Procyclical Finance: The Money View*

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Abstract

This paper offers a monetary theory of financial instability. Financial intermediaries’ debt supports trade by serving as a means of payment (“inside money”). Money creation stimulates growth: resources flow towards productive agents through transactions. Money creation also breeds instability: risk accumulates in booms through procyclical intermediary leverage. Under high leverage, even small shocks can significantly deplete intermediaries’ equity capital and drag the economy into a crisis. As a crisis unfolds, intermediaries deleverage and inside money creation collapses. Since intermediaries face recapitalization frictions, the ensuing recovery is slow. As an alternative form of money (“outside money”), government debt may alleviate money shortage. However, it squeezes intermediaries’ profit from money creation, and thereby, slows down their balance-sheet repairment. The latter effect dominates if the government is too timid: a small increase of outside money prolongs a crisis, because the economy still heavily relies on inside money and intermediaries’ recovery. The calibrated model provides quantitative implications on the quantity and price of money and credit, intermediary leverage cycle, and the impact of government debt on the frequency and duration of a crisis.

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1 Introduction

The years leading up to the Great Recession saw rapid growth in the financial sector and significant build-up of its leverage. Spreads narrowed considerably across asset classes, enhancing liquidity provision and stimulating real activity. A booming real economy in turn fueled financial intermediaries’ expansion and leverage. During the crisis, the spiral flipped.

Much progress has been made in recent years in developing dynamic macro models with an intermediation sector that generate financial crises (e.g., He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)), but a complete account of procyclical finance remains a challenge. This paper argues that at the heart of this procyclicality is financial intermediaries’ role as money creators. The monetary aspect of financial intermediation is so ubiquitous that we often fail to notice. Intermediary debt (e.g. bank deposits) is a store of value, but more importantly, it supports trade by serving as a means of payment (“inside money”). In an economy where future income is not fully pledgeable, money facilitates spot transactions and resource reallocation. However, the money demand in the real sector feeds leverage to the financial sector, which generates instability.

I build a continuous-time dynamic model of a macroeconomy that crystallizes this money view of financial intermediation. The model explains a variety of phenomena, such as procyclical intermediary leverage, countercyclical money premium and credit spread. The model features fragile booms and stagnant financial crises. In good times, risk accumulates as intermediaries increase leverage. The longer the boom lasts, the more likely small shocks can trigger a big crisis. In bad times, intermediaries deleverage, and without sufficient money to support trade, the economy falls into a stagnant recession. The calibrated model can predict a ten-year period of recovery.

The idea that financial intermediaries affect the real economy through money supply goes back at least to the classic account of the Great Depression by Friedman and Schwartz (1963). One may argue that this money view is less relevant today in the context of modern countervailing expansion of the monetary base in a crisis (Woodford (2010)). However, the model reveals a quantitatively important channel through which the increase in government-issued money can actually destabilize the financial sector, amplify the leverage cycle, and prolong financial crises.

A monetary theory of financial intermediation requires three ingredients: money demand from the real sector, bank debt that circulates as money, and bank loans extended to the real sector that back money. Accordingly, there are three types of decision makers: households, firms, and banks. Unlike the typical setup in which firms borrow and households lend, the model emphasizes
a money demand of firms who need to hoard liquidity for investment and spot transactions. House-
holds borrow from banks against a collateral asset (housing). This structure intends to capture the
recent experience in the U.S., where nonfinancial corporations hold huge liquidity savings in the
form of money-like securities, and where financial intermediaries issue these securities backed by
mortgage-related assets.

The economy has two types of capital that produce a generic nondurable good for consump-
tion and investment. Households own the “tangible capital”, which we can think of as houses, and
entrepreneurs (“firms”) own the “intangible capital”, which we can think of as professional skills,
proprietary technology, or brand names. Households can also hold firms’ and banks’ equity.

Every instant, a fraction of firms are hit by a liquidity shock: their production halts, and
their capital can either grow – if further investment is made – or perish, if not. Investment is not
pledgeable, so firms must obtain investment inputs (i.e. goods) via spot transactions. In anticipa-
tion, firms hold means of payment in the form of bankers’ risk-free debt (“deposits”), the inside
money.1 This setup captures the savings glut in R&D-intensive sectors.2 In effect, firms face a
deposits-in-advance constraint on investment. Those that do not experience the liquidity shock
operate as in a frictionless economy; in particular, they trade intangible capital in a competitive
market, and raise equity freely from households.3

To back deposits, bankers invest in loans extended to households that are collateralized by
tangible capital. I simplify the model by letting households represent the borrowing demand in
general, including that from industries with physical collateralizable assets (e.g. manufacturing).

Lending is risky. The aggregate shock in my setup is modeled as an exogenous diffusion
process that determines how much collateral is made worthless every instant. Following a bad
shock, more loans default, and bank equity falls. In the spirit of Myers and Majluf (1984), I assume
that banks face an equity dilution cost when issuing new shares, so that they do not recapitalize
immediately. As a result, their capacity to issue risk-free deposits shrinks. In equilibrium, firms
hold less liquidity, so that they invest less. In sum, the real consequence of an intermediation

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1The term is borrowed from Gurley and Shaw (1960). From the private sector’s perspective, fiat money and
government securities are in positive supply (“outside money”), while deposits, as bank liabilities, are in zero net

2The corporate savings glut has been documented in Bates, Kahle, and Stulz (2009) among others. The increase in
nonfinancial firms’ liquidity holdings in the last three decades is largely driven by the rise of new-economy industries,
such as technology, healthcare, and R&D-intensive industries in general (Begenau and Palazzo (2015); Graham and
Leary (2015); Pinkowitz, Stulz, and Williamson (2016)). The phenomenon arises against a general landscape of
transformation towards an intangible-intensive economy (Corrado and Hulten (2010)).

3In reality, patents can be traded, and product design, brand, a division, or a whole firm can be bought and sold.
disruption is the collapse of money creation, which feeds into lower investment.

Consider now the effects of a good shock. Fewer loans default than expected. Due to the dilution cost, one dollar inside the bank is worth more than a dollar paid out, so banks optimally hoard this windfall. Therefore, the shock’s positive impact on bank equity will only dissipate gradually, so that firms expect bankers to issue more inside money going forward, which they can hold to fund future investments. This expectation raises the equilibrium price of intangible capital. Since intangible capital is more valuable, firms’ current money demands increase in anticipation of the opportunities to create more capital.

Firms’ money demands are procyclical due to this intertemporal complementarity that works through the price response of intangible capital. Following a good shock, both money supply and money demand increase, so that equilibrium bank deposits may grow faster than bank equity, giving rise to procyclical bank leverage. This fundamental prediction of my model contrasts with the countercyclical leverage predicted by models that feature a static demand for intermediaries’ debt and focus exclusively on the asset-side of intermediary balance sheets (e.g. He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014); Phelan (2015); Klimenko et al. (2016)).

Another critical aspect of banks’ balance-sheet dynamics is equity issuance and payout. The state variable in my model is the size of bank equity relative to GDP. Good shocks result in higher bank equity. However, the intensified competition in the deposit and loan markets causes banks’ investment opportunity set to become less attractive. Therefore, if the economy keeps experiencing good shocks, banks will eventually choose to pay out dividends. Similarly, when bad shocks significantly deplete bank equity, the investment opportunity set becomes more attractive, so that banks may choose to pay the dilution cost to raise more equity. The state variable, and the economy, thus vacillate between an issuance and a payout boundary. Near the issuance boundary, banks are undercapitalized and create less deposits; near the payout boundary, banks are well capitalized, so that firms can hold more deposits for investment.\textsuperscript{4}

When the economy is close to either boundary, shocks have an asymmetric impact on banks’ equity, and on their deposit creation capacity. Near the issuance boundary, the impact of bad shocks is limited, because when equity declines too much, banks will issue new shares. In contrast, the impact of good shocks can be large. Near the payout boundary, good shocks can trigger dividend payments, so bank equity cannot grow further. In contrast, the impact of bad shocks can be large.

