_{fb} = \frac{1+r+\delta_r-y_l}{(y_h-y_l)}$, so

$$V_{fb}(\omega_{reg}=0, r+\delta_r) = \int_{\hat{p}_{fb}}^{1}\left[p\left(y_h-y_l\right)+y_l-(1+r+\delta_r)\right]h(p)dp \tag{8}$$

We impose by assumption that: $\int_{\hat{p}_{fb}}^{1}\left[p\left(y_h-y_l\right)+y_l-(1+r+\delta_r)\right]h(p)dp > C$, such that

$$\hat{\omega}_{fb}\left(\omega_{reg}=0, r+\delta_r\right) = 0 \tag{9}$$

In the first-best, with no asymmetry of information between households and banks, every bank screens in the absence of any policy intervention.

3.3 Market for Funds

Having analysed the prudence and the participation thresholds ($\hat{p}$ and $\hat{\omega}$), we can now write banks' aggregate demand for funds as:

$$\mu \int_{\hat{\omega}}^{\omega_{ub}} (1-\omega)(1-H(\hat{p}))\, dF(\omega)$$

The aggregate supply of funds comes from three sources: households; banks that did not screen; and banks that did screen but discovered a p which is less than their prudence threshold:

$$(1-\mu) S^* + \mu \int_{\omega_{lb}}^{\hat{\omega}} \omega dF(\omega) + \mu \int_{\hat{\omega}}^{\omega_{ub}} \omega H(\hat{p})\, dF(\omega)$$

where S^* is the savings decision made by the optimising representative household.

Banks' net demand for funds from households is therefore $\mu \int_{\hat{\omega}}^{\omega_{ub}} [1-H(\hat{p})]\, dF(\omega) - \mu E[\omega]$. When the supply of funding exceeds banks' demand for them, any surplus is placed in the safe deposit facility in the central bank. When banks' demand for funds exceed the supply from households, δ_r (the excess expected return on bank deposits over the safe rate r) adjusts upwards until demand equals supply. Therefore:

- if $\frac{\mu}{1-\mu}\left[\int_{\hat{\omega}(\delta_r=0)}^{\omega_{ub}} [1-H(\hat{p}(\delta_r=0))]\, dF(\omega) - E[\omega]\right] \le S^*(\delta_r=0)$, then $\delta_r = 0$;
- if $\frac{\mu}{1-\mu}\left[\int_{\hat{\omega}(\delta_r=0)}^{\omega_{ub}} [1-H(\hat{p}(\delta_r=0))]\, dF(\omega) - E[\omega]\right] > S^*(\delta_r=0)$, then δ_r is given by the implicit equation: $\frac{\mu}{1-\mu}\left[\int_{\hat{\omega}(\delta_r)}^{\omega_{ub}} [1-H(\hat{p}(\delta_r))]\, dF(\omega) - E[\omega]\right] = S^*(\delta_r)$

In general, therefore, we say that the **market for funds** clears when there are no excess net demand for funds, or when

$$D \equiv \frac{\mu}{1-\mu}\left[\int_{\hat{\omega}}^{\omega_{ub}} [1-H(\hat{p})]\, dF(\omega) - E[\omega]\right] \le S^* \tag{10}$$

3.4 Household Optimisation

We assume that the representative household can calculate the expected rate of return on its savings. Specifically, the **households' expected return on savings**, $E_t(1+r_{hh})$, is a weighted average of the expected return from the financial products available to it (the safe deposit facility at the central bank, as well as debt and equity contracts with banks). Note that in equilibrium the expected return on deposits is $(1+r+\delta_r)$ with $\delta_r > 0$ only if banks' demand for household deposits at $\delta_r = 0$ exceeds what households want to hold in debt, so that banks offer a higher expected return than the safe rate available at the central bank. Households may also have the opportunity to hold bank equity, so $\tilde{\omega}$ could be positive. We assume that if banks offer equity to households, which they will only do if forced by regulation, the equity is offered equally to all households. Effectively this is a rationing scheme, which is necessary because equity offers an excess expected return over debt. Thus:

$$
\begin{aligned}
E_t\left(1+r_{hh}\right) &= \left(1+r+\delta_r\right)+\left[\frac{\mu}{(1-\mu)\,S}\right]\int_{\hat{\omega}}^{\omega_{ub}}\left(1-H\left(\hat{p}\right)\right)\frac{\tilde{\omega}}{\omega+\tilde{\omega}}Z\left(\hat{p}\right)dF\left(\omega\right) \\
\text{where } Z\left(\hat{p}\right) &= \left[y_l+A\left(\hat{p}\right)\left(y_h-y_l\right)-\left(1+r+\delta_r\right)\right] \\
\text{and } A\left(\hat{p}\right) &\equiv E\left[p|p>\hat{p}\right]=\frac{1}{1-H\left(\hat{p}\right)}\int_{\hat{p}}^{1}ph\left(p\right)dp
\end{aligned}
\tag{11}
$$

Note that when $\tilde{\omega}=0$ for all banks (from corollary 1: no banks would offer equity contracts in the absence of capital requirements), then $E_t\left(1+r_{hh}\right)=\left(1+r+\delta_r\right)$.

The representative household's optimisation problem can be characterised as:

$$
\begin{aligned}
\max_S U_t &= \left[c_t^{\rho}+\beta\left(E_t\left[c_{t+1}\right]\right)^{\rho}\right]^{1/\rho} \\
\text{subject to} &: \quad c_t=W-S;\text{ and} \\
E_t\left[c_{t+1}\right] &= E_t\left(1+r_{hh}\right)\times S \\
&= \left(1+r+\delta_r\right)S+\frac{\mu}{(1-\mu)}\int_{\hat{\omega}}^{\omega_{ub}}\left(1-H\left(\hat{p}\right)\right)\frac{\tilde{\omega}}{\omega+\tilde{\omega}}Z\left(\hat{p}\right)dF\left(\omega\right)
\end{aligned}
\tag{12}
$$

Differentiate w.r.t. S to give the familiar Euler equation:

$$
\frac{E_t\left[c_{t+1}^{*}\right]}{c_t^{*}}=\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}
\tag{13}
$$

And re-arrange to find an explicit solution for the optimal saving decision for households:

$$
\begin{aligned}
S^{*} &= \frac{\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}}{\left(1+r+\delta_r\right)+\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}}\left[W-\frac{transfer}{\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}}\right] \\
\text{where } transfer &= \frac{\mu}{(1-\mu)}\int_{\hat{\omega}}^{\omega_{ub}}\left(1-H\left(\hat{p}\right)\right)\frac{\tilde{\omega}}{\omega+\tilde{\omega}}Z\left(\hat{p}\right)dF\left(\omega\right)
\end{aligned}
\tag{14}
$$

Transfer is the expected excess return on household saving that is invested in bank equity.

Having characterised the above first-order conditions for the optimal household decision, we can express the **value function for households** (the maximum level of utility achievable for given starting endowment W) as[8]:

[8]Derivation:
Substituting the FOC $\frac{E_t\left[c_{t+1}^{*}\right]}{c_t^{*}}=\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}$ into the utility function gives

$U_h^{*}=\left[\left(W-S^{*}\right)^{\rho}+\beta\left(\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}\left(W-S^{*}\right)\right)^{\rho}\right]^{\frac{1}{\rho}}$ (* superscript suppressed henceforth for simplicity)

or: $U_t=\left[\frac{\left(\left(1+r+\delta_r\right)+\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}\right)}{\left(1+r+\delta_r\right)}\left(W-S\right)^{\rho}\right]^{\frac{1}{\rho}}$.

$$\begin{aligned} U_h^* &= \alpha\left(\left(1+r+\delta_r\right)W + transfer\right) \\ \text{where } \alpha &= \left(\frac{\left(\left(1+r+\delta_r\right)+\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}\right)}{\left(1+r+\delta_r\right)}\right)^{\frac{1}{\rho}}\left(\frac{1}{\left(1+r+\delta_r\right)+\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}}\right); \end{aligned}$$

3.5 Banks' optimisation

To maximise their expected profits, banks proceeds with any project with $p \geq \hat{p}$. Therefore aggregate profits in the banking sector (normalised by the relative proportion of bankers to households, $\frac{\mu}{(1-\mu)}$) is given by:

$$\begin{aligned} \Pi_t &= \frac{\mu}{(1-\mu)}\int_{\hat{\omega}}^{\omega_{ub}}\left[1-H\left(\hat{p}\right)\right]\left[Z\left(\hat{p}\right)\right]dF\left(\omega\right)+\frac{\mu}{(1-\mu)}E\left(\omega\right)\left(1+r+\delta_r\right) \\ &\quad -transfer-\frac{\mu}{(1-\mu)}C\int_{\hat{\omega}}^{\omega_{ub}}dF\left(\omega\right) \\ \text{where } Z\left(\hat{p}\right) &= \left[y_l+A\left(\hat{p}\right)\left(y_h-y_l\right)-\left(1+r+\delta_r\right)\right] \\ transfer &= \frac{\mu}{(1-\mu)}\int_{\hat{\omega}}^{\omega_{ub}}\left(1-H\left(\hat{p}\right)\right)\frac{\tilde{\omega}}{\omega+\tilde{\omega}}Z\left(\hat{p}\right)dF\left(\omega\right) \end{aligned} \tag{15}$$

3.6 Aggregate Welfare

We assume that the policymaker cares about the sum of household and bank utility.

Recall that the value function for households is given by the form:

$$U_h^* = \alpha\left[\left(1+r+\delta_r\right)W + transfer\right]$$

To translate banking profits into the utility space, we assume the value function for banks takes an identical form[9]: $U_b^* = \alpha\left(\Pi_t\right)$,

such that social welfare, the sum of the U_h^* and U_b^*, can be expressed as:

substitute in $S^* = \frac{\left[\beta(1+r+\delta_r)\right]^{\frac{1}{1-\rho}}}{(1+r+\delta_r)+\left[\beta(1+r+\delta_r)\right]^{\frac{1}{1-\rho}}}\left[W-\frac{transfer}{\left[\beta(1+r+\delta_r)\right]^{\frac{1}{1-\rho}}}\right]$ and simplify to yield:

$$U_t = \left(\frac{\left((1+r+\delta_r)+\left[\beta(1+r+\delta_r)\right]^{\frac{1}{1-\rho}}\right)}{(1+r+\delta_r)}\right)^{\frac{1}{\rho}}\left(\frac{1}{(1+r+\delta_r)+\left[\beta(1+r+\delta_r)\right]^{\frac{1}{1-\rho}}}\right)\left(\left(1+r+\delta_r\right)W+transfer\right)$$

[9] So the utility generated by the owners of banks from consuming the value of their investments is treated in the same way as other households consuming their endowment.

$$\begin{aligned}
welfare &= U_h^* + U_b^* \\
&= \alpha \left[\begin{array}{c} (1+r+\delta_r)\left(W + \frac{\mu}{(1-\mu)} E(\omega)\right) + \frac{\mu}{(1-\mu)} \int_{\hat{\omega}}^{\omega_{ub}} [1 - H(\hat{p})] [Z(\hat{p})] \, dF(\omega) \\ - \frac{\mu}{(1-\mu)} C \int_{\hat{\omega}}^{\omega_{ub}} dF(\omega) \end{array} \right] \\
\text{where } \alpha &= \left(\frac{\left((1+r+\delta_r) + [\beta(1+r+\delta_r)]^{\frac{1}{1-\rho}}\right)}{(1+r+\delta_r)} \right)^{\frac{1}{\rho}} \left(\frac{1}{(1+r+\delta_r) + [\beta(1+r+\delta_r)]^{\frac{1}{1-\rho}}} \right); \text{ and} \\
Z(\hat{p}) &= [y_l + A(\hat{p})(y_h - y_l) - (1+r+\delta_r)] \qquad (16)
\end{aligned}$$

α (a decreasing function of r) captures the intertemporal substitution elements of the utility function.

3.7 Summary

In the first-best households can observe individual bank's probability of success (p_i). This is sufficient to ensure that banks operate in the most appropriate and prudent manner, regardless of the level of their capitalisation/leverage. Specifically, every bank sets the prudence threshold at $\hat{p}_{fb} = \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}$, the level which ensures that every lending project is expected to deliver at least the risk-free return. In addition, with the participation threshold ($\hat{\omega}$) equal to zero, all banks screen so no profitable lending opportunities are wasted. In such a state, there is no need for any policy intervention.

4 The model with asymmetry of information

Frictions in the model arise because of the interaction between asymmetric information and limited liability. When households cannot observe banks' probability of success (p_i), moral hazard becomes prevalent in the banking sector. This problem manifests itself in a sub-optimal level of screening for the risky project; and for those banks that do screen, a lower level of prudence in deciding whether to undertake the project.

Analytically, the structure of the model remains unchanged under asymmetric information (the majority of the equations are identical). The main distinction is that $\hat{p}$ becomes a function of ω, and therefore can no longer be evaluated outside of the integral over ω.

4.1 The Prudence Threshold

As with the first-best, we start by examining the prudence threshold (at Stage 2 of the game) for those banks that have discovered their individual realisation of p_i through screening. Under asymmetric information, the analogues of equations 2-5 are:

1. The condition that a bank with initial capital ω and additional equity injection $\tilde{\omega}$ is indifferent between lending and depositing:

$$\frac{\omega}{\omega+\tilde{\omega}}\left\{\hat{p}\left(y_h-R\right)+\left(1-\hat{p}\right)\max\left[0,y_l-R\right]\right\}=\omega(1+r+\delta_r)$$

or:

$$\hat{p}\left(y_h-R\right)+\left(1-\hat{p}\right)\max\left[0,y_l-R\right]=\left(\omega+\tilde{\omega}\right)\left(1+r+\delta_r\right) \tag{17}$$

this is identical to equation 2.