\textsuperscript{4}As in Phelan (2015) and Klimenko et al. (2016), banks issue equity in bad times, and pay out dividends in good times, which is consistent with the evidence in Baron (2014) and Adrian, Boyarchenko, and Shin (2015).
Combined with this asymmetric impact of shocks, the procyclicality of bank leverage implies that booms are unstable and crises are stagnant. In a boom, high leverage only makes payout more frequent following goods shocks, while bad shocks have an amplified impact on bank equity. Thus, as the economy grows closer to the payout boundary, it also becomes more fragile. Near the issuance boundary, where banks are undercapitalized, low leverage means shocks have a relatively small impact on bank equity. Extremely bad shocks are rare, so issuance is relatively infrequent. More importantly, banks need a sequence of *sufficiently large* good shocks to speed up the accumulation of equity. As a result, the economy is stuck with undercapitalized banks.

Banks add value to the economy by creating deposits that firms can hold as a liquidity buffer. Aside from banks’ balance-sheet capacity, another limit on deposit creation is household borrowing capacity. Since deposits are backed by loans, if households are not borrowing, bankers cannot issue deposits. And because bank loans take the form of collateralized debt, what limits household borrowing is the total value of tangible capital in the economy. Therefore, when banks are well capitalized and willing to extend loans and issue deposits, household borrowing constraint binds, which gives rise to a collateral shadow value of tangible capital.\(^5\)

This collateral value ultimately comes from the convenience yield that firms assign to bank deposits. In other words, it is an *intermediated collateral premium*, and only arises when the banking sector is well capitalized. This link between intermediary balance-sheet capacity to asset prices differs from the typical mechanism that is based on intermediaries’ exclusive access to assets (e.g. He and Krishnamurthy (2013)). In sum, the model provides an anatomy of money shortage: in equilibrium, the convenience yield of deposits (or money premium), is split into a risk compensation required by banks, and a collateral premium of tangible capital.

Finally, I consider the effects of government-issued money, which can be interpreted as including a broad range of government-issued debt securities, not just those that back central bank’s liabilities, but also those actively traded in secondary markets or serving as repo collateral with minimal margins.\(^6\) I assume that there is a fixed government debt capacity. The issuance proceeds are distributed as lump-sum transfer, and debt is repaid with lump-sum tax.

By expanding the money supply, government debt alleviates the money shortage faced by firms. However, it raises the yield on bank debt through the competition in the money market,

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\(^5\) Adrian, Colla, and Shin (2012) and Becker and Ivashina (2014) document the procyclicality of bank lending.

\(^6\) The monetary services of government debt is an old theme (e.g., Patinkin (1965); Friedman (1969)). Recent contributions include Bansal and Coleman (1996), Bansal, Coleman, and Lundblad (2011), Krishnamurthy and Vissing-Jorgensen (2012), Greenwood, Hanson, and Stein (2015), Bolton and Huang (2016), and Nagel (2016).
and thereby, government debt reduces banks’ profits and return on equity. In good times, banks are more willing to pay out dividends, because retaining equity has become less profitable. In bad times, banks delay recapitalization, waiting for a severe money shortage and a large money premium that boosts return on equity and justifies the issuance cost. As a result, banks’ leverage can become more procyclical, leading to more frequent crises.

Government debt can also lengthen crises. By squeezing banks’ profit, government debt slows down the rebuilding of bank equity. Unless the government issues a large amount of debt that nearly satiates firms’ money demand, economic recovery still relies on banks’ capacity to create deposits. Therefore, the economy can get stuck with underinvestment for an extended period of time. The calibrated model shows that raising government debt-to-output ratio from 50% to 100% adds 10% to the probability of recession, and doubles the duration of recession.

**Related literature.** Financial intermediaries provide liquidity – the ease of transferring resources over time and between people. They finance projects (credit supply), and issue securities that facilitate trade (money supply). The cost of liquidity is zero in the frictionless world of Modigliani and Miller (1958), but in reality, we rely on intermediaries to supply money and credit, and due to their limited balance-sheet capacity, they earn a spread. This paper focuses on money supply. It advances an old tradition by taking a corporate-finance approach that emphasizes money as a store of value and a means of payment rather than a unit of account, which has been the critical ingredient of models with nominal rigidities (e.g. Christiano, Eichenbaum, and Evans (2005)).

A recent set of papers has revived the money view of financial intermediaries by emphasizing bank debt as a medium of exchange (e.g., Hart and Zingales (2014); Quadrini (2014); Piazzesi and Schneider (2016)). This paper builds on this money view to explain procyclical finance, and in particular, the procyclical leverage of financial intermediaries, and resulting financial instability and economic stagnation.

My modeling approach is closely related to macroeconomic models on the balance-sheet channel. A key variable in these models is productive agents’ financial slack, mostly measured by net worth (e.g., Bernanke and Gertler (1989); Kiyotaki and Moore (1997)). Recently, the focus has

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7There are several branches of literature that provide a microfoundation for bank debt serving as a medium of exchange. Limited commitment (Kiyotaki and Moore (2002)) and imperfect record keeping (Kocherlakota (1998)) restrict credit, so trades must engage in *quid pro quo*, involving a transaction medium. Banks overcome these problems and supply money (e.g., Kiyotaki and Moore (2000); Cavalcanti and Wallace (1999)). Ostroy and Starr (1990) and Williamson and Wright (2010) review the literature of monetary theories. Another approach relates asset resalability to information sensitivity. Intermediaries create liquidity by issuing information-insensitive claims (e.g., Gorton and Pennacchi (1990); DeMarzo and Duffie (1999); Holmström (2012); Dang et al. (2014)).
been on financial intermediaries’ balance sheets. Models with a static demand for intermediary debt typically predict countercyclical leverage because intermediary equity is more responsive to shocks than their assets (e.g., He and Krishnamurthy (2012); Brunnermeier and Sannikov (2014)). This paper features an endogenously procyclical money demand that leads to procyclical leverage.

The existing studies on procyclical leverage focus on the impact of asset price variations on collateral or risk constraints (e.g., Brunnermeier and Pedersen (2009); Geanakoplos (2010); Danielsson, Shin, and Zigrand (2012); Adrian and Shin (2014); Moreira and Savov (2014); Bolton, Santos, and Scheinkman (2016)). This paper offers a complementary explanation based on money demand. More importantly, it unveils a feedback mechanism between the real and financial sectors that sets the stage for a formal analysis of the financial stability implications of government debt.

It has long been recognized that government debt provides monetary services (e.g., Patinkin (1965); Friedman (1969)). Recent literature documents a money premium that lowers the yield on government securities (e.g., Bansal and Coleman (1996); Krishnamurthy and Vissing-Jorgensen (2012); Nagel (2016)). Financial intermediaries capture the money premium by issuing short-term debt, such as deposits (Drechsler, Savov, and Schnabl (2016)) and asset-backed commercial paper (Sunderam (2015)). Many have emphasized the risk of excessive leverage (e.g., Gorton (2010); Stein (2012)), and point out that increasing government debt supply stabilizes the economy by crowding out intermediary debt (e.g., Greenwood, Hanson, and Stein (2015); Krishnamurthy and Vissing-Jorgensen (2015); Woodford (2016)). However, these existing studies leave out the dynamic equity management of banks, which in my model, is a key dimension of their response to government debt supply. By squeezing intermediaries’ profit, government debt also crowds out bank equity, and thereby, can amplify the leverage cycle and destabilize the economy.