2. The condition that the households are indifferent between providing debt and depositing:

$$A\left(\hat{p}\right)R+\left(1-A\left(\hat{p}\right)\right)\left(\min\left[y_l,R\right]\right)=\left(1-\omega-\tilde{\omega}\right)\left(1+r+\delta_r\right) \tag{18}$$

where $A\left(\hat{p}\right)\equiv E[p\mid p>\hat{p}]\equiv\frac{1}{1-H(\hat{p})}\int_{\hat{p}}^{1}ph\left(p\right)dp$ denotes the conditional expectation of p given that $p\geq\hat{p}$. Note that the key difference here (relative to equation 3) is that instead of observing the p directly, households now need to form a conditional expectation of p.

3. The condition that for households providing additional equity to the bank their expected return from the capital injection is at least equal to the safe rate of return:

$$\frac{\tilde{\omega}}{\omega+\tilde{\omega}}\left\{A\left(\hat{p}\right)\left(y_h-R\right)+\left(1-A\left(\hat{p}\right)\right)\left(\max\left[0,y_l-R\right]\right)\right\}\geq\tilde{\omega}\left(1+r+\delta_r\right)$$

or:

$$A\left(\hat{p}\right)\left(y_h-R\right)+\left(1-A\left(\hat{p}\right)\right)\left(\max\left[0,y_l-R\right]\right)\geq\left(\omega+\tilde{\omega}\right)\left(1+r+\delta_r\right) \tag{19}$$

This last condition is always satisfied given equation 17 and the fact that $A\left(\hat{p}\right)\geq\hat{p}$.

Proposition 3. *With asymmetric information, banks that are not 'fully capitalised' will take excessive risks:*

1. As before, let $\omega^*=\omega+\tilde{\omega}$ denote the level of capital such that $R\left(\omega^*\right)=y_l$, so $\omega^*=1-\frac{y_l}{(1+r+\delta_r)}$. We refer to any bank with $\omega_i+\tilde{\omega}_i\geq\omega^*$ as a "fully capitalised" bank.

2. Banks become more prudent as they increase their capital, up to a cap when they become fully capitalised.

Case 1. For $\omega+\tilde{\omega}\geq\omega^*$, we have $\hat{p}=\frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}$; $R=\left(1-\left(\omega+\tilde{\omega}\right)\right)\left(1+r+\delta_r\right)$;

Case 2. For $\omega+\tilde{\omega}<\omega^*$, we have $\frac{\partial\hat{p}}{\partial\omega}=\frac{(A(\hat{p})-\hat{p})(1+r+\delta_r)}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)}$; and $\hat{p}\leq\frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}=\hat{p}_{fb}$.

The function for $\hat{p}(\omega + \tilde{\omega})$ kinks at ω^*, with right-hand derivative given by $\partial_+ \hat{p}(\omega^*) = 0$ and left-hand derivative given by $\partial_- \hat{p}(\omega^*) = \frac{(A(\hat{p}) - \hat{p})(1+r+\delta_r)}{(y_h - R)A(\hat{p}) + \hat{p}A'(\hat{p})(R - y_l)} > 0$.

An increase in initial endowment has the same marginal impact on prudence as an increase in external equity. Banks lending behaviour is influenced by the size of their equity rather than its source.

Proof. We give an outline here. A full proof is in the annex.

In the first part of the proposition, ω^* is defined as the level of endowments such that $y_l = R(\omega^*, r)$, so $\omega^* = 1 - \frac{y_l}{(1+r+\delta_r)}$ follows directly from re-arranging equation 18. Banks with capital above this level can borrow at the risk free rate because the liquidation value of the lending project is sufficient to cover the bank's debt obligations (again from re-arranging equation 18 after setting $R(\omega^*) = y_l$).

The second part of the proposition says that banks that are not fully capitalised will take excessive risks. Limited liability encourages banks to take excessive risks when the liquidation value of the project is less than the bank's debt obligations. Households can anticipate this, so will increase the cost of debt $\frac{R(\omega+\tilde{\omega},r)}{(1-\omega-\tilde{\omega})}$ for banks with small capital (and thus high funding needs). The fact that banks cannot credibly communicate their success rate (p_i) to households exacerbates this issue as the debt contract can only be formulated as a function of the bank's observable capital $(\omega+\tilde{\omega})$. This means two banks with the same level of capital will face the same borrowing costs regardless of the success probabilities on their projects - the bank with the better project is effectively subsidising the borrowing costs of the one with the poorer project. Taken together, limited liability and this cross-subsidisation towards less attractive projects give rise to moral hazard in bank lending. The fact that prudence is an increasing function of capital reflects the observation that the moral hazard issue becomes less pronounced when banks have more 'skin in the game'. □

Proposition 4. *Preference for Debt*

After screening banks invest in the risky lending project if and only if $p \geq \hat{p}(\omega + \tilde{\omega}, r + \delta_r)$; and

when banks undertake the project they prefer to use as much debt finance as possible (i.e. banks will choose to set $\tilde{\omega} = 0$) .

Proof. Part (1) follows from the definition of $\hat{p}(\omega + \tilde{\omega})$: banks with success probability $p < \hat{p}(\omega + \tilde{\omega})$ would receive a higher expected return from depositing their endowment with other banks than from lending with debt finance.

Part (2) holds because banks expect to pay out a higher rate of return to external equity providers than to debt providers. We show this formally in the annex. Qualitatively, equity is more costly in this model because debt is available at a rate which makes households indifferent (in expectation) between providing the debt and using the safe deposit facility. Equity, on the other hand, allows the household to share the surplus from bank lending, and therefore delivers a rate of return for households that is greater than the risk-free facility. The original owners of bank equity have the value of their claims diluted by raising new equity from outsiders. Consequently banks stick with debt in the absence of any policy intervention, because this is the cheapest way for them to raise funds at the margin. This reflects an important assumption that banks with access to the potential to screen loans and find out their chance of success have a valuable option which is not competed away by free entry of new banks. □

Corollary 2. *In the absence of capital regulations, banks will not seek external equity injections when they undertake risky lending.*

4.2 The Participation Threshold

As in the first-best case, the value of screening for banks is still given by equation 6:

$$\begin{aligned} &V(\omega,\tilde{\omega},r+\delta_r) \\ &\equiv E_p\left[\max\left\{0\ ,\ \frac{\omega}{\omega+\tilde{\omega}}\begin{pmatrix} p\,(y_h-R)+ \\ (1-p)\max\left[0,y_l-R\right]\end{pmatrix} - \ \omega(1+r+\delta_r)\right\}\right] \\ &=\frac{\omega}{\omega+\tilde{\omega}}\int_{\hat{p}(\omega+\tilde{\omega})}^{1}\left[p\,(y_h-y_l)+y_l-(1+r+\delta_r)\right]h(p)dp \end{aligned} \tag{20}$$

And the participation threshold $\hat{\omega}$ is still given implicitly by equation 7:

$$\frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}}\int_{\hat{p}(\omega+\tilde{\omega})}^{1}\left[p\,(y_h-y_l)+y_l-(1+r+\delta_r)\right]h(p)dp=C \tag{21}$$

with the only difference that $\hat{p}$ is now a function of $\omega+\tilde{\omega}$. Since $\hat{p}\,(\omega+\tilde{\omega})\leq\hat{p}_{fb}$, this means that even when $\tilde{\omega}=0$, the participation threshold under asymmetric information will in general be a positive number.

Proposition 5. $\frac{\partial V(\omega,\tilde{\omega}=0,r)}{\partial\omega}\begin{cases} >0 \ for\ \omega<\omega^* \\ =0\ for\ \omega\geq\omega^*\end{cases}$. *In the absence of capital regulations (and $\tilde{\omega}=0$), the expected value of screening is increasing in the bank's initial endowments (up to a cap when the bank becomes fully well capitalised). (see proof in the annex)*

Since we have established through corollary 2 that banks will only use debt funding in the absence of capital regulations, for now we can only concern ourselves with the case where $\tilde{\omega}=0$.

The equations for the rest of the model (market for funds, households and bank optimisation) are unchanged from the generalised forms presented in the first-best case.

4.3 Summary and Implications

The presence of moral hazard in the model give rise to sub-optimal outcomes compared to the first-best scenario. Banks have no incentives to top-up their capital. Consequently, the hurdle for participation, $\hat{\omega}>0$, is too high;

and the level of prudence, $\hat{p}(\omega, r) \leq \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)} = \hat{p}_{fb}$ is too low. As a result aggregate welfare is lower than it is in the first-best.

5 Policy tools and their transmission

Monetary policy and capital regulations can be used to drive outcomes closer to the first-best level. Both policy tools function through their effects on the participation and prudence thresholds.

5.1 Monetary Policy

We introduce monetary policy by allowing the central bank to affect the risk-free rate of return on the deposit facility (r). So the stance of monetary policy can affect the attractiveness of risky bank lending relative to the outside option. The central bank remunerates all resources placed in its deposit facilities at the rate r, the cost of which is covered by taxation of households[10]. The amount of tax paid by the representative household is denoted by τ:

$$\tau = r\left(S^{*} - D\right) \tag{22}$$

where S^{*} is the optimal saving decision made by the representative household; and
$D \equiv \frac{\mu}{1-\mu}\left[\int_{\hat{\omega}}^{\omega_{ub}}\left[1 - H\left(\hat{p}\right)\right] dF\left(\omega\right) - E\left[\omega\right]\right]$ is the banks' net demand for funds (per household).

We assume further that using monetary policy to address financial stability concerns incurs a deadweight cost, which reflects the tax that need to be raised when r is greater than zero. For standard tax distortion reasons, we assume this cost has the quadratic form:

$$\theta r^{2}\left(S^{*} - D\right) \tag{23}$$

with θ being a small number in our subsequent numerical simulations.

The monetary policy stance (r) is revealed to all agents in Stage 1, as soon as banks find out their realisation of ω_i.

We normalise $r = 0$ as the neutral stance of monetary policy. In the first-best scenario, there is no need for monetary policy intervention, so $r = 0$ and the first-best level of prudence become:

$$\hat{p}_{fb} = \frac{1 + \delta_r - y_l}{(y_h - y_l)} \tag{24}$$

The first-best level of the participation threshold $\hat{\omega}_{fb}\left(\omega_{reg} = 0, \delta_r\right) = 0$ remains the same as before (see equation

[10] The tax is borne by households because in the majority of cases only households have spare funds to place at the risk-free deposit facility (i.e. banks' net demand for funds are in general positive). Under some specific parameterisation / policy settings, it is possible that banks' net demand for funds becomes negative (banks can get more than enough funds from other bankers who decided against their own lending projects). In such cases, the tax is borne by both households and banks, in proportion to the aggregate amount each places in the central bank facility.

9)

Note that the possibility of monetary policy intervention introduces a wedge between the socially optimal outcome and what is privately optimal in the absence of asymmetric information. In particular, the privately optimal level of prudence is given by

$$\hat{p}^*(r) = \frac{1 + r + \delta_r - y_l}{(y_h - y_l)} \tag{25}$$

which will be different to $\hat{p}_{fb}$ (the socially optimal level of prudence) for non-zero r.

The effect of monetary policy on banks' behaviour is summarised in the proposition below.

Proposition 6. *Monetary policy tightening improves 'prudence' at the cost of decreasing 'participation'. Monetary policy loosening achieves the converse. In other words:*

1. $\frac{\partial \hat{p}}{\partial r} > 0$; and

2. $\frac{\partial \hat{\omega}}{\partial r} > 0$

Proof. We describe the intuition for the proposition here; formal proof is in the annex.

Monetary policy tightening improves prudence for all banks by increasing the attractiveness of the outside option. A quick way to illustrate the result $\frac{\partial \hat{p}}{\partial r} > 0$ is by appealing to proposition 3: $\hat{p}(\omega + \tilde{\omega}) \leq \hat{p}_{fb}$. Recall from equation 24: $\hat{p}_{fb} = \frac{1+\delta_r - y_l}{(y_h - y_l)}$; and from Proposition 3: $\hat{p}(\omega + \tilde{\omega}, r) \leq \frac{1+r+\delta_r - y_l}{(y_h - y_l)} = \hat{p}^*(r)$. So by increasing r, a central bank can increase $\hat{p}^*(r)$ and thus push $\hat{p}(\omega + \tilde{\omega}, r)$ closer to $\hat{p}_{fb}$ for all ω.

Money policy tightening decreases the level of participation because it increases the relative attractiveness of the outside option. Recall that $\hat{\omega}(\tilde{\omega}, r)$ is implicitly defined by

$$V(\hat{\omega}, \tilde{\omega}, r) = \frac{\hat{\omega}}{\hat{\omega} + \tilde{\omega}} \int_{\hat{p}(\hat{\omega}+\tilde{\omega}, r)}^{1} \left[p\left(y_h - y_l\right) + y_l - \left(1 + r + \delta_r\right)\right] h(p) dp = C$$

Increasing r reduces the value of screening, so for a given fixed cost of screening C, banks need a higher amount of initial capital to make the screening process worthwhile. □

5.2 Prudential Policy

We model capital regulations in the simple form of a leverage ratio. Regulators impose a minimum capital requirement ω_{reg} such that banks with endowments $\omega_i < \omega_{reg}$ must seek outside equity of at least $(\omega_{reg} - \omega_i)$ or are barred from lending. ω_{reg} is announced as soon as banks find out their individual realisations of ω_i. A strengthening of capital requirements achieves the same qualitative effect as a tightening of monetary policy on banks' lending decisions, albeit through slightly different means.

Proposition 7. *Capital regulations improve prudence by forcing some banks to seek additional equity funding, but do so at the cost of decreased participation:*

1. $\tilde{\omega} = \max[0, \omega_{reg} - \omega]$. Only banks that are constrained by the capital regulations will seek equity injections from households. And when they do, they will only top-up to the minimum capital standard.

2. $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega_{reg}} \geq 0$ with inequality as long as $\hat{\omega}(\tilde{\omega}=0,r) < \omega_{reg} < \omega^*$. A capital requirement will improve the prudence level of banks for which it binds, so long as they are not already 'fully capitalised'.