As in Woodford (1990b) and Holmström and Tirole (1998), government debt facilitates investment by allowing entrepreneurs to transfer wealth over time. Government has an additional effect in my model as a intermediaries’ competitor in liquidity supply. Public liquidity crowds out intermediated liquidity, especially during bad times when intermediaries become more reluctant to raise capital in response. Government’s ability to commit non-pledgeable income through taxation may not always improve efficiency, even from a pure liquidity provision perspective.

This paper contributes to the literature on the causes of procyclicality in the financial sector

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8For instance, net worth resolves agency issues (e.g., Holmström and Tirole (1997); Diamond and Rajan (2000)).
9In this paper, government debt affects banks’ profit and equity capital by narrowing the money premium. Also highlight the dilution cost of equity issuance, Bolton and Freixas (2000) analyze the effects of monetary policy on banks’ profit and equity capital through changes in the lending spread.
and how government should respond to financial instability and economic stagnation. After the financial crisis, central banks expanded balance sheets in advanced economies through quantitative easing programs, which has raised concerns about moral hazard and inflation (Fischer (2009)). This paper highlights a particular channel through which an expanding government balance sheet can be counterproductive from a financial stability perspective.

Several studies have noted the enormous amount of corporate cash holdings in recent decades (e.g. Bates, Kahle, and Stulz (2009)). This paper shows that corporate savings can feed leverage to the financial sector and contribute to its procyclicality. In the model, money demand arises from firms’ investment needs.\(^\text{10}\) R&D is a typical form of intangible investment that heavily relies on internal liquidity, and exhibits strong procyclicality.\(^\text{11}\) A number of studies have shown that the increase of corporate cash holdings in the last few decades is driven in particular by the entry of R&D-intensive firms (e.g., Begenau and Palazzo (2015); Graham and Leary (2015); Pinkowitz, Stulz, and Williamson (2016)). As the U.S. economy becomes more intangible capital intensive (Corrado and Hulten (2010)), we would expect the model’s mechanism, and more generally, the money view of financial intermediation, to become increasingly relevant.

The remainder of the paper is organized as follows. Section 2 lays out the frictions that give rise to the special role of bank debt as money, and provides an anatomy of money shortage. Section 3 extends the model to a continuous-time setting to explore the implications on bank leverage cycle and boom-bust dynamics in general. Section 4 introduces government debt. Section 5 concludes. Appendix I contains proofs and the solution algorithm. Appendix II shows the model’s performance after adding new features, such as financial innovation (collateral velocity), regulatory leverage constraint, banks’ holdings of long-term assets, banks’ expertise in restructuring bad loans that captures the credit view of banking (e.g., Bernanke (1983)), and state-contingent gov-

\(^\text{10}\) Eisfeldt and Rampini (2009) show that the liquidity premium tends to increase when asynchronicity between cash flow and investment opportunities in the corporate sector becomes more severe, which is consistent with the prediction in Holmström and Tirole (2001). Eisfeldt (2007) shows that liquidity demand driven by household consumption smoothing cannot explain the liquidity premium on Treasury bills. Investment need is a key determinant of corporate cash holdings (e.g., Denis and Sibilkov (2010); Duchin (2010)), especially for firms with less collateral (e.g., Almeida and Campello (2007); Li, Whited, and Wu (2016)) and more intensive R&D activities (Falato and Sim (2014)).

\(^\text{11}\) The procyclicality of R&D expenditures, as measured by the NSF (U.S. National Science Foundations), has been documented by many studies, including Griliches (1990), Fatas (2000), and Comin and Gertler (2006). Using data from the NSF and Compustat, Barley (2007) finds a significant positive correlation between real GDP growth and the growth rate of R&D. Ouyang (2011) documents procyclical R&D at the industry level. Using French firm-level data, Aghion et al. (2012) show that the procyclicality of R&D investment (with respect to sales growth) is found among firms that are financially constrained, and in particular, those with a low degree of asset tangibility. Fabrizio and Tsolmon (2014) find that R&D investments are more procyclical in industries with faster obsolescence. The setup of firms’ liquidity shock is largely motivated by these findings.
government debt supply. Appendix III provides preliminary evidence of firms’ money demand, and the impact of government debt on the cyclicality of intermediary leverage.

2 Static Model

This section lays out the key economic forces in a two-period model. There are three types of agents (households, firms, and bankers) and two types of capital (tangible and intangible). Households own the tangible capital and invest in firms’ and banks’ equity. Firms own the intangible capital. They can raise equity from households at date 0, but some are hit by a liquidity shock at date 1 as in Holmström and Tirole (1998): without further investment, they lose their capital, and investment has to be financed internally. Therefore, firms carry deposits issued by bankers as a liquidity buffer. In turn, bankers back deposits by loans extended to households that are collateralized by tangible capital. Banks’ ability to create deposits depends on their equity capital, and on the amount of collateral in the economy that can support household borrowing.

2.1 Setup

There are two dates, \( t = 0 \) and \( t = 1 \). All agents consume a non-storable, generic good, and have the same risk-neutral utility with discount rate \( \rho \):

\[
U = c_i^0 + \frac{E[c_i^1]}{1 + \rho}, \quad i \in B, I, T
\]

\( c_i^0 \) and \( c_i^1 \) are consumption at \( t = 0 \) and \( t = 1 \), respectively. \( i \) is the agent’s type. Throughout the paper, I use subscripts for time, and superscripts for type (“B” for bankers, “I” for firms who own intangible capital, and “T” for households who own tangible capital).

At \( t = 0 \), a unit mass of households own \( K_0^T \) units of tangible capital (e.g. houses). One unit of capital produces \( \alpha \) units of goods at \( t = 1 \). The household sector can be broadly interpreted to also include industries with tangible assets.\(^{12}\) At \( t = 0 \) tangible capital is traded in a competitive market at price \( q_0^T \) (denominated in goods). Denote the units of capital a representative household carries from \( t = 0 \) to \( t = 1 \) by \( k_0^T \), so that \( K_0^T = \int_{s \in [0,1]} k_0^T(s) \, ds \), where “s” indexes households and will be suppressed going forward.

\(^{12}\)Merging the balance sheets of households and firms who own tangible capital together assumes there is no financial friction that limits the flow of funds between households and these firms.
Households are endowed with a sufficient amount of goods at $t = 0$, so that they need to consume in order to clear the goods market. In equilibrium, $q_0^T$ adjusts to deliver an expected return equal to $\rho$, so households are indifferent with regard to holding capital and consuming. Households also require an expected return of $\rho$ for investments in firms’ and banks’ equity.

The economy has $K_0^I$ units of intangible capital owned by a unit mass of entrepreneurs (“firms”). Intangible capital has the same productivity $\alpha$ as tangible capital. It is only productive in the hands of entrepreneurs. Intangible capital can be traded at $t = 0$, at price $q_0^I$. Let $k_0^I$ denote the units of capital a firm carries from $t = 0$ to $t = 1$, so that $K_0^I = \int_{s \in [0,1]} k_0^I(s) \, ds$. The term “intangible capital” is intended to capture a broad range of non-physical assets – such as knowledge and proprietary technology – the investment in which relies heavily on internal liquidity.13

There is also a unit mass of bankers. Each has $e_0^B$ units of goods, so their aggregate endowment is $E_0^B = \int_{s \in [0,1]} e_0^B(s) \, ds$. Bankers add value to the economy by issuing securities at $t = 0$ that firms can carry for investment at $t = 1$. Next, I will elaborate on firms’ liquidity needs.