3. $\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r) > \hat{\omega}(\tilde{\omega}=0,r)$ for all ω_{reg} such that $\omega_{reg} > \hat{\omega}(\tilde{\omega}=0,r)$. The participation threshold is higher for banks that require external equity injections (i.e. banks that falls short of the capital requirement), than for banks that are not bound by the capital requirement.

4. $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$. A tightening of capital standards increases the threshold for participation.

Proof. We outline the proof here, details are in the annex. □

1. $\tilde{\omega} = \max[0, \omega_{reg} - \omega]$ follows directly from the observation that banks weakly prefer debt to equity (proposition 4). Consequently, banks for which the regulations are binding will only top-up their capital to the regulatory minimum, and seek the remaining funds in the form of debt.

2. $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega_{reg}} \geq 0$ is a corollary of proposition 3 ($\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \tilde{\omega}} \geq 0$) and the previous observation that $\tilde{\omega} = \max[0, \omega_{reg} - \omega]$. In particular, $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial \omega_{reg}} = 0$ for banks with $\omega_i > \omega_{reg}$ or $\omega_i \geq \omega^*$. In the former case, banks with $\omega_i > \omega_{reg}$ are not in violation of the capital requirement, and thus will remain unaffected by a marginal increase in ω_{reg}. In the latter case, banks with $\omega_i \geq \omega^*$ already operate at the optimal level of prudence, so pushing ω_{reg} above ω^* does not bring any further gains in prudence.

3. $\hat{\omega}(\omega_{reg}-\omega,r) > \hat{\omega}(0,r)$ holds because equity funding is more costly than debt funding due to the dilution effect (proposition 4). Therefore, the value of screening is lower for banks that are constrained by the capital requirement. (ω_{reg} is announced in Stage 1, so banks can anticipate capital needs in advance of screening.) Consequently, the presence of binding capital regulations increases the participation threshold for a given fixed cost of screening.

4. $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$ follows from part 1 and 3. Higher capital requirements increase the size of equity injection required, and thus reduce the value of screening and increase the threshold for participation.

5.3 Summary of Policy Implications

A tightening of monetary policy – an increase in the risk-free interest rate - increases the opportunity cost of risky bank lending. This pushes up prudence for all banks at all levels of initial endowments. So poorly endowed (or highly leveraged) banks become more prudent, but very well capitalised banks may become too cautious[11]. A

[11] This side effect falls away if we restrict the upper support of ω to a value significantly below 1.

strengthening of capital standards forces more banks to seek out more external equity, inducing them to behave as if they are a better endowed bank. Unlike monetary policy, capital regulations will not lead to some banks being too cautious, but it only affects banks for which the regulation is binding.

The two policy instruments will also affect participation. Because external equity is more costly than debt, tougher capital requirements make the break-even level of initial endowment higher for a given fixed cost of screening. So more banks will drop out from screening due to a low level of initial endowments. This is the disadvantage of using capital regulations. A tightening of monetary policy shares the same drawback. A higher r raises the participation threshold for all banks.

So the trade-off between 'prudence' and 'participation' is similar across both monetary policy and capital regulations. But there are two important differences. First, monetary policy distorts incentives for all banks; whereas capital regulations only affect those banks for which it is binding (i.e. poorly capitalised banks). Second, a loosening in the central bank policy rate can increase participation beyond the level possible in the absence of any intervention (and thus gets closer to the state of universal participation under the first-best). In contrast, capital regulations can be at best non-binding. This means capital requirements alone can never correct the participation distortions relative to the first best.

Lastly, variations in the benchmark interest rate distorts households' intertemporal substitution decisions. This is an additional cost in using the interest rate to address financial stability concerns.

5.4 Aggregate Welfare

In calculating aggregate welfare, we need to take into account the impact of the tax/subsidy τ as r varies across different policy regimes.

We assume the government can work out the required tax τ in advance and levy the household at the start of the game, such that households optimise over post-tax wealth:

$$\begin{aligned} W' &= W - \tau - deadweight \\ &= W - r\left(S^*\left(W'\right) - D\right) - \theta r^2 \left(S^*\left(W'\right) - D\right) \\ \text{where } S^*\left(W'\right) &= \frac{\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}}{\left(1+r+\delta_r\right) + \left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}} \left[W' - \frac{transfer}{\left[\beta\left(1+r+\delta_r\right)\right]^{\frac{1}{1-\rho}}}\right] \end{aligned} \tag{26}$$

Aggregate Welfare is given by:

$$
\begin{aligned}
welfare &= \alpha(r)\left[(1+r+\delta_r)W' + transfer\right] + \\
&\quad \alpha(0)\left[\begin{array}{c} \frac{\mu}{(1-\mu)}\left[1-H(\hat{p}_{fb})\right]\left[Z_{fb}\right]\int_{\hat{\omega}}^{\omega_{ub}} dF(\omega) + \frac{\mu}{(1-\mu)}E(\omega)(1+r+\delta_r) \\ -transfer - \frac{\mu}{(1-\mu)}C\int_{\hat{\omega}}^{\omega_{ub}} dF(\omega) \end{array}\right] \\
\text{where } \alpha(r) &= \left(\frac{\left((1+r+\delta_r)+\left[\beta(1+r+\delta_r)\right]^{\frac{1}{1-\rho}}\right)}{(1+r+\delta_r)}\right)^{\frac{1}{\rho}}\left(\frac{1}{(1+r+\delta_r)+\left[\beta(1+r+\delta_r)\right]^{\frac{1}{1-\rho}}}\right) \\
\text{and } \alpha(0) &= \alpha(r=0) \qquad (27)
\end{aligned}
$$

Comparing equation 27 with equation 16, the key difference is that households optimise over post-tax endowment, and bank profits are transformed into the utility space assuming a neutral rate of monetary policy, $\alpha(r=0)$.

6 Numerical Simulations

Numerical simulations can help to illustrate the degree of trade-off between monetary policy and capital requirements. But the model we have outlined above is stylised and a precise calibration of the model is not straight-forward because there are no direct empirical counterparts to some of the key parameters of the model. Nevertheless, the numerical examples we provide here demonstrate the key features of the model, which are robust across a broad spectrum of possible calibrations.

The key parameters of the model, and our benchmark calibration, can be summarised as follows:

- **The fundamentals of the bank lending project**: This includes: (i) the payoff of the risky lending project in the good state and in the bad state, y_h and y_l respectively; (ii) the random variable for the probability of success of the project p; and (iii) the fixed cost of screening C (for banks to find out its realisation of p). As described in the previous sections, the model imposes a few restrictions on the value these parameters can take. First, the unconditional expected return from the project must be less than the risk-free return, $E(p)(y_h - y_l) + y_l < 1 + \delta_r$, such that it would not be optimal for banks to undertake lending in the absence of screening. Second, the expected value-added from the project, conditional on all banks acting prudently, needs to exceed the cost of screening: $C < V(\hat{p}_{fb}) = \int_{\hat{p}_{fb}}^{1} \left[p(y_h - y_l) + y_l - (1+\delta_r)\right] h(p)\,dp$. This second condition ensures that it would be socially optimal for every lending project to be screened.

- **The distribution of banks' initial endowments** ω: Specifically, ω represents the distribution of banks' equity capital when there are no capital regulations. We assume for the benchmark case that ω is distributed uniformly between 0 and 0.1; in other words, in the absence of any regulatory requirements, the most well-capitalised bank would hold equity equals to 10% of its total assets (or be leveraged 10 times).

- **The relative size of aggregate bank endowments and household endowments** $\{\mu, E(\omega), W, \gamma\}$: we set-up the benchmark case such that were households to divide their consumption equally between the two periods, there will still be enough deposits for every bank should they all wish to engage in risky lending: $\mu(1 - E(\omega)) = \frac{1}{2}(1-\mu)W$. This set-up ensures that as long as we are content with the distribution of ω, the precise calibration of μ and W does not matter for the optimal policy combination derived by the model, as long as μ is small and W is set according to the relationship: $W = 2\frac{\mu}{(1-\mu)}(1 - E(\omega))$. As we will show below, if we let the general form of W to be $W = \gamma\frac{\mu}{(1-\mu)}(1 - E(\omega))$, varying γ will only have an significant impact on the optimal policy combination if the resulting W is small enough to lead to an excess demand for funds from banks, and thus require adjustments in the market for funds via an increase in δ_r.

- **The shape of the household utility function** $\{\rho, \beta\}$: the elasticity of intertemporal substitution $\frac{1}{1-\rho}$ and the rate of time preference β, affect the households' optimal saving decisions. Our robustness checks show that as long as there is still an excess supply of funds from households, variations in ρ and β do not lead to a change in the optimal policy setting. When ρ and β are set such that there is an excess demand for funds, then δ_r will need to adjust in order to clear the market for funds. The mechanics involved are very similar to the case of a small W (household endowment). In both cases, bank funding dries up as the result of the reduction in household savings, and policies must adapt accordingly.

- **Frictions in financial intermediation and policy intervention**: For the benchmark case, we set the fixed cost of screening C to 0.01, or 1% of the funds lent. We assumed that the central bank could affect banks' lending behaviour through influencing the attractiveness of the outside option (by changing the remuneration rate r on the safe deposit facility) and the cost is recouped through taxation. In practice taxes create costs, which we assume take the form:

$$\theta r^2 (S^* - D) \tag{28}$$

  where $(S^* - D)$ is the total amount of funds deposited with the central bank in the safe facility, and r is the policy rate (or the rate of remuneration on these deposits). The deadweight cost associated with monetary policy action is quadratic in the policy rate used, and we set θ to 2.5% in the benchmark case. This ensures that the cost of using monetary policy is negligible when r is close to the 'neutral stance' of monetary policy (when $r = 0$), but increase progressively when r deviates from its neutral level.

We find that for most parameter calibrations the model can produce two local optima: one where the stance of monetary policy is close to neutral and the level of capital requirements is substantial and binding; the other where monetary policy is substantially tighter and capital regulations are not binding for any banks engaged in lending activities. In some cases, where θ is very small (or zero), the latter combination of policy tools may be the global optimum. The location of the global optimum aside, the broader finding that the optimal level of central bank

Table 2: Benchmark Calibrations

Parameters	Benchmark Calibration	Parameters	Benchmark Calibration
Good state payoff from risky bank lending (y_h)	1.4	Relative size of household endowment (γ)	2
Bad state payoff from risky bank lending (y_l)	0.5	Intertemporal substitution (ρ)	0.5
Fixed cost of screening (C)	0.01	Time preference (β)	0.9
Probability of success (p)	$U\,[0,1]$	Distortionary cost of monetary policy (θ)	0.025
Initial endowment of banks (ω)	$U\,[0,0.1]$		

policy rate falls as bank capital requirements are tightened is very robust to alternative calibrations.

6.1 Main results under benchmark calibration

We use a numerically calibrated version of the model to analyze the effects on welfare of different combinations of the monetary policy stance and regulatory capital requirements. Figure A1 and Table A1 in the annex illustrate the full spectrum of results for ω_{reg} between 0% and 40% (step size of 2.5 percentage point), and r between -2% and +8% (step size of 50 basis points). Table 3 below shows the outcomes under a selection of these possible policy combinations. Figure A1 gives a graphical representation of the same information and shows the surface of welfare for combinations of interest rates and capital requirements. Each element in Table 3 shows outcomes under different policy settings. The columns illustrate how expected aggregate welfare changes as the stance of monetary policy varies, for a given level of capital requirements. This allows us to trace a path for the optimal stance of monetary policy conditional on the level of capital requirement (cells in bold are column maxima). The table shows that the optimal stance of monetary policy loosens as regulatory capital standards tighten.

Under the benchmark calibration, the central bank would optimally set interest rates at 6.5 percentage points above the 'neutral stance' ($r = 0$) if there are no capital requirement for banks. The optimal stance of monetary policy falls to $r = 3\%$ when capital requirement increases to 10% of total assets; given that ω is assumed to be distributed between 0 and 0.1, a 10% level for required capital would mean that the leverage ratio is binding for all but the most highly capitalised bank in the population. At a capital requirement of 20% it would be optimal to set a neutral monetary policy ($r = 0$).

Looking across the rows of Table 3 allows us to trace out a path for the optimal level of bank capital requirements for a given setting of monetary policy (cells in italic and underscored are row maxima). We find that the optimal level of bank capital requirement falls as monetary policy tightens.

Under the benchmark calibration, the globally optimal outcome is achieved when the capital requirement is set to 18% of total assets and the monetary policy stance is at very close to the neutral level ($r = 0.52\%$). This point

Table 3: Aggregate Welfare under Asymmetric Information (abridged version)

Aggregate Welfare		Capital Requirement							
		0.0%	5.0%	10.0%	15.0%	20.0%	25.0%	40.0%	Max
Interest Rate	-2.0%	1.8852	1.8916	1.8999	1.9052	1.9071	1.9070	**1.9005**	1.9072
	-1.5%	1.8872	1.8930	1.9009	1.9059	1.9075	1.9072	1.9002	1.9075
	-1.0%	1.8890	1.8944	1.9019	1.9065	*1.9078*	**1.9072**	1.8997	1.9078
	-0.5%	1.8908	1.8957	1.9027	1.9070	*1.9080*	1.9072	1.8992	1.9080
	0.0%	1.8926	1.8970	1.9035	1.9074	***1.9081***	1.9071	1.8985	1.9081
	0.5%	1.8942	1.8981	1.9042	1.9077	1.9081	1.9068	1.8976	1.9082
	1.0%	1.8958	1.8992	1.9048	1.9078	1.9079	1.9064	1.8966	1.9082
	1.5%	1.8973	1.9002	1.9053	**1.9079**	1.9077	1.9058	1.8955	1.9080
	2.0%	1.8986	1.9011	1.9056	*1.9078*	1.9072	1.9051	1.8942	1.9078
	2.5%	1.8999	1.9019	1.9058	*1.9076*	1.9067	1.9043	1.8927	1.9076
	3.0%	1.9010	1.9025	**1.9060**	*1.9072*	1.9060	1.9033	1.8911	1.9072
	3.5%	1.9020	1.9031	1.9059	1.9067	1.9051	1.9022	1.8892	1.9068
	4.0%	1.9029	1.9035	1.9058	1.9061	1.9041	1.9009	1.8873	1.9063
	4.5%	1.9036	1.9037	1.9055	1.9053	1.9030	1.8994	1.8851	1.9058
	5.0%	1.9042	**1.9039**	1.9050	1.9044	1.9016	1.8977	1.8827	1.9050
	5.5%	*1.9046*	1.9038	1.9044	1.9032	1.9001	1.8959	1.8802	1.9046
	6.5%	***1.9048***	1.9033	1.9027	1.9005	1.8966	1.8917	1.8745	1.9048
	8.0%	*1.9037*	1.9011	1.8987	1.8950	1.8898	1.8840	1.8643	1.9037
	Max	1.9048	1.9039	1.9060	1.9079	1.9081	1.9072	1.9005	1.9082

Table 4: Outcomes under different scenarios

Scenarios	Participation Threshold	Prudence Threshold	Probability of failure for bank loans made	Aggregate Lending	Aggregate Welfare
First-best	0	0.56	0.22	0.44	1.93
Optimal Policy combination	0.0239	0.41	0.29	0.45	1.91
No Policy Intervention	0.0037	varies with ω; average = 0.26	0.37	0.71	1.89

is marked with a cross in Figure A1.