**Money demand.** Firms face a liquidity shock at $t = 1$ in the spirit of Holmström and Tirole (1998). At the beginning of date 1, the capital of a fraction $\lambda$ of firms becomes obsolete. These firms either quit or create new capital. By investing $i_1^I$ units of goods per unit of capital, firms can preserve their current production scale and grow at a rate of $F'(i_1^I)$, with $F''(\cdot) > 0$, $F'''(\cdot) < 0$. Following investment, production takes place at the expanded scale, $\alpha \left[ 1 + F'(i_1^I) \right] k_0^I$, but at first, these firms need to purchase goods as investment inputs from other firms, which has to be internally financed.14 Therefore, firms need to carry liquidity to date 1. They do this by holding bank deposits. Let $m_0^I$ denote deposits per unit of capital. Intangible investment is thus directly tied to bank deposits carried from date 0.

$$i_1^I k_0^I \leq m_0^I k_0^I. \quad (1)$$

Equation (1) resembles a cash-in-advance constraint (e.g., Svensson (1985); Lucas and Stokey (1987)), except that what firms hold for transaction purposes is not fiat money, but bank liabilities,

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13Since Hall (1992) and Himmelberg and Petersen (1994), it has been well documented that R&D investment heavily relies on internal financing (see Hall and Lerner (2009) for a survey on innovation financing). A difficulty of external financing is that the knowledge asset created by R&D is intangible, partly embedded in human capital, and ordinarily very specialized to the particular firm in which it resides. It is difficult for investors to repossess such intangible assets in case of default.

14This assumption can be motivated by a typical moral hazard problem as in Holmström and Tirole (1998) or asymmetric information. Investment here can also be interpreted as a reduced form of the effort in product competition where internal liquidity serves a strategic purpose, as in Bolton and Scharfstein (1990) (see evidence in Frésard (2010)).

15To be consistent with the continuous-time expressions, deposits’ interest payments are ignored in Equation (1).
or “inside money.” The key inefficiency that bankers address is supplying deposits that can be held by firms in order to relax this cash-in-advance constraint on intangible investment.

**Money supply.** At \( t = 0 \), bankers issue risk-free debt (deposits) at the competitive deposit rate \( r_0 \), and lend to households against tangible collateral at the competitive loan rate \( R_0 \). Loans are repaid at \( t = 1 \), as long as the collateral is intact.\(^{16}\) At \( t = 1 \), a fraction \( \pi(Z_1) = \delta - \sigma Z_1 \) of tangible capital is destroyed. Households default on loans backed by such capital. For simplicity, I assume that these are all no-recourse loans. The shock \( Z_1 \) is an aggregate shock that cannot be diversified away. It is a binary random variable that takes the value 1 or \(-1\) with equal probability \((\delta - \sigma \geq 0 \text{ and } \delta + \sigma \leq 1)\). Therefore, bankers’ loan portfolio has a return equal to \((1 - \pi(Z_1))(1 + R_0)\). To match the expressions in the continuous-time analysis, I approximate this return with \(1 + R_0 - \pi(Z_1)\), ignoring the product of the two percentages \(\pi(Z_1)R_0\).

Because of the aggregate shock, bankers’ risk-free debt capacity (deposit creation capacity) depends on their equity cushion. Let \( x^B_0 \) denote bank leverage (asset-to-equity ratio). A banker will not default if her net worth is still positive in a bad state, i.e. \( x^B_0 e^B_0 (1 + R_0 - \pi_D(-1)) \geq (x^B_0 - 1) e^B_0 (1 + r_0) \). This incentive constraint – or solvency requirement – can be rewritten as a limit on leverage:

\[
x^B_0 \leq \frac{r_0 + 1}{r_0 + \delta + \sigma - R_0} := \bar{x}_0
\]

At \( t = 0 \), bankers may issue equity to households subject to a proportional dilution cost \( \chi \). To raise one dollar, a bank needs to give \( 1 + \chi \) worth of equity to outside shareholders.\(^{17}\) For simplicity, this static model assumes \( \chi = \infty \). In the continuous-time setting, banks are allowed to issue equity. Thus, the maximum amount of deposits is the aggregate bank equity \( E^B_0 \) multiplied by \((\bar{x}_0 - 1)\).

Figure 1 shows how events unfold. First, deposits are created through lending. When banks lend to households at date 0, they transfer goods to households. Then, when lending exceeds their endowments, bankers credit deposits on households’ accounts.\(^{18}\) Households can use deposits to buy tangible capital or goods from each other, but in aggregate, deposits are paid to firms for their equity. At date 1, when hit by the liquidity shock, firms use these deposits to buy investment inputs. All financial contracts are settled at the end of date 1 after investments are made, so that investing firms cannot hold deposits until maturity, and, must exchange deposits for investment inputs. This timing emphasizes that deposits, beyond being a store of value, facilitate reallocation by serving

\(^{16}\)In a richer setting, collateralized debt may arise as optimal contract (e.g., Townsend (1979); Lacker (2001)).

\(^{17}\)\( \chi \) may arise from information friction in settings such as Myers and Majluf (1984) or Dittmar and Thakor (2007).

\(^{18}\)McLeay, Radia, and Thomas (2014) describe how banks create money through lending in the modern economy.
as a means of payment. The amount of deposits created by banks depends on both bank equity $E_B^0$ (due to the solvency constraint), and tangible capital $K_T^0$ that backs household borrowing. Next, the equilibrium analysis highlights these two limits of inside money creation.

2.2 Equilibrium

Lemma 1 and 2, and Proposition 1 below summarize the optimal choices for households, firms, and banks at $t = 0$. We consider the case where firms’ cash-in-advance constraint binds. A marginal increase of deposits expands investment by $F' \left( m_0^I \right)$, which has an expected net value of $\lambda \left[ \alpha F' \left( m_0^I \right) - 1 \right]$. This convenience yield makes firms willing to accept a return lower than their cost of equity $\rho$. The spread, $\rho - r_0$, is the “money premium” and firms’ cost of carrying money.

**Lemma 1 (Money Demand)** Firms’ equilibrium deposits, $m_0^I$, satisfy the condition

$$\lambda \left[ \alpha' F' \left( m_0^I \right) - 1 \right] = \rho - r_0.$$
expected loan repayment is \((1 - \delta)(1 + R_0)\) per dollar of borrowing, approximated by \(1 + R_0 - \delta\). When \(R_0 - \delta = \rho\), households are indifferent; when \(R_0 - \delta < \rho\), households strictly prefer to borrow to the maximum feasible amount. The spread, \(\rho - (R_0 - \delta)\), is the collateral shadow value; in other words, the Lagrange multiplier of the borrowing constraint.