The model also allows us to describe banks' behaviour under each policy setting. Under the globally optimal combination of $r = 0.52\%, \omega_{reg} = 18\%$, the capital regulation is binding for all banks in the population. The participation threshold equals 0.0239, meaning that 24% of banks would choose not to screen and forego the prospect of engaging in risky bank lending. For banks that do screen, they will proceed with the lending project if and only if the probability of success exceeds the prudence threshold of 0.41 (the first-best is 0.56). The conditional expectation of success is therefore $E\left(p|p \geq \hat{p}\left(\omega_{reg}\right) = 0.41\right) = 0.71$, such that around 30% of the projects that are taken forward are expected to fail. In aggregate about 45% of potential lending projects are undertaken.

Table 4 shows that across a broad range of measures - including aggregate output, lending and probability of failure - the outcomes under optimal policy are between first-best levels (when there is no asymmetry of information, no capital requirements and monetary policy is neutral) and levels under asymmetric information but with no policy intervention. The table shows that aggregate lending is significantly lower in the first-best case and under optimal policy (with asymmetric information) than compared to the case of no-intervention; free market outcomes with asymmetric information generate too much bank lending.

Note that although the banks in our model are heterogeneous at the start (they take independent draws from the distribution of ω), having a minimum capital requirement that exceeds the upper bound of the ω distribution (i.e. $\omega_{reg} \geq \omega_{ub}$) imposes homogeneity of leverage amongst those banks that do proceed with risky lending. All banks that proceed will top-up their equity capital to the minimum level required (ω_{reg}=18% in the optimal case) and fund the remainder through debt. These banks therefore share a common prudence threshold: $\hat{p}(\omega_{reg}) = 0.41$. In contrast, if ω_{reg} was lower than ω_{ub} (e.g. $\omega_{reg} = 5\%$), then we would observe a cluster of banks with capital equal to the regulatory minimum, as well as some banks with a surplus of capital for regulatory purposes (having started with a better endowment). Since the prudence threshold $\hat{p}$ is a function of capital ω, banks would no longer behave in a homogeneous fashion.

Table 3 reveals that a policy combination of no capital regulation but optimally set monetary policy $r = 6.5\%\%$ delivers a better outcome than cases where monetary policy responds optimally to very low capital requirements (e.g. $r = 5\%$). In fact, if monetary policy always responds optimally to the level of capital requirements, we need ω_{reg} greater than 7.5% under the benchmark calibration for aggregate welfare to exceed the case of no capital regulations. That is not to say that in practice low-to-modest capital requirements are worse than useless. When the official interest rates is not set optimally to address the excessive risk-taking behavior in the banking sector (e.g. when a loose policy stance is set) then a low capital requirement can be better than no capital requirements at all. For instance, if $r = 0.5\%$ then moving from $\omega_{reg} = 0\%$ to $\omega_{reg} = 5\%$ increases aggregate welfare. It is also possible that we have underestimated the distortionary costs of using monetary policy under the benchmark calibration. If we had instead taken a value for θ (the parameter which reflects the distortionary costs of raising revenue to finance the central bank paying interest on reserves) greater than 2.5%, then the optimal interest rate in the absence of capital regulations falls below 6.5%, and small increases in capital requirements make a bigger impact. A higher θ gives a bigger role to capital requirements.

6.2 Robustness Checks

We check the robustness of results against a broad range of alternative calibrations.

Table 5 shows optimal combinations of the interest rate and the capital requirement when the payoffs from successful and unsuccessful lending vary. Table 6 shows the impact of varying the level of household wealth relative to the size of the banking sector. Table 7 shows what happens when we vary the rate of inter-temporal substitution and the rate of time preference.

In many cases there exist two local optima in terms of policy choices. One where capital requirements play the

Table 5: Robustness Check I - Varying good and bad outcomes for lending (bold = base case)

Optimal Policy under Asymmetric Information (r, ω_{reg})		y_h					
		1.2	1.3	1.4	1.5	1.6	1.7
y_l	0	(0.000, 0.000)	(0.001, 0.139)	(0.004, 0.207)	(0.006, 0.258)	(0.007, 0.300)	(0.009, 0.329)
	0.1	(0.000, 0.000)	(0.002, 0.150)	(0.004, 0.211)	(0.007, 0.253)	(0.008, 0.290)	(0.010, 0.314)
	0.2	(0.000, 0.000)	(0.002, 0.160)	(0.004, 0.212)	(0.006, 0.250)	(0.008, 0.277)	(0.010, 0.297)
	0.3	(-0.001, 0.101)	(0.003, 0.165)	(0.005, 0.208)	(0.006, 0.238)	(0.009, 0.259)	-
	0.4	(0.001, 0.117)	(0.003, 0.165)	(0.005, 0.198)	(0.007, 0.222)	-	-
	0.5	(0.014, 0.000)	(0.004, 0.159)	**(0.005, 0.183)**	-	-	-
	0.6	(0.021, 0.000)	(0.004, 0.146)	-	-	-	-
	0.7	(0.026, 0.000)	-	-	-	-	-

Table 6: Robustness Check II - Varying the relative size of household endowments

Household Endowment Multiplier (γ)	(r, ω_{reg})	δ_r under no intervention
3	(0.0029,0.1873)	0.0000
2	**(0.0052, 0.1832)**	**0.0000**
1.5	(0.0086,0.1792)	0.0110
1.3	(0.0800,0.0000)	0.0454
1.1	(0.0854,0.0000)	0.0778

Table 7: Robustness Check III - Varying the Rate of Time Preference and Intertemporal Substitution

Time preference (β)	(r, ω_{reg})	δ_r under no intervention
1	(0.0045, 0.1850)	0.0000
0.9	**(0.0052, 0.1832)**	**0.0000**
0.75	(0.0073, 0.1805)	0.0000
0.6	(0.0080, 0.0000)	0.0624

Intertemporal substitution (ρ)	(r, ω_{reg})	δ_r under no intervention
0.75	(0.0045, 0.1851)	0.0000
0.5	**(0.0052, 0.1832)**	**0.0000**
0.25	(0.0055, 0.1832)	0.0000

significant role, and another where monetary policy is relied upon instead. There are two broad scenarios where favouring monetary policy is the globally optimal solution:

1. when the difference between the good and bad state outcome for bank lending is low (so the underlying incentive issue is lessened); and

2. when there is a distinct shortage of savings from households, relative to demand from banks, such that the shadow rate of funds (δ_r) is high in the absence of policy intervention. (This can be brought about by: (i) small household endowments relative to banks' endowments; or (ii) specific calibrations of the household utility function such that households significantly favour current consumption over savings and future consumption).

Clearly numerical results differ across alternative calibrations, but our qualitative results are robust, namely that: monetary policy and capital regulations function as imperfect substitutes; these policy levers create trade-offs between 'prudence' and 'participation' of banks; and as long as there is a non-negligible cost to using monetary policy, the global optimum more often than not tends to be the one where r is close to neutral, and capital regulation is binding for the vast majority of banks.

7 Concluding Remarks

We find that monetary policy and prudential capital requirements operate as imperfect substitutes. A tightening of either instruments can improve 'prudence' (by dis-incentivising banks against undertaking projects with low probability of success); but only at the cost of decreased 'participation' (as more banks will choose to forego the lending opportunity even before they discover its probability of success through costly screening).

Numerical simulations of our model help to illustrate, and to broadly quantify, the magnitude of this interaction between monetary policy and capital regulation. We find that in general there exist two sets of policy choices that are locally optimal: one with high capital requirements and little monetary policy intervention; and the other where the converse is true. In our base calibration, and across a broad range of plausible alternatives, the global optimum is one where interest rates should be slightly higher than a neutral setting and capital requirements should be substantial and binding.

Looking at the overall relation between welfare and policy instruments, and not just the two local optima, we find that the trade-off between 'prudence' and 'participation' is such that the optimal level of the central bank policy rate falls as prudential capital requirements are tightened. This does not imply that there will never be a case where it is optimal to raise both interest rates and capital requirements at the same time. If the economy starts from a point far away from the optimal policy setting, with both sub-optimally low capital requirements and ultra loose monetary policy, then aggregate welfare can be improved by tightening both. It may be that monetary policy has been set at a very loose level for reasons that are not captured in our stylised model which abstracts from fluctuations that require counter-cyclical monetary policy. So our results are consistent with the view that central banks should increase policy rates from the lows they had fallen to after the 2007-08 financial crisis as economies return to normal, even if prudential requirements strengthen at the same time.

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A Annex

A.1 Proof of Proposition 2.2 - Weak preference for debt in the First-Best

1. Case A: For a bank with $\omega \geq \omega^*$, and thus $y_l > R(\omega)$, we show that the bank has at least a weak preference for external finance using debt, through proof by contradiction:

   (a) Statement A: suppose that the bank with $\omega \geq \omega^*$ strictly prefers using some amount of external equity as opposed to only using debt:

   $p(y_h - R(\omega)) + (1-p)(y_l - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - R(\omega+\tilde{\omega})) + (1-p)(y_l - R(\omega+\tilde{\omega}))]$ for some $\tilde{\omega} \in [0, 1-\omega]$.

   Then this implies:

   $p(y_h - y_l) + (y_l - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - y_l)(y_l - R(\omega+\tilde{\omega}))]$

   From proposition 1: $R(\omega+\tilde{\omega}) = (1-(\omega+\tilde{\omega}))(1+r+\delta_r)$, so:

   $p(y_h - y_l) + (y_l - (1-\omega)(1+r+\delta_r)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - y_l) + (y_l - (1-(\omega+\tilde{\omega}))(1+r+\delta_r))]$

   $\tilde{\omega}[p(y_h - y_l) + y_l] < -\omega(1-(\omega+\tilde{\omega}))(1+r+\delta_r) + (\omega+\tilde{\omega})(1-\omega)(1+r+\delta_r)$

   $\tilde{\omega}[p(y_h - y_l) + y_l] < [(\omega+\tilde{\omega}) - \omega(\omega+\tilde{\omega}) - \omega + \omega(\omega+\tilde{\omega})](1+r+\delta_r)$

   $p(y_h - y_l) + y_l < (1+r+\delta_r)$

   $p < \frac{(1+r+\delta_r)-y_l}{y_h-y_l} = \hat{p}_{fb}$. Contradiction - since we know from the definition of the prudence threshold $\hat{p}_{fb}$, that banks will only proceed with lending if $p \geq \hat{p}_{fb}$.

2. Case B: Consider banks with $\omega < \omega^*$, and thus $y_l < R(\omega)$. As before, we show weak preference for debt through proof by contradiction.

   (a) Statement B1: banks strictly prefer to use some external equity when the equity injection means they are still not 'fully capitalised':

   $p(y_h - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - R(\omega+\tilde{\omega}))]$ for some $\tilde{\omega} \in [0, \omega^* - \omega)$

   (b) Statement B2: banks strictly prefer to use some external equity when the equity injection means they become 'fully capitalised':

   $p(y_h - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - R(\omega+\tilde{\omega})) + (1-p)(y_l - R(\omega+\tilde{\omega}))]$ for some $\tilde{\omega} \in [\omega^* - \omega, 1-\omega]$

   Consider statement B1, which simplifies to

   $y_h - R(\omega) < \frac{\omega}{\omega+\tilde{\omega}}[(y_h - R(\omega+\tilde{\omega}))]$

   From equation 2 and proposition 1, we have: $\hat{p}_{fb}(y_h - R(\omega)) = \omega(1+r+\delta_r) = \frac{\omega}{\omega+\tilde{\omega}}[\hat{p}_{fb}(y_h - R(\omega+\tilde{\omega}))]$

So $(y_h - R(\omega)) = \frac{\omega}{\omega+\tilde{\omega}}(y_h - R(\omega+\tilde{\omega}))$. Contradiction. In fact, the same proof shows that a 'not-fully-capitalised bank' would be indifferent between using all debt, or using a mixture of debt and additional equity (provided that the additional equity injection does not make the bank 'fully-capitalised').

Consider statement B2, which simplifies to:

$p(y_h - R(\omega)) < \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - y_l) + y_l - R(\omega+\tilde{\omega})]$,

where $R(\omega < \omega^*) = \frac{(1-\omega)(1+r+\delta_r)-(1-p)y_l}{p}$, and

$R(\omega+\tilde{\omega} > \omega^*) = (1-(\omega+\tilde{\omega}))(1+r+\delta_r)$.