**Lemma 2 (Credit Demand)** The equilibrium loan rate is given by: \(R_0 = \delta + \rho - \kappa_0^T\).

Competitive bankers take as given the market loan rate \(R_0\) and deposit rate \(r_0\). At \(t = 0\), a representative banker chooses consumption \(c_0^B\) (and retained equity \(e_0^B - c_0^B\)), and the asset-to-equity ratio \(x_0^B\). Each dollar of retained equity is worth \(x_0^B [1 + R_0 - \pi (Z_1)] - (x_0^B - 1) (1 + r_0)\) at \(t = 1\), which is the difference between asset and liability value. Note that \(\mathbb{E}[\pi_D (Z_1)] = \delta\), so that the expected return on retained equity is \(1 + r_0 + x_0^B (R_0 - \delta - r_0)\), and the return in a bad state is \(1 + r_0 + x_0^B (R_0 - \delta - \sigma - r_0)\). Let \(\xi_0\) denote the Lagrange multiplier of the solvency constraint; in other words, the shadow value of bank equity at \(t = 1\). The value function of a banker is

\[
v(e_0^B; R_0, r_0) = \max_{c_0^B \geq 0, x_0^B \geq 0} c_0^B + \frac{(e_0^B - c_0^B)}{(1 + \rho)} \{1 + r_0 + x_0^B (R_0 - \delta - r_0) \\
+ \xi_0 [1 + r_0 + x_0^B (R_0 - \delta - \sigma - r_0)]\}.
\]

**Proposition 1 (Bank Optimization)** The first-order condition (F.O.C.) for bank leverage \(x_0^B\) is

\[
R_0 - r_0 = \delta + \gamma_0^B \sigma,
\]

where \(\gamma_0^B = \left(\frac{\xi_0}{1 + \xi_0}\right) = \frac{R_0 - \delta - r_0}{\sigma} \in [0, 1)\) is the banker’s “effective risk aversion”. The solvency constraint binds if and only if \(\gamma_0^B > 0\). Substituting the F.O.C. into the value function, we have

\[
v(e_0^B; q_0^B) = c_0^B + q_0^B (e_0^B - c_0^B), \text{ where, } q_0^B = \frac{(1 + r_0) (1 + \xi_0)}{(1 + \rho)}.
\]

The banker consumes if \(q_0^B \leq 1\); if \(q_0^B > 1\), \(c_0^B = 0\) so that the entire endowment are lent out.

The equilibrium credit spread, \(R_0 - r_0\), has two components: the expected default probability \(\delta\) and the risk premium \(\gamma_0^B \sigma\). Each dollar lent adds \(\sigma\) units of downside risk at date 1, thus, tightening the capital adequacy constraint. Let \(\gamma_0^B\) be the price of risk charged by bankers, which is equal to the market Sharpe ratio of risky lending financed by risk-free deposits.

Let \(q_0^B\) be the marginal value of bank equity (Tobin’s Q). Retained equity has a compounded payoff of \((1 + r_0) (1 + \xi_0)\) from reducing the debt cost and relaxing the solvency constraint, so its
present value is \( \frac{(1+r_0)(1+\xi_0)}{(1+\rho)} \). When \( q_0^B > 1 \) bankers lend all of their endowment to households and carry all of their net worth to \( t = 1 \); this allows them to maximize profitable deposit creation.

Substituting the equilibrium loan rate into Equation (3), we can solve for the money premium \( \rho - r_0 \), as the sum of \( \gamma_0^B \sigma \), banks’ risk compensation, and \( \kappa_0^T \), the collateral value of tangible capital. Corollary 1 summarizes the economic forces behind equilibrium money supply.

**Corollary 1 (Money Premium Decomposition)** The equilibrium money premium is given by

\[
\rho - r_0 = \gamma_0^B \sigma + \kappa_0^T.
\]  

Equation (5) decomposes the money premium into measures of the scarcities of bank equity and collateral respectively. Since the money premium is also equal to the expected value of foregone marginal investment (Lemma 1), Equation (5) provides an anatomy of equilibrium investment constraint. First-best investment, \( i_{FB}^l \), given by the condition \( \alpha F' (i_{FB}^l) = 1 \), requires that each firm to carry at least \( i_{FB}^l \) deposits per unit of capital; or \( i_{FB}^l K_0^I \) deposits in total, which in turn requires a minimum level of bank equity:

\[
E_{FB} := \frac{i_{FB}^l K_0^I}{\tau_{FB} - 1} = \frac{i_{FB}^l}{1+\rho - 1} K_0^{I},
\]

where \( \tau_{FB} \) is solved as follows: under the first-best investment, the money premium is zero, so \( \kappa_0^T = 0 \). Substituting \( r_0 = \rho \) and \( R_0 = \delta + \rho \) into the solvency constraint yields \( \tau_{FB} = \frac{1+\rho}{\sigma} \).

First-best deposit creation requires a minimum stock of tangible capital to back household borrowing. The required total bank lending is \( \tau_{FB} E_{FB} \), so that collateral must be sufficient to cover households’ debt obligation: \( \alpha^T K_0^T \geq \tau_{FB} E_{FB} (1 + R_0) = \tau_{FB} E_{FB} (1 + \delta + \rho) \), or,

\[
K_0^T \geq K_{FB}^T := \frac{\tau_{FB} E_{FB} (1 + \delta + \rho)}{\alpha^T} = \left( \frac{1+\rho}{\sigma} \right) \left( \frac{i_{FB}^l (1 + \delta + \rho)}{\alpha^T} \right) K_0^{I}.
\]

I summarize this discussion by noting that the following corollary must hold.

**Corollary 2 (Sufficient Conditions for a Money Shortage)** The equilibrium money premium is positive, and intangible investment is below the first-best level, if \( E_0^B < E_{FB} \) or \( K_0^T < K_{FB}^T \).

Money creation is constrained at two margins: the scarcity of collateral, and bank equity. The collateral margin reflects a particular form of asset shortage (Caballero (2006); Giglio and
Severo (2012)). Bankers specialize in collateralized lending. More specifically, according to Jordà, Schularick, and Taylor (2016), mortgages account for somewhere between one-half and two-thirds of a typical bank’s balance sheet in developed countries. They also find that the upward trend in credit-to-GDP in the late 20th century is primarily driven by banks’ mortgage lending.

In order to create safe deposits, bankers need equity as a buffer against negative aggregate shocks. In principle, firms can hold other assets as a liquidity buffer; these may include other firms’ equity, households’ liabilities, or tangible capital. If firms have the asset management expertise, and if these assets can be either resold or pledged when the liquidity shock hits, firms would not need to rely exclusively on deposits.\textsuperscript{19} The model rules out these possibilities for simplicity and only allow for a corporate demand for short-term, safe assets, which has been noted to feed the growth and leverage of the financial sector (Pozsar (2011)).\textsuperscript{20}

3 Continuous-time Model

In this section, I extend the model into a continuous-time setting in which banks make dynamic decisions on leverage, dividend payout, and equity issuance. The anatomy of money shortage still holds, with the intermediary and collateral margins evolving endogenously and investment inefficiencies arising in both static and dynamic forms. A key feature is the procyclicality of bank leverage: in boom, the expansion of firms’ money demand feeds leverage to bankers with growing risk appetite; triggered by the loss of households’ collateral, a crisis features banks’ deleveraging and the collapse of inside money creation, which exacerbates investment inefficiencies.

3.1 Setup

**Households.** Risk-neutral households maximize life-time utility: $\mathbb{E} \left[ \int_{t=0}^{\infty} e^{-\rho t} dc^T_t \right]$, where $c^T_t$ is the cumulative consumption.\textsuperscript{21} Negative consumption ($dc^T_t < 0$) is allowed to fix households’...
required return to $\rho$.\(^{22}\) Households can hold firms’ and banks’ equity, and they trade tangible capital at the unit price $q_t^T$. One unit of tangible capital produces $\alpha$ units of goods per unit of time.

**Firms.** Risk-neutral entrepreneurs maximize life-time utility: $E\left[\int_{t=0}^{\infty} e^{-\rho t} dc_t\right].$ Negative consumption means that firms issue equity to households. Positive consumption means paying dividends to outside shareholders (households), and to entrepreneurs themselves for consumption. Allowing equity issuance fixes firms’ cost of capital to $\rho$, so the deposit carry cost (i.e. the money premium) is defined by the spread between $\rho$ and deposit rate $r_t$ as in the static model.\(^{23}\) Firms trade intangible capital at price $q_t^I$. One unit of intangible capital produces $\alpha$ units of goods per unit of time.

Firms cannot raise equity when hit by liquidity shocks that arrive at idiosyncratic Poisson times with intensity $\lambda$. When hit by the shock, a firm can either quit or invest. Let $k_t^I$ denote its capital holdings. By investing $i_t^I k_t^I$ units of goods (i.e. $i_t^I$ units per unit of capital) it can preserve the existing capital and create $F(i_t^I) k_t^I$ units of new capital. Investment is constrained by the firm’s deposit holdings, $i_t^I \leq m_t^I$, where $m_t^I$ is the deposits held per unit of capital.\(^{24}\) I assume the investment technology $F(\cdot)$ is sufficiently productive, so that this constraint always binds.

**Aggregate shock.** The aggregate shock $Z_t$ is a standard Brownian motion. Every instant, $\delta dt - \sigma dZ_t$ fraction of tangible capital is destroyed. Households default when the collateral is gone. Let $R_t$ denote the market loan rate. For one dollar borrowed from banks at $t$, households pay back

$$
\underbrace{1 + R_t dt}_{\text{principal + interest payments}} \underbrace{[1 - (\delta dt - \sigma dZ_t)]}_{\text{default probability}} = 1 + R_t dt - (\delta dt - \sigma dZ_t),
$$

where high-order infinitesimal terms are ignored. The default probability is a random variable that loads on $dZ_t$.\(^{25}\) Both loans and deposits are short-term contracts, initiated at $t$ and settled at $t + dt$.\(^{26}\)

\(^{22}\)It also fixes households’ marginal value of wealth to one, which considerably simplifies the analysis. $dc_t^T < 0$ can be interpreted as dis-utility from additional labor to produce extra goods as in Brunnermeier and Sannikov (2014).

\(^{23}\)Nagel (2016) emphasizes the variation in illiquid return (i.e. $\rho$ in the model) as a driver of the money premium dynamics in data. This paper provides an alternative model that focuses on the yield on money-like securities, $r_t$.

\(^{24}\)The idiosyncratic and independent nature of liquidity shock implies that firms’ money demand does not contain hedging motive that complicates model mechanism. Bolton, Chen, and Wang (2013) model the market timing motive of corporate liquidity holdings in the presence of technological illiquidity and state-dependent external financing costs. He and Kondor (2016) examine how the market timing motive of liquidity holdings amplifies investment cycle through pecuniary externality in the market of productive capital that in turn results from firms’ financial constraints.

\(^{25}\)Probit transformation can make $\pi(dZ_t) \in (0, 1)$, but complicates expressions. See also Klimenko et al. (2016).

\(^{26}\)For short-term deposits, in order to highlight the resale of deposits, I assume that banks repay deposits after intangible investment takes place, so that investing firms cannot wait for goods paid by banks until the maturity of deposits, and thus, have to actually pay them out in exchange for goods. Long-term deposits avoid this assumption, but would introduce other sources of instability, such as the Fisherian deflationary spiral in Brunnermeier and Sannikov.
Banks. Let \( r_t \) denote the market deposit rate, and \( x_t^B \) banks’ asset-to-equity ratio. Let \( c_t^B \) denote the representative banker’s cumulative dividend. \( dc_t^B > 0 \) means consuming and paying dividends to outside shareholders (households); \( dc_t^B < 0 \) means raising equity. The banker’s equity \( e_t^B \) follows a regulated diffusion process, reflected at payout and issuance (\( dc_t^B \neq 0 \)):

\[
d e_t^B = e_t^B \left( R_t dt - (\delta dt - \sigma dZ_t) \right) - e_t^B (x_t^B - 1) r_t dt - dc_t^B - e_t^B \mu_t dt. \tag{6}
\]

Because in equilibrium, banks earn a positive expected return on equity, the operation cost \( \iota \) is introduced to motivate payout so banks will not outgrow the economy.\(^{27}\)

Bankers maximize life-time utility, subject to a proportional equity issuance cost:

\[
\mathbb{E} \left\{ \int_{t=0}^T e^{-rt} \left[ \mathbb{I}_{\{dc_t^B \geq 0\}} - (1 + \chi) \mathbb{I}_{\{dc_t^B < 0\}} \right] dc_t^B \right\}.
\]

\( \mathbb{I}_A \) is the indicator function of event \( A \).\(^{28}\) The solvency constraint in the static setting boils down to the requirement of non-negative equity. Unlike the static setting, in equilibrium, bankers always preserve a slackness, so \( \tau := \inf \{ t : e_t^B \leq 0 \} = \infty \). As will be shown later, even in the absence of a binding solvency constraint, bankers will still be risk-averse due to the issuance cost \( \chi \).

State variable. At time \( t \), the economy has \( K_t^T \) units of tangible capital, \( K_t^I \) units of intangible capital, and aggregate bank equity \( E_t^B = \int_{s \in [0,1]} e_t^B (s) \, ds \). In principle, a time-homogeneous Markov equilibrium would have three state variables: \( E_t^B, K_t^I \), and \( K_t^T \). Four assumptions reduce the dimensions. First, intangible capital is also destroyed by a fraction \( \delta dt - \sigma dZ_t \). Second, the intangible capital’s growth from investment spills over to tangible capital. Therefore, we have

\[
d K_t^T = \frac{dK_t^I}{K_t^I} \mu_t^K dt + \sigma dZ_t, \quad \text{where} \quad \mu_t^K = \lambda F(m_t^I) - \delta, \tag{7}
\]

so the capital mixture is fixed at \( \phi := \frac{K_t^I}{K_t^0 + K_t^T} = \frac{K_t^I}{K_t^0 + K_t^T} \). Moreover, production has constant return-to-scale, and as in Hayashi (1982), the investment technology is homogeneous of degree

\(^{27}\)The cost of operations is equivalent to a higher time-discount rate for bankers, common in the literature (e.g. Kiyotaki and Moore (1997)). It can also be interpreted as an agency cost, a constant flow stolen by insider bankers.

\(^{28}\)While focusing on different questions, Van den Heuvel (2002), Phelan (2015), and Klimenko et al. (2016) also introduce issuance frictions in dynamic models of banking in macroeconomy. Dilution cost is just one form of frictions that give rise to the endogenous variation of intermediaries’ risk-taking capacity. He and Krishnamurthy (2012) achieve the same goal with a minimum requirement of insiders’ stake that resolves the principal-agent problem between inside and outside equity holders.
one in capital, so that we do not need to track the capital stock $K^T_0 + K^I_0$; the economy is scale-free. Under these assumptions, the only state variable is aggregate bank equity scaled by capital:

$$\eta_t = \frac{E^B_t}{K^I_t + K^T_t}.$$  

Because there is a unit mass of homogeneous bankers, $E^B_t$ follows the same dynamics as $e^B_t$. From Equation (6), the instantaneous expectation and standard deviation of $\frac{dE^B_t}{E^B_t}$ are $\mu^e_t = r_t + x^B_t (R_t - \delta - r_t) - \iota$ and $\sigma^e_t = x^B_t \sigma$ respectively. Let $dy^B_t$ denote banks’ payout/issuance rate $\frac{dc^B_t}{e^B_t}$, which is an impulse variable. By Itô’s lemma, $\eta_t$ follows a regulated diffusion process

$$\frac{d\eta_t}{\eta_t} = \mu^\eta_t dt + \sigma^\eta_t dZ_t - dy^B_t,$$

where $\mu^\eta_t = \mu^e_t - \mu^K_t - \sigma^e_t \sigma + \sigma^2$, and since banks issue deposits (i.e. $x^B_t > 1$), $\sigma^\eta_t = (x^B_t - 1) \sigma > 0$. Positive shocks increase $\eta_t$, so banks become richer relative to the real side of the economy; negative shocks make banks relatively undercapitalized.