From equation 2 and proposition 1, we have:

$\hat{p}_{fb}(y_h - R(\omega)) = \omega(1+r+\delta_r) = \frac{\omega}{\omega+\tilde{\omega}}[\hat{p}_{fb}(y_h - R(\omega+\tilde{\omega})) + (1-\hat{p}_{fb})(y_l - R(\omega+\tilde{\omega}))]$, so when $p = \hat{p}_{fb}$, the statement is false.

For the remainder of the proof we just need to show that the statement is also false for $p > \hat{p}_{fb}$.

The derivative of the LHS of statement B2 w.r.t. p is: $(y_h - R(\omega)) - p\frac{\partial R(\omega)}{\partial p} = (y_h - R(\omega)) - p\left(-\frac{R(\omega)}{p} + \frac{y_l}{p}\right) = (y_h - y_l)$

The derivative of the RHS of statement B2 w.r.t. of p is $\frac{\omega}{\omega+\tilde{\omega}}(y_h - y_l) < (y_h - y_l)$. The LHS increases faster than the RHS when p increases from $\hat{p}_{fb.}$

Therefore $p(y_h - R(\omega)) \geq \frac{\omega}{\omega+\tilde{\omega}}[p(y_h - y_l) + y_l - R(\omega+\tilde{\omega})]$ for any $p \geq \hat{p}_{fb}$. Contradiction reached.

A.2 Proof of Proposition 3 - Prudence Threshold under Asymmetric Information

A.2.1 Proposition 3.1

The proof for $\omega^* = 1 - \frac{y_l}{(1+r+\delta_r)}$ follows directly from re-arranging equation 18, which also shows that banks with $\omega_i + \tilde{\omega}_i = \omega^*$ can borrow at the risk-free rate $1+r+\delta_r$. For banks with $\omega_i + \tilde{\omega}_i > \omega^*$, their funding shortfall is smaller and thus their gross debt obligation is lower: $R(\omega_i + \tilde{\omega}_i) < R(\omega^*) = y_l$ (since $\frac{\partial R(\omega_i+\tilde{\omega}_i, r)}{\partial \omega} < 0$, a result we will show in the proof to part 2 of the proposition below). The rate at which the debt is provided is still floored at $1+r+\delta_r$, again from re-arranging equation 18.

A.2.2 Proposition 3.2

1. Consider case A where $y_l \geq R$.

   Re-arrange the banks' and the households' indifference equations to give: $R = (1-(\omega+\tilde{\omega}))(1+r+\delta_r)$ and $\hat{p} = \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}$.

   Note that under this case: $\frac{\partial R}{\partial \omega} < 0$ so we can deduce that $y_l > R$ holds whenever we have $(\omega+\tilde{\omega}) > \omega^*$.

2. Consider case B where $y_l \leq R$.

   Re-arranging the indifference equations gives:

   $\hat{p}\left[y_h - R\right] = (\omega + \tilde{\omega})\left(1 + r + \delta_r\right)$; and $A\left(\hat{p}\right) R + \left(1 - A\left(\hat{p}\right)\right) y_l = \left(1 - (\omega + \tilde{\omega})\right)\left(1 + r + \delta_r\right)$.

   Partially differentiate both w.r.t. ω, [or $\tilde{\omega}$], and solve the system of linear equations to give:
$$\begin{bmatrix} \frac{\partial \hat{p}}{\partial \omega} \\ \frac{\partial R}{\partial \omega} \end{bmatrix} = \begin{bmatrix} \frac{(A(\hat{p}) - \hat{p})(1+r+\delta_r)}{(y_h - R)A(\hat{p}) + \hat{p}A'(\hat{p})(R - y_l)} \\ -\frac{\left[A'(\hat{p})(R-y_l) + (y_h - R)\right](1+r+\delta_r)}{(y_h - R)A(\hat{p}) + \hat{p}A'(\hat{p})(R - y_l)} \end{bmatrix}$$
   Since $\left(A\left(\hat{p}\right) - \hat{p}\right) > 0$ and $A'\left(\hat{p}\right) > 0$ unless $\hat{p} = 1$ (i.e. the degenerate case where no banks lend - which we will rule out by parameterisation), and $y_h \geq R \geq y_l$, we can deduce that $\frac{\partial \hat{p}}{\partial \omega} > 0$ and $\frac{\partial R}{\partial \omega} < 0$. Also, $y_l < R$ whenever we have $(\omega + \tilde{\omega}) < \omega^*$.

3. To summarise: whenever $(\omega + \tilde{\omega}) \geq \omega^*$, we have $\hat{p}\left(\omega + \tilde{\omega}\right) = \frac{(1+r+\delta_r) - y_l}{(y_h - y_l)}$; and whenever $(\omega + \tilde{\omega}) < \omega^*$, we have $\frac{\partial \hat{p}}{\partial \omega} > 0$. Therefore $\hat{p} \leq \frac{(1+r+\delta_r) - y_l}{(y_h - y_l)}$ for all $(\omega + \tilde{\omega})$.

A.3 Proof of Proposition 4 - Preference for Debt under Asymmetric Information.

Part 1 of the proposition follows directly from the definition of $\hat{p}(\omega + \tilde{\omega}, r)$ as the 'prudence threshold' for banks (see discussion in main body).

For part 2 of the proposition, we show that for any given level of initial endowment banks weakly prefer to fund the remainder of the project through debt rather than through any other funding arrangements:

- Case A: consider a bank with $\omega \geq \omega^*$, and thus $y_l \geq R$, we show that the bank has a weak preference to top-up the remainder using debt through proof by contradiction:

  Statement A: suppose that the bank with $\omega \geq \omega^*$ strictly prefers using some amount of external equity as opposed to only using debt:

  $p\left(y_h - R\left(\omega\right)\right) + (1-p)\left(y_l - R\left(\omega\right)\right) < \frac{\omega}{\omega + \tilde{\omega}}\left(p\left(y_h - R\left(\omega + \tilde{\omega}\right)\right) + (1-p)\left(y_l - R\left(\omega + \tilde{\omega}\right)\right)\right)$ for some $0 < \tilde{\omega} \leq 1 - (\omega + \tilde{\omega})$.

  Then this implies $y_l - R\left(\omega\right) + p\left(y_h - y_l\right) < \frac{\omega}{\omega + \tilde{\omega}}\left[y_l - R\left(\omega + \tilde{\omega}\right) + p\left(y_h - y_l\right)\right]$

  Recall from Proposition 3, for $\omega \geq \omega^*$: $R\left(\omega\right) = (1-\omega)\left(1 + r + \delta_r\right)$; $R\left(\omega + \tilde{\omega}\right) = \left(1 - (\omega + \tilde{\omega})\right)\left(1 + r + \delta_r\right)$

  So $\left(1 - \frac{\omega}{\omega + \tilde{\omega}}\right)\left(y_l + p\left(y_h - y_l\right)\right) < R\left(\omega\right) - \frac{\omega}{\omega + \tilde{\omega}} R\left(\omega + \tilde{\omega}\right)$

  $\frac{\tilde{\omega}}{\omega + \tilde{\omega}}\left(y_l + p\left(y_h - y_l\right)\right) < (1-\omega)\left(1 + r + \delta_r\right) - \frac{\omega}{\omega + \tilde{\omega}}\left(1 - (\omega + \tilde{\omega})\right)\left(1 + r + \delta_r\right)$

  $\tilde{\omega}\left(y_l + p\left(y_h - y_l\right)\right) < (\omega + \tilde{\omega})(1-\omega)\left(1 + r + \delta_r\right) - \omega\left(1 - (\omega + \tilde{\omega})\right)\left(1 + r + \delta_r\right)$

  $\tilde{\omega}\left(y_l + p\left(y_h - y_l\right)\right) < \left[(\omega + \tilde{\omega})(1-\omega) - \omega\left(1 - (\omega + \tilde{\omega})\right)\right]\left(1 + r + \delta_r\right)$

  $\tilde{\omega}\left(y_l + p\left(y_h - y_l\right)\right) < \left[(\omega + \tilde{\omega}) - \omega(\omega + \tilde{\omega}) - \omega + \omega(\omega + \tilde{\omega})\right]\left(1 + r + \delta_r\right)$

$\tilde{\omega}\left(y_l + p\left(y_h - y_l\right)\right) < \tilde{\omega}\left(1 + r + \delta_r\right)$

$y_l + p\left(y_h - y_l\right) < \left(1 + r + \delta_r\right)$

$p < \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}$

But recall from Proposition 1 that for any bank with $\omega \geq \omega^*$, $\hat{p} = \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}$.

Therefore, given banks will proceed with the project if and only if $p > \hat{p} = \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}$, statement A cannot be true, and the bank will weakly prefer to use debt for any $\tilde{\omega} \in [0, 1 - (\omega + \tilde{\omega})]$. [For simplicity, we assume that when banks are indifferent between debt and equity, they will proceed with the project using debt finance].

- Case B: consider a bank with $\omega < \omega^*$, and thus $y_l < R(\omega)$, we show a weak preference for debt through proof by contradiction:

  1. Statement B1: the bank strictly prefers to use some external equity:

     $p\left(y_h - R\left(\omega\right)\right) < \frac{\omega}{\omega+\tilde{\omega}} p\left(y_h - R\left(\omega + \tilde{\omega}\right)\right)$ for some $\tilde{\omega} \in [0, \omega^* - \omega)$; and

  2. Statement B2: the bank strictly prefers to use some external equity:

     $p\left(y_h - R\left(\omega\right)\right) < \frac{\omega}{\omega+\tilde{\omega}_2}\left[p\left(y_h - R\left(\omega + \tilde{\omega}_2\right)\right) + (1-p)\left(y_l - R\left(\omega + \tilde{\omega}_2\right)\right)\right]$ for some $\tilde{\omega}_2 \in [\omega^* - \omega, 1 - \omega]$

Consider Statement B1, which simplifies to $y_h - R\left(\omega\right) < \frac{\omega}{\omega+\tilde{\omega}}\left(y_h - R\left(\omega + \tilde{\omega}\right)\right)$.

Let $w = \omega + \tilde{\omega}$ denote total capital. Recall from the bank's indifference condition $\hat{p}\left(w\right)\left[y_h - R\left(w\right)\right] = w(1+r)$; so $y_h - R\left(w\right) = \frac{w(1+r)}{\hat{p}(w)}$;

So Statement B1 becomes $\frac{\omega(1+r)}{\hat{p}(\omega)} < \frac{\omega}{\omega+\tilde{\omega}} \frac{(\omega+\tilde{\omega})(1+r)}{\hat{p}(\omega+\tilde{\omega})}$; or $\hat{p}\left(\omega + \tilde{\omega}\right) < \hat{p}\left(\omega\right)$ when $\omega + \tilde{\omega} < \omega^*$

i.e. a better capitalised bank is less prudent. Contradiction [see Proposition 3].

Consider Statement B2, which simplifies to: $p\left(y_h - R\left(\omega\right)\right) < \frac{\omega}{\omega+\tilde{\omega}_2}\left[\left(y_l - R\left(\omega + \tilde{\omega}_2\right)\right) + p\left(y_h - y_l\right)\right]$, for some $\tilde{\omega}_2 \in [\omega^* - \omega, 1 - \omega]$, where $R\left(\omega + \tilde{\omega}_2\right) = \left(\omega + \tilde{\omega}_2\right)\left(1 + r + \delta_r\right)$

But from bank's indifference condition we know:

$\hat{p}\left(\omega\right)\left[y_h - R\left(\omega\right)\right] = \omega(1 + r + \delta_r)$; and $\frac{\omega}{\omega+\tilde{\omega}_2}\left(\left(y_l - R\left(\omega + \tilde{\omega}_2\right)\right) + \hat{p}\left(\omega + \tilde{\omega}_2\right)\left(y_h - y_l\right)\right) = \omega(1 + r + \delta_r)$

So $\hat{p}\left(\omega\right)\left[y_h - R\left(\omega\right)\right] = \frac{\omega}{\omega+\tilde{\omega}_2}\left(\left(y_l - R\left(\omega + \tilde{\omega}_2\right)\right) + \hat{p}\left(\omega + \tilde{\omega}_2\right)\left(y_h - y_l\right)\right)$

Since $\omega < \omega^* \leq \omega + \tilde{\omega}_2$ by assumption, we know from proposition 3 that $\hat{p}\left(\omega\right) < \hat{p}\left(\omega + \tilde{\omega}_2\right)$

Therefore we have $p\left(y_h - R\left(\omega\right)\right) \geq \frac{\omega}{\omega+\tilde{\omega}_2}\left[\left(y_l - R\left(\omega + \tilde{\omega}_2\right)\right) + p\left(y_h - y_l\right)\right]$ for any $p \in \left[\hat{p}\left(\omega\right), \hat{p}\left(\omega + \tilde{\omega}_2\right)\right]$. Contradiction.

For the last part of the proof by contradiction, just need to show that as p increases, the return from debt financed investments increases faster than that from mixed financed investments:

i.e. $(y_h - R(\omega)) \geq \frac{\omega}{\omega+\tilde{\omega}_2}(y_h - y_l)$. $\begin{bmatrix} \frac{\partial \hat{p}}{\partial \omega} \\ \frac{\partial R}{\partial \omega} \end{bmatrix} = \begin{bmatrix} \frac{(A(\hat{p})-\hat{p})(1+r+\delta_r)}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)} \\ -\frac{\left[A'(\hat{p})(R-y_l)+(y_h-R)\right](1+r+\delta_r)}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)} \end{bmatrix}$

Recall again $y_h - R(\omega) = \frac{\omega(1+r+\delta_r)}{\hat{p}(\omega)}$, and $\hat{p}(\omega) < \frac{(1+r+\delta_r)-y_l}{(y_h-y_l)}$, we have $y_h - R(\omega) > \omega(1+r+\delta_r)\frac{(y_h-y_l)}{(1+r+\delta_r)-y_l} = \omega(y_h - y_l)\frac{(1+r+\delta_r)}{(1+r+\delta_r)-y_l}$

so to complete the contradiction to statement B2, just need to show $\omega(y_h - y_l)\frac{(1+r+\delta_r)}{(1+r+\delta_r)-y_l} \geq \frac{\omega}{\omega+\tilde{\omega}_2}(y_h - y_l)$; or $\frac{(1+r+\delta_r)}{(1+r+\delta_r)-y_l} \geq \frac{1}{\omega+\tilde{\omega}_2}$

Recall $\omega^* = 1 - \frac{y_l}{(1+r+\delta_r)} = \frac{(1+r+\delta_r)-y_l}{(1+r+\delta_r)}$, so $\frac{1}{\omega^*} = \frac{(1+r+\delta_r)}{(1+r+\delta_r)-y_l}$. And given $\omega + \tilde{\omega}_2 \geq \omega^*$, we have $\frac{1}{\omega^*} = \frac{(1+r+\delta_r)}{(1+r+\delta_r)-y_l} \geq \frac{1}{\omega+\tilde{\omega}_2}$. QED.