As $\eta_t$ evolves over time, the economy repeats the timeline in Figure 1 with date 0 replaced by $t$ and date 1 replaced by $t + dt$. The endowments of the static model serve as the initial condition. Let intervals $\mathbb{T} = [0, 1]$, $\mathbb{I} = [0, 1]$, and $\mathbb{B} = [0, 1]$ denote the set of households, firms, and bankers respectively. The Markov equilibrium is formally defined as follows.

**Definition 1 (Markov Equilibrium)** For any initial endowments of tangible capital $\{k^T_0(s), s \in \mathbb{T}\}$, intangible capital $\{k^I_0(s), s \in \mathbb{I}\}$, and goods (initial bank equity) $\{e^B_0(s), s \in \mathbb{B}\}$ such that

$$\int_{s \in \mathbb{T}} k^T_0(s) ds = K^T_0, \quad \int_{s \in \mathbb{I}} k^I_0(s) ds = K^I_0, \quad \text{and} \quad \int_{s \in \mathbb{B}} e^B_0(s) ds = E^B_0,$$

a Markov equilibrium is described by the stochastic processes of agents’ choices and price variables on the filtered probability space generated by the Brownian motion $\{Z_t, t \geq 0\}$, such that:

(i) Agents know and take as given the processes of price variables, such as the price of tangible capital $q^T_t$, the price of intangible capital $q^I_t$, the loan rate $R_t$, and the deposit rate $r_t$;

(ii) Households optimally choose consumption $dc^T_t$, tangible capital holdings $k^T_t$, bank loan $l^T_t$, deposit holdings, and holdings of firms’ and banks’ equity;

(iii) Firms optimally choose intangible capital holdings $k^I_t$, deposits per unit of capital $m^I_t$, and payout and equity issuance policies;

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(iv) Bankers optimally choose leverage $x_t^B$, and payout and issuance policies;

(v) Price variables adjust to clear the markets;

(vi) All the choice variables and price variables are functions of $\eta_t$, so Equation (8) is an autonomous law of motion that maps any path of shocks \( \{Z_s, s \leq t\} \) to the current state $\eta_t$.

3.2 Markov Equilibrium

Anatomy of money shortage. The model puts banks at the center. To characterize the Markov equilibrium, we start with firms’ demand for bank deposits and households’ decision to take bank loans. Lemma 1’ gives firms’ optimal deposit demand in analogy to Lemma 1, with one modification that capital is valued at the market price $q_t^I$ instead of the terminal value in the static setting.\(^{29}\)

**Lemma 1’ (Money Demand)** Firms’ equilibrium deposits, $m_t^I$, satisfy the condition

\[
\rho - r_t = \lambda \left[ q_t^I F' \left( m_t^I \right) - 1 \right].
\]

As in the static model, households do not hold deposits when the deposit rate is lower than their discount rate, so firms’ deposit demands are the aggregate demand for bank debt. Let $l_t^T$ denote a representative household’s borrowing from banks, which is subject to the collateral constraint, $l_t^T \leq q_t^T k_t^T$.\(^{30}\) Households are indifferent if the expected loan repayment is equal to the discount rate (i.e. $1 + R_t dt - \delta dt = 1 + \rho dt$). When $1 + R_t dt - \delta dt < 1 + \rho dt$, households pledge all of their collateral to borrow. The wedge, $\kappa_t^T$, is the collateral shadow value of tangible capital.

**Lemma 2’ (Credit Demand)** The equilibrium loan rate is given by: $R_t = \delta + \rho - \kappa_t^T$.

Bankers solve a fully dynamic problem. Following Proposition 1, I conjecture that bankers’ value function is linear in equity, $v \left( e_t^B, q_t^B \right) = q_t^B e_t^B$, where $q_t^B$ summarizes the investment opportunity set. Define $\epsilon_t^B$ as the elasticity of $q_t^B$, $\epsilon_t^B : = \frac{dq_t^B}{q_t^B} \frac{d\eta_t}{\eta_t}$. Intuitively, $q_t^B$ signals the scarcity of bank equity, so I look for an equilibrium in which $\epsilon_t^B \leq 0$. Individual bankers take as given the equilibrium dynamics of $q_t^B$. Let $\mu_t^B$ and $\sigma_t^B$ denote the instantaneous expectation and standard deviation of $\frac{dq_t^B}{q_t^B}$ respectively. The Hamilton-Jacobi-Bellman (HJB) equation can be written as

\(^{29}\)To be precise, the liquidity shock hits at $t + dt$, and by then the new capital created will be worth $q_t^I + dq_t^I = q_t^I + \delta dt$

In equilibrium, $q_t^I$ is a diffusion process with continuous sample paths, so $dq_t^I$ is infinitesimal, and thus, ignored.

\(^{30}\)Infinitesimal terms, such as the interest payment and the capital price appreciation, are ignored in the constraint.
\[
\rho = \max_{dy_B} \left\{ \frac{(1 - q^B)}{q^B} I\{dy^B > 0\} dy^B + \frac{(q^B - 1 - \chi)}{q^B} I\{dy^B < 0\} (-dy^B) \right\} + \mu^B + \max_{x^B \geq 0} \left\{ r_t + x^B (R_t - \delta - r_t) - x^B \gamma_t I\{dy^B < 0\} \right\} - \iota,
\]

where the effective risk aversion is defined by \(\gamma^B_t := -\sigma^B_t\). By Itô’s lemma, \(\gamma^B_t = -\epsilon^B_t \sigma^\eta_t \geq 0\).

Bankers are risk-averse because of the equity issuance cost. From an individual banker’s perspective, the issuance cost causes her marginal value of equity \(q^B_t\) to be negatively correlated with shocks. Following a negative shock, bankers will not raise equity unless \(q^B_t\) reaches \(1 + \chi\), so the whole industry shrinks (i.e. the aggregate bank equity decreases), and the Tobin’s Q, \(q^B_t\) increases. Following a positive shock, bankers will not immediately pay out dividends unless \(q^B_t\) drops to 1, so the whole industry expands, and \(q^B_t\) decreases. Thus, bankers require a risk premium for holding any asset whose return is positively correlated with the aggregate shock (i.e. negatively correlated with \(q^B_t\)). In particular, bankers require a risk compensation from extending loans.

**Proposition 1’ (Bank Optimization)** The first-order condition for equilibrium leverage \(x^B_t\) is

\[
R_t - \delta - r_t = \gamma^B_t \sigma,
\]

The banker pays dividends \(dy^B_t > 0\) if \(q^B_t < 1\), and raises equity \(dy^B_t < 0\) if \(q^B_t > 1 + \chi\).

On the left-hand side of Equation (10) is the net interest margin \(R_t - \delta - r_t\), the marginal benefit of issuing deposits backed by risky loans. The right-hand side is the marginal cost, \(\sigma\) units of risk exposure, priced at \(\gamma^B_t\) per unit.\(^{31}\) Alternatively, we can interpret \(\gamma^B_t\) as the expected profit per unit of risk (i.e. the Sharpe ratio), from creating deposits backed by risky loans:

\[
\gamma^B_t = \frac{R_t - \delta - r_t}{\sigma}.
\]

Banks face two markets, the loan market and the money market. With the loan rate \(R_t\) determined by the loan market clearing condition, there is a one-to-one mapping between the deposit rate \(r_t\) and \(\gamma^B_t\). Therefore, we can say the money market is cleared by \(r_t\) or by \(\gamma^B_t\).