A.4 Value of Screening (equation 6)

We present the proof for the last line of equation 6:

$$\begin{aligned} & V(\omega,\tilde{\omega},r+\delta_r) \\ & \equiv E_p\left[\max\left\{0\ ,\ \frac{\omega}{\omega+\tilde{\omega}}\begin{pmatrix} p\,(y_h - R) + \\ (1-p)\max\left[0, y_l - R\right] \end{pmatrix} -\ \omega(1+r+\delta_r)\right\}\right] \\ & = \frac{\omega}{\omega+\tilde{\omega}}\int_{\hat{p}}^{1}\left[p\,(y_h - y_l) + y_l - (1+r+\delta_r)\right]h(p)dp \end{aligned}$$

The last equality follows because household provide debt at the risk-free rate:

1. For fully capitalised banks with $\omega + \tilde{\omega} \geq \omega^* = 1 - \frac{y_l}{(1+r+\delta_r)}$:

$$V(\omega,\tilde{\omega},r+\delta_r) = E_p\left[\max\left(0\ ,\ \frac{\omega}{\omega+\tilde{\omega}}\left\{\begin{matrix} p\,(y_h - R) + \\ (1-p)\,(y_l - R) \end{matrix}\right\} - \omega(1+r+\delta_r)\right)\right]$$

So from equation 17 we have:

$$V(\omega,\tilde{\omega},r+\delta_r) = \int_{\hat{p}(\omega+\tilde{\omega},r+\delta_r)}^{1}\left[\frac{\omega}{\omega+\tilde{\omega}}\left\{\begin{matrix} p\,(y_h - R) + \\ (1-p)\,(y_l - R) \end{matrix}\right\} - \omega(1+r+\delta_r)\right]h(p)dp$$

Which can be simplified to:

$V(\omega,\tilde{\omega},r+\delta_r) = \frac{\omega}{\omega+\tilde{\omega}}\int_{\hat{p}}^{1}\left[p\,(y_h - y_l) + y_l - R\,(\omega+\tilde{\omega}) - (\omega+\tilde{\omega})\,(1+r+\delta_r)\right]h(p)dp$

But when $\omega + \tilde{\omega} \geq \omega$, we know from proposition 3 that $R\,(\omega+\tilde{\omega}) = (1 - (\omega+\tilde{\omega}))\,(1+r+\delta_r)$

So $V(\omega,\tilde{\omega},r+\delta_r) = \frac{\omega}{\omega+\tilde{\omega}}\int_{\hat{p}(\omega+\tilde{\omega},r)}^{1}\left[p\,(y_h - y_l) + y_l - (1+r+\delta_r)\right]h(p)dp.$

2. For banks with $\omega + \tilde{\omega} < \omega^*$:

   $V(\omega, \tilde{\omega}, r + \delta_r) = E_p \left[\max\Big(\ 0 \ , \ \frac{\omega}{\omega+\tilde{\omega}} p \left(y_h - R\left(\omega + \tilde{\omega}\right)\right) - \omega(1 + r + \delta_r)\Big)\right]$

   Which can be simplified again to:

   $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}}^{1} \left[p \left(y_h - R\left(\omega + \tilde{\omega}, r\right)\right) - \left(\omega + \tilde{\omega}\right)\left(1 + r + \delta_r\right)\right] h(p)dp$

   Recall from equation 18 $\ A\left(\hat{p}\right) R\left(\omega + \tilde{\omega}\right) + \left(1 - A\left(\hat{p}\right)\right) y_l = \left(1 - \omega - \tilde{\omega}\right)\left(1 + r + \delta_r\right)$,

   so $A\left(\hat{p}\right)\left(R\left(\omega + \tilde{\omega}, r\right) - y_l\right) + y_l = \left(1 - \omega - \tilde{\omega}\right)\left(1 + r + \delta_r\right)$.

   Substituting in the definition of $A\left(.\right)$ as the conditional expectation:

   $\left[\frac{1}{1-H(\hat{p})} \int_{\hat{p}}^{1} ph\left(p\right) dp\right] \left(R\left(\omega + \tilde{\omega}\right) - y_l\right) + y_l = \left(1 - \left(\omega + \tilde{\omega}\right)\right)\left(1 + r + \delta_r\right)$

   Re-arrange to give:

   $\int_{\hat{p}}^{1} p\left(R\left(\omega + \tilde{\omega}\right) - y_l\right) h\left(p\right) dp = \left[\left(1 - \left(\omega + \tilde{\omega}\right)\right)\left(1 + r + \delta_r\right) - y_l\right] \left[\int_{\hat{p}}^{1} h\left(p\right) dp\right]$

   ... because $R\left(.\right)$ is not a function of p, and ω and p are independently distributed.

   So $\int_{\hat{p}}^{1} pR\left(\omega + \tilde{\omega}\right) h\left(p\right) dp = \int_{\hat{p}}^{1} \left[py_l - y_l + \left(1 - \omega - \tilde{\omega}\right)\left(1 + r + \delta_r\right)\right] h\left(p\right) dp$

   And $V(\omega, \tilde{\omega}, r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}(\omega+\tilde{\omega},r)}^{1} \left[p\left(y_h - y_l\right) + y_l - \left(1 + r + \delta_r\right)\right] h(p)dp$ Q.E.D.

3. Note that when $\tilde{\omega} = 0$ (i.e. in the absence of capital regulation):

   $V(\omega, 0, r + \delta_r) = \int_{\hat{p}}^{1} \left[p\left(y_h - y_l\right) + y_l - \left(1 + r + \delta_r\right)\right] h(p)dp$.

   Note further that when $\tilde{\omega} = \omega_{reg} - \omega$ (i.e. when a bank tops-up to the regulatory minimum - see proposition 7):

   $V(\omega, \tilde{\omega} = \omega_{reg} - \omega, r + \delta_r) = \frac{\omega}{\omega_{reg}} \int_{\hat{p}(\omega_{reg},r)}^{1} \left[p\left(y_h - y_l\right) + y_l - \left(1 + r + \delta_r\right)\right] h(p)dp = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg} \ , \tilde{\omega} = 0 \ , r + \delta_r)$.

   So the value of screening for a bank which falls short of the regulatory capital requirement ($\omega_i < \omega_{reg}$) is a $\frac{\omega_i}{\omega_{reg}}$ fraction of the value of screening for a bank that is just meeting the regulatory requirement:

   $$V(\omega, \tilde{\omega} = \omega_{reg} - \omega) = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0) \tag{29}$$

   and the participation threshold for a capital constrained bank is given by:

   $$\hat{\omega}\left(\tilde{\omega} = \omega_{reg} - \omega\right) = \frac{\omega_{reg} C}{V(\omega = \omega_{reg}, \tilde{\omega} = 0)} \tag{30}$$

   (see definition 7).

A.5 Proof of Proposition 5 - Participation Threshold

Recall: $V(\omega, \tilde{\omega}, r+\delta_r) = \frac{\omega}{\omega+\tilde{\omega}} \int_{\hat{p}}^{1} [p(y_h - y_l) + y_l - (1+r+\delta_r)] h(p)dp$

Using the Product Rule and the Leibniz Integral Rule[12] we get:

$$\begin{aligned} \frac{\partial V(\omega,\tilde{\omega})}{\partial \omega} &= \frac{\tilde{\omega}}{(\omega+\tilde{\omega})^2} \int_{\hat{p}}^{1} [p(y_h - y_l) + y_l - (1+r+\delta_r)] h(p)dp \\ &\quad - \frac{\omega}{\omega+\tilde{\omega}} [\hat{p}(y_h - y_l) + y_l - (1+r+\delta_r)] h(\hat{p}) \frac{\partial \hat{p}}{\partial \omega} \end{aligned}$$

or $\frac{\partial V(\omega,\tilde{\omega})}{\partial \omega} = \frac{1}{(\omega+\tilde{\omega})} \frac{\tilde{\omega}}{\omega} V - \frac{\omega}{\omega+\tilde{\omega}} [\hat{p}(y_h - y_l) + y_l - (1+r+\delta_r)] h(\hat{p}) \frac{\partial \hat{p}}{\partial \omega}$.

Recall $\tilde{\omega} = \max[0, \omega_{reg} - \omega]$ (see proposition 7):

- So when $\tilde{\omega} = 0$, $\frac{\partial V(\omega,\tilde{\omega}=0,r)}{\partial \omega} = -[\hat{p}(y_h - y_l) + y_l - (1+r+\delta_r)] h(\hat{p}) \frac{\partial \hat{p}}{\partial \omega} \geq 0$

  where the inequality holds because $\frac{\partial \hat{p}}{\partial \omega} \geq 0$ with equality when $(\omega+\tilde{\omega}) \geq \omega^*$; and $\hat{p}(y_h - y_l) + y_l - (1+r+\delta_r) \leq 0$ with equality when $(\omega+\tilde{\omega}) \geq \omega^*$.

- When $\tilde{\omega} = \omega_{reg} - \omega$, equation 29 shows $V(\omega, \tilde{\omega} = \omega_{reg} - \omega) = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0)$, so $\frac{\partial V(\omega,\tilde{\omega}=\omega_{reg}-\omega)}{\partial \omega} = \frac{1}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0) \geq 0$.

A.6 Proof of Proposition 6 - Transmission of Monetary Policy

A.6.1 Proposition 6.1: $\frac{\partial \hat{p}}{\partial r} > 0$

1. For banks with $\omega + \tilde{\omega} \geq \omega^*$, recall that $\hat{p}(\omega+\tilde{\omega}) = \frac{(1+r+\delta_r)-y_l}{(y_h - y_l)}$, so $\frac{\partial \hat{p}}{\partial r} > 0$.

2. For banks with $\omega + \tilde{\omega} < \omega^*$ (and thus $y_l < R(\omega+\tilde{\omega})$) we have from equation 17 and equation 18:

   $\hat{p}(y_h - R(\omega+\tilde{\omega})) = (\omega+\tilde{\omega})(1+r+\delta_r)$; and

   $A(\hat{p}) R(\omega+\tilde{\omega}) + (1 - A(\hat{p})) y_l = (1-\omega-\tilde{\omega})(1+r+\delta_r)$.

   Partially differentiate both equations with respect to r, and solving the resulting system of linear equations to give:

   $\frac{\partial \hat{p}(\omega+\tilde{\omega},r)}{\partial r} = \frac{A(\hat{p})(\omega+\tilde{\omega}) + \hat{p}(1-\omega-\tilde{\omega})}{(y_h - R)A(\hat{p}) + \hat{p}A'(\hat{p})(R - y_l)} > 0$. Q.E.D.

[12] Leibniz Integral Rule:
$\frac{d}{dx} \int_{y_0}^{y_1} f(x,y)dy = \int_{y_0}^{y_1} f_x(x,y)dy + f(x,y_1)\frac{dy_1}{dx} - f(x,y_0)\frac{dy_0}{dx}$ provided f and f_x are both continuous over a region in the form $[x_0, x_1] \times [y_0, y_1]$

A.6.2 Proposition 6.2: $\frac{\partial \hat{\boldsymbol{\omega}}}{\partial r} > 0$

Recall that $\hat{\omega}\left(\tilde{\omega}, r+\delta_r\right)$ is implicitly defined by:

$$V(\hat{\omega}, \tilde{\omega}, r+\delta_r) = \frac{\hat{\omega}}{\hat{\omega}+\tilde{\omega}} \int_{\hat{p}}^{1} \left[p\left(y_h - y_l\right) + y_l - \left(1+r+\delta_r\right)\right] h(p) dp = C$$

We know also from proposition 5 that $\frac{\partial V(\omega,\tilde{\omega},r+\delta_r)}{\partial \omega} \begin{cases} > 0 & \text{for} \omega+\tilde{\omega}<\omega^* \\ = 0 & \text{for } \omega+\tilde{\omega} \geq \omega^* \end{cases}$. So to show $\frac{\partial \hat{\omega}}{\partial r} > 0$ for $\hat{\omega} < \omega^{*}$[13], it would be sufficient to show that $\frac{\partial V(\omega,\tilde{\omega},r)}{\partial r} < 0$ for $\omega+\tilde{\omega} < \omega^*$ (by the implicit function theorem).

Using the Leibniz Integral Rule we get:

$$\frac{\partial V(\omega, \tilde{\omega}, r+\delta_r)}{\partial r} = -\frac{\omega}{\omega+\tilde{\omega}}\left[1 - H\left(\hat{p}\right)\right] + \frac{\omega}{\omega+\tilde{\omega}}\left[\left(1+r+\delta_r\right) - y_l - \hat{p}\left(y_h - y_l\right)\right] h\left(\hat{p}\right) \frac{\partial \hat{p}}{\partial r}$$

So $\frac{\partial V}{\partial r} < 0$ for $\omega+\tilde{\omega} < \omega^*$ iff $\frac{\partial \hat{p}}{\partial r}\left(\omega+\tilde{\omega} < \omega^*\right) < \frac{1-H(\hat{p})}{[(1+r+\delta_r)-y_l-\hat{p}(y_h-y_l)]h(\hat{p})}$ [14]

For $\omega+\tilde{\omega} < \omega^*$, recall from the first part of this proposition that:

$\frac{\partial \hat{p}}{\partial r}\left(\omega+\tilde{\omega} < \omega^*\right) = \frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(1-\omega-\tilde{\omega})}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)}$

Substituting this into the inequality above means we need to show:

$\frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(1-\omega-\tilde{\omega})}{(y_h-R)A(\hat{p})+\hat{p}A'(\hat{p})(R-y_l)} < \frac{1-H(\hat{p})}{[(1+r+\delta_r)-y_l-\hat{p}(y_h-y_l)]h(\hat{p})}$

Re-arranging equation 17 and 18 (when $\omega+\tilde{\omega} < \omega^*$) gives:

$y_h - R = \frac{(\omega+\tilde{\omega})(1+r+\delta_r)}{\hat{p}}$; $R = \frac{(1-(\omega+\tilde{\omega}))(1+r+\delta_r)-y_l}{A(\hat{p})} + y_l$; and $(y_h - y_l) = \frac{(\omega+\tilde{\omega})(1+r+\delta_r)}{\hat{p}} + \frac{(1-(\omega+\tilde{\omega}))(1+r+\delta_r)-y_l}{A(\hat{p})}$.

It can also be shown generically that $\frac{1-H(\hat{p})}{h(\hat{p})} = \frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$ (to be shown in Annex # below).

So substituting these results into the inequality above we get:

$\frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(1-\omega-\tilde{\omega})}{\left(\frac{(\omega+\tilde{\omega})(1+r+\delta_r)}{\hat{p}}\right)A(\hat{p})+\hat{p}A'(\hat{p})\left(\frac{(1-(\omega+\tilde{\omega}))(1+r+\delta_r)-y_l}{A(\hat{p})}\right)} < \frac{1}{\left[(1+r+\delta_r)-y_l-\hat{p}\left(\frac{(\omega+\tilde{\omega})(1+r+\delta_r)}{\hat{p}}+\frac{(1-(\omega+\tilde{\omega}))(1+r+\delta_r)-y_l}{A(\hat{p})}\right)\right]} \frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$

Re-arrange and simplify the RHS to give:

$\frac{A(\hat{p})(\omega+\tilde{\omega})+\hat{p}(1-\omega-\tilde{\omega})}{\left(\frac{(\omega+\tilde{\omega})(1+r+\delta_r)}{\hat{p}}\right)A(\hat{p})+\hat{p}A'(\hat{p})\left(\frac{(1-(\omega+\tilde{\omega}))(1+r+\delta_r)-y_l}{A(\hat{p})}\right)} < \frac{1}{[(1-(\omega+\tilde{\omega}))(1+r+\delta_r)-y_l]} \frac{A(\hat{p})}{A'(\hat{p})}$

So:

$$\left[A\left(\hat{p}\right)\left(\omega+\tilde{\omega}\right) + \hat{p}(1-(\omega+\tilde{\omega}))\right]\left[(1-(\omega+\tilde{\omega}))\left(1+r+\delta_r\right) - y_l\right] < \left\{ \begin{array}{c} \left(\frac{(\omega+\tilde{\omega})(1+r+\delta_r)}{\hat{p}}\right) A\left(\hat{p}\right) \frac{A(\hat{p})}{A'(\hat{p})} + \\ \hat{p}\left(\left(1-(\omega+\tilde{\omega})\right)\left(1+r+\delta_r\right) - y_l\right) \end{array} \right\}$$

... which can be further simplified to:

$A\left(\hat{p}\right)\left(1-(\omega+\tilde{\omega})\right)\left(1+r+\delta_r\right) - A\left(\hat{p}\right) y_l - \hat{p}\left[\left(1-(\omega+\tilde{\omega})\right)\left(1+r+\delta_r\right) - y_l\right] < \frac{A(\hat{p})}{\hat{p}} \frac{A(\hat{p})}{A'(\hat{p})}\left(1+r+\delta_r\right)$.

[13] $\frac{\partial \hat{\omega}}{\partial r}$ for $\omega+\tilde{\omega} \geq \omega^*$ is not well defined.

[14] We are looking at cases where $\omega+\tilde{\omega} < \omega^*$, so $\left[\left(1+r+\delta_r\right) - y_l - \hat{p}\left(y_h - y_l\right)\right] > 0$.

Recall from equation 18 : $(1-(\omega+\tilde{\omega}))(1+r+\delta_r)-y_l = A(\hat{p})(R-y_l) > 0$, so for the above inequality to hold it is sufficient to show that:

$A(\hat{p})(1-(\omega+\tilde{\omega}))(1+r+\delta_r) < \frac{A(\hat{p})}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})}(1+r+\delta_r)$; or $(1-(\omega+\tilde{\omega})) < \frac{1}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})}$.

For $p \sim U[0,1]$, $\frac{1}{\hat{p}}\frac{A(\hat{p})}{A'(\hat{p})} = \frac{(\hat{p}+1)}{\hat{p}} > 1 > (1-(\omega+\tilde{\omega}))$. Therefore, $\frac{\partial V}{\partial r} < 0$ for $\omega+\tilde{\omega} < \omega^*$, and $\frac{\partial \hat{\omega}}{\partial r} > 0$ for $\hat{\omega} < \omega^*$. Q.E.D.

A.7 Relationship between conditional expectation and probability density function

Proposition 8. $\frac{1-H(\hat{p})}{h(\hat{p})} = \frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$ *, where* $A(\hat{p}) \equiv \frac{1}{[1-H(\hat{p})]}\int_{\hat{p}}^{p_{ub}} ph(p)\,dp$ *and* p_{ub} *is the upper bound of the p distribution with pdf* $h(.)$ *and cdf* $H(.)$*.*

Proof. Let $g(p) = \int ph(p)\,dp$, then $A(\hat{p}) = \frac{1}{[1-H(\hat{p})]}[g(p_{ub})-g(\hat{p})]$, and

$A'(\hat{p}) = \frac{h(\hat{p})}{[1-H(\hat{p})]^2}[g(p_{ub})-g(\hat{p})] + \frac{1}{[1-H(\hat{p})]}g'(\hat{p}) = A(\hat{p})\frac{h(\hat{p})}{[1-H(\hat{p})]} + \frac{1}{[1-H(\hat{p})]}\hat{p}h(\hat{p})$

So $A'(\hat{p}) = (A(\hat{p})-\hat{p})\frac{h(\hat{p})}{[1-H(\hat{p})]}$, and $\frac{1-H(\hat{p})}{h(\hat{p})} = \frac{A(\hat{p})-\hat{p}}{A'(\hat{p})}$ □

A.8 Proof of Proposition 7

Part 1 and 2 of the proposition are proved in the main text (these are corollaries of previous propositions). We start with the proof for part 4 of the proposition here, and use the result $\frac{\partial\hat{\omega}(\omega_{reg}-\omega,r)}{\partial\omega_{reg}} > 0$ to prove part 3 of the proposition.

A.8.1 Proof of Proposition 7.4: $\frac{\partial\hat{\omega}(\omega_{reg}-\omega,r)}{\partial\omega_{reg}} \geq 0$

Given equation 30 in Annex A.4, we have $\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega) = \frac{\omega_{reg}C}{V(\omega=\omega_{reg},\tilde{\omega}=0)}$, so

$$\frac{\partial\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r)}{\partial\omega_{reg}} = \frac{C}{V(\omega=\omega_{reg},\tilde{\omega}=0)} - \frac{\omega_{reg}C}{[V(\omega=\omega_{reg},\tilde{\omega}=0)]^2}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}} = C\left\{\frac{V(\omega=\omega_{reg},\tilde{\omega}=0)-\omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}}}{[V(\omega=\omega_{reg},\tilde{\omega}=0)]^2}\right\}$$

implying that for non-zero C and $V(\omega=\omega_{reg},\tilde{\omega}=0)$, $\frac{\partial\hat{\omega}(\tilde{\omega}=\omega_{reg}-\omega,r)}{\partial\omega_{reg}} \geq 0$ iff $V(\omega=\omega_{reg},\tilde{\omega}=0) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}} \geq 0$.

Recall that $V(\omega,\tilde{\omega}) \geq 0$ by definition (V is the value of an option in risky bank lending), so when $\omega_{reg}=0$: $V(\omega=\omega_{reg},\tilde{\omega}=0) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}} = V(0,0) \geq 0$. All that remains is to show $V(\omega=\omega_{reg},\tilde{\omega}=0) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}}$ is non-decreasing for $\omega_{reg} \in [0,1]$.

Differentiating with respect to ω_{reg} gives:

$$\begin{aligned}\frac{\partial\left\{V(\omega=\omega_{reg},\tilde{\omega}=0)-\omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}}\right\}}{\partial\omega_{reg}} &= \left\{\begin{array}{c}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}} + ...\\ -\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}} - \omega_{reg}\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}^2}\end{array}\right\}\\ &= -\omega_{reg}\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial\omega_{reg}^2}\end{aligned}$$

So we need to show that $\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}^2} \leq 0$.

Recall from the proof to proposition 5 that:

$$\frac{\partial V(\omega, \tilde{\omega}=0)}{\partial \omega} = -\left[\hat{p}\left(y_h - y_l\right) + y_l - \left(1 + r + \delta_r\right)\right] h(\hat{p}) \frac{\partial \hat{p}}{\partial \omega}$$

So we have:

$$\begin{aligned} \frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}^2} &= \frac{\partial^2 V(\omega,0)}{\partial \omega^2}\left(\omega=\omega_{reg}\right) \\ &= -\left\{ \begin{array}{c} \left[\frac{\partial \hat{p}(\omega_{reg})}{\partial \omega}\left(y_h - y_l\right)\right] h(\hat{p}\left(\omega_{reg}\right)) \frac{\partial \hat{p}(\omega_{reg})}{\partial \omega} + \\ h'(\hat{p}\left(\omega_{reg}\right)) \left[\hat{p}\left(\omega_{reg}\right)\left(y_h - y_l\right) + y_l - \left(1 + r + \delta_r\right)\right] \frac{\partial \hat{p}(\omega_{reg})}{\partial \omega} + \\ \left[\hat{p}\left(\omega_{reg}\right)\left(y_h - y_l\right) + y_l - \left(1 + r + \delta_r\right)\right] h(\hat{p}\left(\omega_{reg}\right)) \frac{\partial^2 \hat{p}(\omega_{reg})}{\partial^2 \omega} \end{array} \right\} \end{aligned}$$

For uniformly distributed p, $h'(.) = 0$. So $\frac{\partial^2 \hat{p}(\omega_{reg})}{\partial^2 \omega} \leq 0$ is sufficient for $\frac{\partial^2 V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}^2} \leq 0$.[15] This in turn implies that $\frac{\partial \left\{V(\omega=\omega_{reg},\tilde{\omega}=0) - \omega_{reg}\frac{\partial V(\omega=\omega_{reg},\tilde{\omega}=0)}{\partial \omega_{reg}}\right\}}{\partial \omega_{reg}} \geq 0$ and finally $\frac{\partial \hat{\omega}(\omega_{reg}-\omega,r)}{\partial \omega_{reg}} \geq 0$.

A.8.2 Proof of Proposition 7.3: $\hat{\omega}\left(\tilde{\omega} = \omega_{reg} - \omega, r\right) > \hat{\omega}\left(\tilde{\omega} = 0, r\right)$ for all $\omega_{reg} > \hat{\omega}\left(0, r\right)$

From the (implicit) definition of $\hat{\omega}$, we have:

$V\left(\hat{\omega}\left(0, r\right), \tilde{\omega} = 0\right) = C$; and

$V\left(\hat{\omega}\left(\omega_{reg} - \omega, r\right), \tilde{\omega} = \omega_{reg} - \omega\right) = C$.

We also know from equation 29: $V(\omega, \tilde{\omega} = \omega_{reg} - \omega) = \frac{\omega}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0)$ [i.e. the value of screening for a bank which falls short of the regulatory capital requirement ($\omega_i < \omega_{reg}$) is a $\frac{\omega_i}{\omega_{reg}}$ fraction of the value of screening for a bank that is just meeting the regulatory requirement].

Taken together, the above equations imply

$$V\left(\hat{\omega}\left(\omega_{reg} - \omega, r\right), \tilde{\omega} = \omega_{reg} - \omega\right) = \frac{\hat{\omega}\left(\omega_{reg} - \omega, r\right)}{\omega_{reg}} V(\omega = \omega_{reg}, \tilde{\omega} = 0) = C$$

and therefore $\hat{\omega}\left(\omega_{reg} - \omega, r\right) = \omega_{reg}$ iff $\omega_{reg} = \hat{\omega}\left(0, r\right)$. In other words, when the minimum capital requirement is set at the level of participation threshold which prevails in the absence of the requirement, then the capital requirement has no impact on participation. We can define a 'binding regulatory regime' as one where $\omega_{reg} > min\left[\hat{\omega}\left(\omega_{reg} - \omega, r\right), \hat{\omega}\left(0, r\right)\right]$, because if the capital requirement is less than or equal to the lower of the two

[15] Recall for $\omega + \tilde{\omega} < \omega^*$: $\frac{\partial \hat{p}}{\partial \omega} = \frac{(A(\hat{p}) - \hat{p})(1+r+\delta_r)}{(y_h - R)A(\hat{p}) + \hat{p}A'(\hat{p})(R - y_l)} > 0$. For uniformly distributed p, we have $h\left(p\right) = \frac{1}{p_{ub} - p_{lb}}$, $h'\left(p\right) = 0$ and $A\left(\hat{p}\right) = \frac{1}{2}\left(\hat{p} + p_{ub}\right)$.

So $\frac{\partial \hat{p}}{\partial \omega} = \frac{\left(\frac{1}{2}(\hat{p}+p_{ub}) - \hat{p}\right)(1+r+\delta_r)}{(y_h - R)\frac{1}{2}(\hat{p}+p_{ub}) + \hat{p}\frac{1}{2}(R - y_l)} = \frac{(p_{ub} - \hat{p})(1+r+\delta_r)}{p_{ub}(y_h - R) + \hat{p}(y_h - y_l)} > 0$.

$\frac{\partial^2 \hat{p}}{\partial \omega^2} = \frac{-\frac{\partial \hat{p}}{\partial \omega}(1+r+\delta_r)[p_{ub}(y_h - R) + \hat{p}(y_h - y_l)] - (p_{ub} - \hat{p})(1+r+\delta_r)\left[-p_{ub}\frac{\partial R}{\partial \omega} + \frac{\partial \hat{p}}{\partial \omega}(y_h - y_l)\right]}{[p_{ub}(y_h - R) + \hat{p}(y_h - y_l)]^2} \leq 0$ since $\frac{\partial \hat{p}}{\partial \omega} > 0$ and $\frac{\partial R}{\partial \omega} \leq 0$.

participation thresholds then no banks would be bound by it should their decide to engage in lending.

The rest of the proof is shown by contradiction:

Suppose $\hat{\omega}\left(\tilde{\omega}=\omega_{reg}-\omega,r\right)\leq\hat{\omega}\left(\tilde{\omega}=0,r\right)$ and the regulatory regime is binding $\omega_{reg}>min\left[\hat{\omega}\left(\omega_{reg}-\omega,r\right),\hat{\omega}\left(0,r\right)\right]$. We can then examine the participation thresholds for banks with $\omega_i<\omega_{reg}$ (banks for which the regulatory regime is binding), and show that a contradiction must arise:

1. First consider regulatory regime (i): $\hat{\omega}\left(\tilde{\omega}=0,r\right)\geq\omega_{reg}>\hat{\omega}\left(\tilde{\omega}=\omega_{reg}-\omega,r\right)$.

   Under this regime $\omega_{reg}>\hat{\omega}\left(\tilde{\omega}=\omega_{reg}-\omega,r\right)$. So for the condition shown at the start of the proof

$$\begin{aligned}V\left(\hat{\omega}\left(0,r\right),\tilde{\omega}=0\right) &= V\left(\hat{\omega}\left(\omega_{reg}-\omega,r\right),\tilde{\omega}=\omega_{reg}-\omega\right)\\ &= \frac{\hat{\omega}\left(\omega_{reg}-\omega,r\right)}{\omega_{reg}}V(\omega=\omega_{reg},\tilde{\omega}=0)\end{aligned}$$

   ... to hold we need $V(\omega=\omega_{reg},\tilde{\omega}=0)>V\left(\hat{\omega}\left(0,r\right),\tilde{\omega}=0\right)$, which given $\frac{\partial V(\omega,\tilde{\omega})}{\partial\omega}\geq 0$ (see proof to proposition 5) means $\omega_{reg}>\hat{\omega}\left(0,r\right)$. Contradiction.

2. Now consider regulatory regime (ii): $\omega_{reg}>\hat{\omega}\left(\tilde{\omega}=0,r\right)\geq\hat{\omega}\left(\tilde{\omega}=\omega_{reg}-\omega,r\right)$

   We have already shown that $\hat{\omega}\left(\omega_{reg}-\omega,r\right)=\omega_{reg}$ iff $\omega_{reg}=\hat{\omega}\left(0,r\right)$; which implies $\hat{\omega}\left(\omega_{reg}-\omega,r\right)=\hat{\omega}\left(0,r\right)$ when $\omega_{reg}=\hat{\omega}\left(0,r\right)$. So given $\frac{\partial\hat{\omega}(\omega_{reg}-\omega,r)}{\partial\omega_{reg}}\geq 0$ (Proposition 7.4), when $\omega_{reg}>\hat{\omega}\left(0,r\right)$, $\hat{\omega}\left(\omega_{reg}-\omega,r\right)>\hat{\omega}\left(0,r\right)$. Contradiction.

Therefore $\hat{\omega}\left(\tilde{\omega}=\omega_{reg}-\omega,r\right)>\hat{\omega}\left(\tilde{\omega}=0,r\right)$ for all $\omega_{reg}>\hat{\omega}\left(0,r\right)$. Q.E.D.

Figure A1: Aggregate welfare under different policy choices [Benchmark Calibration]

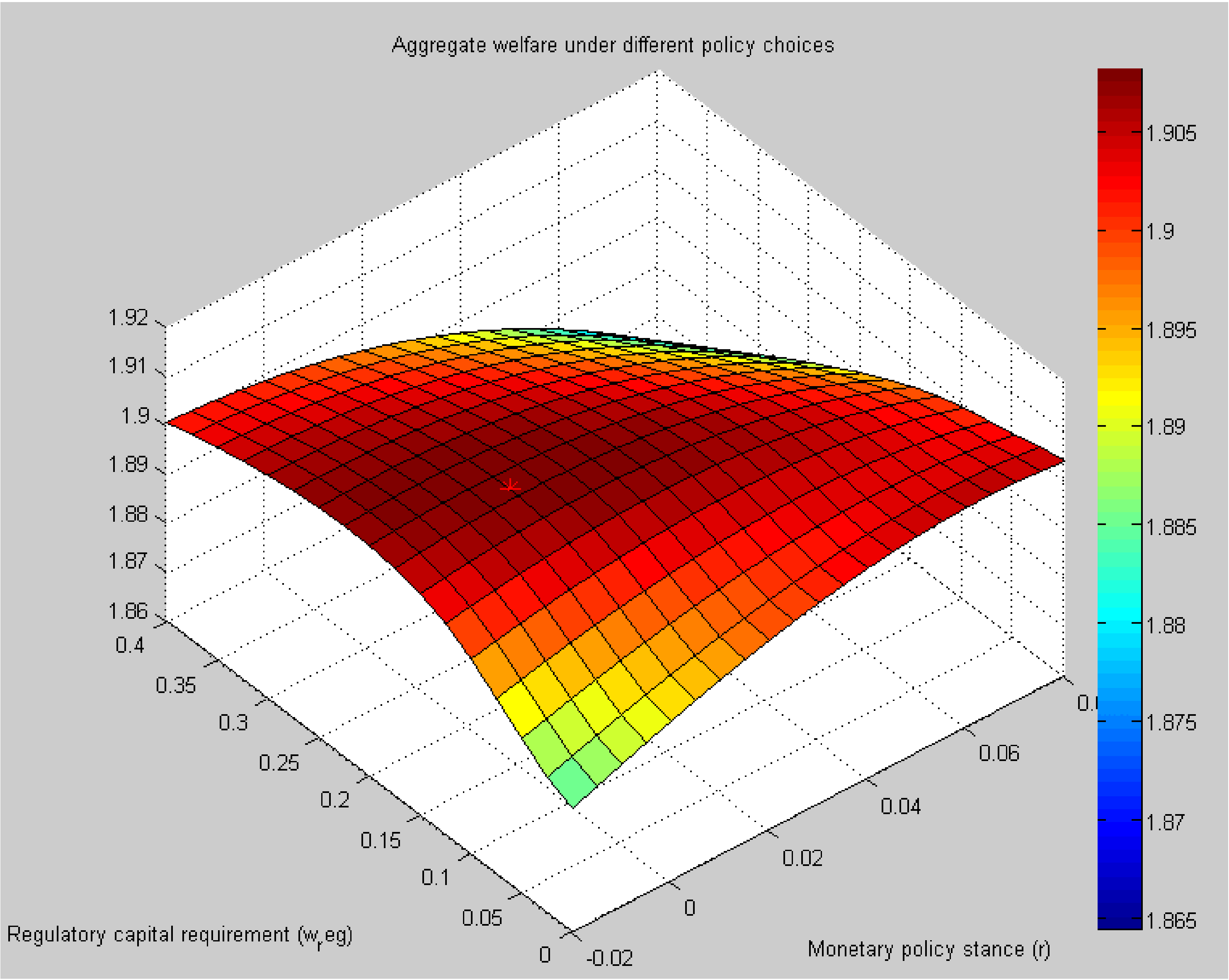

Table A1: Aggregate welfare under different policy choices [Benchmark Calibration]

Aggregate Welfare		**Capital Requirement**																	
		0.0%	**2.5%**	**5.0%**	**7.5%**	**10.0%**	**12.5%**	**15.0%**	**17.5%**	**20.0%**	**22.5%**	**25.0%**	**27.5%**	**30.0%**	**32.5%**	**35.0%**	**37.5%**	**40.0%**	Max
Interest Rate	**-2.0%**	1.8852	1.8876	1.8916	1.8957	1.8999	1.9031	1.9052	1.9065	1.9071	*1.9072*	1.9070	1.9064	1.9056	**1.9045**	**1.9033**	**1.9020**	**1.9005**	1.9072
	-1.5%	1.8872	1.8893	1.8930	1.8970	1.9009	1.9040	1.9059	1.9070	1.9075	*1.9075*	1.9072	**1.9065**	**1.9056**	1.9045	1.9032	1.9017	1.9002	1.9075
	-1.0%	1.8890	1.8909	1.8944	1.8982	1.9019	1.9048	1.9065	1.9075	*1.9078*	1.9077	**1.9072**	1.9065	1.9055	1.9043	1.9029	1.9014	1.8997	1.9078
	-0.5%	1.8908	1.8925	1.8957	1.8993	1.9027	1.9054	1.9070	1.9078	*1.9080*	**1.9078**	1.9072	1.9063	1.9052	1.9039	1.9025	1.9009	1.8992	1.9080
	0.0%	1.8926	1.8940	1.8970	1.9003	1.9035	1.9060	1.9074	1.9080	**1.9081**	1.9078	1.9071	1.9061	1.9049	1.9035	1.9019	1.9003	1.8985	1.9081
	0.5%	1.8942	1.8954	1.8981	1.9012	1.9042	1.9065	1.9077	**1.9082**	1.9081	1.9076	1.9068	1.9057	1.9044	1.9029	1.9012	1.8995	1.8976	1.9082
	1.0%	1.8958	1.8968	1.8992	1.9020	1.9048	1.9068	1.9078	*1.9082*	1.9079	1.9073	1.9064	1.9052	1.9037	1.9022	1.9004	1.8986	1.8966	1.9082
	1.5%	1.8973	1.8980	1.9002	1.9028	1.9053	1.9070	**1.9079**	*1.9080*	1.9077	1.9069	1.9058	1.9045	1.9030	1.9013	1.8994	1.8975	1.8955	1.9080
	2.0%	1.8986	1.8992	1.9011	1.9034	1.9056	**1.9072**	*1.9078*	1.9078	1.9072	1.9063	1.9051	1.9037	1.9020	1.9002	1.8983	1.8963	1.8942	1.9078
	2.5%	1.8999	1.9002	1.9019	1.9039	1.9058	1.9072	*1.9076*	1.9074	1.9067	1.9056	1.9043	1.9027	1.9010	1.8991	1.8970	1.8949	1.8927	1.9076
	3.0%	1.9010	1.9011	1.9025	1.9043	**1.9060**	1.9070	*1.9072*	1.9068	1.9060	1.9048	1.9033	1.9016	1.8997	1.8977	1.8956	1.8934	1.8911	1.9072
	3.5%	1.9020	1.9020	1.9031	1.9045	1.9059	*1.9068*	1.9067	1.9062	1.9051	1.9038	1.9022	1.9003	1.8983	1.8962	1.8940	1.8916	1.8892	1.9068
	4.0%	1.9029	1.9027	1.9035	**1.9046**	1.9058	*1.9063*	1.9061	1.9053	1.9041	1.9026	1.9009	1.8989	1.8968	1.8945	1.8922	1.8898	1.8873	1.9063
	4.5%	1.9036	1.9032	1.9037	1.9046	1.9055	*1.9058*	1.9053	1.9043	1.9030	1.9013	1.8994	1.8973	1.8950	1.8927	1.8902	1.8877	1.8851	1.9058
	5.0%	1.9042	1.9036	**1.9039**	1.9044	1.9050	*1.9050*	1.9044	1.9032	1.9016	1.8998	1.8977	1.8955	1.8931	1.8907	1.8881	1.8854	1.8827	1.9050
	5.5%	*1.9046*	1.9039	1.9038	1.9041	1.9044	1.9042	1.9032	1.9019	1.9001	1.8981	1.8959	1.8935	1.8910	1.8884	1.8857	1.8830	1.8802	1.9046
	6.0%	*1.9048*	**1.9040**	1.9036	1.9036	1.9036	1.9031	1.9020	1.9004	1.8984	1.8963	1.8939	1.8914	1.8888	1.8860	1.8832	1.8803	1.8774	1.9048
	6.5%	**1.9048**	1.9039	1.9033	1.9030	1.9027	1.9019	1.9005	1.8987	1.8966	1.8942	1.8917	1.8891	1.8863	1.8834	1.8805	1.8775	1.8745	1.9048
	7.0%	*1.9047*	1.9037	1.9027	1.9021	1.9015	1.9005	1.8988	1.8968	1.8945	1.8920	1.8893	1.8865	1.8836	1.8806	1.8776	1.8745	1.8713	1.9047
	7.5%	*1.9043*	1.9033	1.9020	1.9011	1.9002	1.8989	1.8970	1.8948	1.8923	1.8896	1.8867	1.8838	1.8807	1.8776	1.8744	1.8712	1.8679	1.9043
	8.0%	*1.9037*	1.9027	1.9011	1.8999	1.8987	1.8971	1.8950	1.8925	1.8898	1.8870	1.8840	1.8808	1.8776	1.8744	1.8711	1.8677	1.8643	1.9037
	Max	1.9048	1.9040	1.9039	1.9046	1.9060	1.9072	1.9079	1.9082	1.9081	1.9078	1.9072	1.9065	1.9056	1.9045	1.9033	1.9020	1.9005	1.9082