\(^{31}\)\(\gamma^B_t\sigma\) opens up a wedge between the credit spread, \(R_t - r_t\), and \(\delta\) the expected default rate. This intermediary premium shares the insight of He and Krishnamurthy (2013), but here, the purpose of intermediation is to create inside money. Bankers need loans to back deposits, and all that households need is to break even as shown in Lemma 2′.
Interpreting $\gamma^B_t$ as the Sharpe ratio of money creation helps us build an intuitive connection between $q^B_t$ and $\gamma^B_t$ – as a summary statistic for banks’ investment opportunity set, $q^B_t$ reflects the expectation of future profits from money creation (i.e. the future paths of $\gamma^B_t$). The Sharpe ratio interpretation also makes it very easy to see that $\gamma^B_t$ decreases in $\eta_t$, which is a key property of the equilibrium that will be confirmed later by the full solution. Intuitively, when the banking sector is relatively large, measured by $\eta_t$, its profit per unit of risk declines.

Substituting the loan rate in Lemma 2 into net the interest margin, we have

$$R_t - \delta - r_t = \rho - r_t - \kappa^T_t.$$  

Firms are willing to pay the money premium $\rho - r_t$ to hold deposits. To earn the money premium, bankers issue deposits backed by loans, so they compete with each other to lend to households. When households’ borrowing constraint binds, the money premium is shared by households and bankers: households take the collateral premium $\kappa^T_t$, the spread between the expected loan repayment and their discount rate, while bankers earn the rest through the net interest margin.

In analogy to Corollary 1, the equilibrium money premium is decomposed to reveal the two limits on deposit creation: the scarcity of collateral, and bankers’ limited risk-taking capacity.

**Corollary 1′ (Money Premium Decomposition)** The equilibrium money premium is given by

$$\rho - r_t = \gamma^B_t \sigma + \kappa^T_t.$$  

Figure 2 takes a snapshot of the deposit market, given $(\gamma^B_t, q^I_t, q^T_t)$. In the Markov equilibrium, these variables evolve continuously with the state variable $\eta_t$. The horizontal axis is $m^I_t$, the representative firm’s deposits per unit of intangible capital. The vertical axis is the money premium. The investment technology $F(\cdot)$ is concave, so firms’ indifference curve from Lemma 1′ gives a downward-sloping demand curve. The supply curve is bankers’ indifference curve $\rho - r_t = \gamma^B_t \sigma$, when households’ borrowing constraint does not bind ($\kappa^T_t = 0$); when it binds, the maximum value of bank loans is $q^T_t K^T_t$, so the maximum value of deposits is $q^T_t K^T_t - E^B_t$, and firms can hold no more than $\frac{q^T_t K^T_t - E^B_t}{K^T_t} = \frac{1}{\phi} \left[ q^T_t (1 - \phi) - \eta_t \right]$ deposits per unit of intangible capital.

Two equilibrium points are circled. When the banking sector is undercapitalized (low $\eta_t$), $\gamma^B_t$ is high, the active margin that limits deposit creation is bankers’ limited risk-taking capacity. The equilibrium money premium must compensate bankers’ risk exposure. Firms’ deposits are below the maximum level, and the collateral premium of tangible capital, $\kappa^T_t$, is zero.
When the banking sector is well capitalized (high $\eta_t$), $\gamma_t^B$ is low, so bankers risk-taking capacity is large but households’ borrowing constraint arises as the active limit on deposit creation, and thus, a positive collateral premium emerges. In analogy to Corollary 2 of the static model, Corollary 2’ links the investment inefficiency to the limits on money creation.

**Corollary 2’ (Investment Inefficiency)** From Lemma 1’ and Corollary 1’, we have

$$\lambda \left[ q_t^I F' (m_t^I) - 1 \right] = \gamma_t^B \sigma + \kappa_t^T. \quad (12)$$

Given the price of intangible capital $q_t^I$, the net present value of the foregone marginal investment is equal to the sum of the risk compensation required by bankers and the collateral premium.

**State variable dynamics.** So far, the continuous-time model has provided the counterparts of the static model’s mains results. The key insight is still the anatomy of money shortage. The new feature is that bankers always preserve financial slackness in this dynamic setting ($e_t^B > 0$), even though they are still risk-averse due to the equity issuance cost.

Next, I will focus on the dynamic aspects of the model. The main results are laid out in propositions and corollaries with intuitive explanations that will be later confirmed by the performances of the fully solved model. Let us start with the dynamics of the state variable $\eta_t$. 

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**Figure 2: Money Market.**
Bankers’ issuance and payout policies imply that $\eta_t$ is bounded by two reflecting boundaries: the issuance boundary $\eta$, given by $q^B(\eta) = 1 + \chi$, and the payout boundary $\bar{\eta}$ given by $q^B(\bar{\eta}) = 1$. When $\eta_t$ falls to $\eta$, banks raise equity and $\eta_t$ never decreases further; When $\eta_t$ rises to $\bar{\eta}$, banks pay out dividends and $\eta_t$ never increases further. When $\eta_t \in (\eta, \bar{\eta})$, bankers neither issue equity nor pay out dividends, because $q_t^B \in (1, 1 + \chi)$ by monotonicity.

**Corollary 3 (Reflecting Boundaries)** The economy moves within bank issuance boundary $\eta$ and payout boundary $\bar{\eta}$. In $[\eta, \bar{\eta}]$, the law of motion of the state variable $\eta_t$ is given by Equation (8).

**Procyclical intangible capital price.** The market price of intangible capital plays several important roles in the dynamic analysis. The trading of intangible capital can be interpreted as mergers and acquisitions. Firms can raise equity from households to purchase intangible capital, so households’ required return, $\rho$, is firms’ marginal cost of capital and firms’ required return for any investments. Proposition 2 shows firms’ indifference condition as a capital pricing formula.

**Proposition 2 (Intangible Capital Valuation)** Given firms’ optimal deposits-to-intangible capital ratio $m_{It}$, the equilibrium price of intangible capital satisfies

$$q^I_t = \frac{\text{Production}}{\alpha} + \lambda \left[ m_{It} F (m_{It} - m_{It}^I) - (\rho - r_t) m_{It}^I \right] - \frac{\text{Deposit carry cost}}{\mu_t^I} + \frac{\text{Expected price appreciation}}{\delta} - \frac{\text{Expected capital destruction}}{\delta} + \frac{\text{Quadratic covariation}}{\sigma^2 t},$$  \hspace{1cm} (13)

where $\mu_t^I$ and $\sigma_t^I$ are defined in the equilibrium capital price dynamics: $dq^I_t = \mu^I_t q^I_t dt + \sigma^I_t q^I_t dZ_t$.

The equilibrium price of intangible capital is procyclical. Consider an interior state of the economy, $\eta_t \in (\eta, \bar{\eta})$, and a positive shock, $dZ_t > 0$. Since fewer loans default than expected, banks receive a windfall. Because the equity issuance cost creates a wedge between $q_t^B$ and one, $q_t^B$ does not immediately jump to one and triggers payout. Therefore, banks’ equity increases, and in expectation, the shock’s positive impact on the bank equity will only dissipate gradually into the future. Thus, a positive shock increases the current bank equity, and through the persistence of its impact, it lifts up the expectation of future bank equity.

The positive shock increases the price of intangible capital through two channels. As banks’ equity increases, they charge a lower price of risk for deposit creation. By Corollary 1’, firms pay
a lower deposit carry cost and hold more deposits from $t$ to $t + dt$. Through more investments financed by these deposits, intangible capital is expected to grow faster in $dt$, which directly leads to a higher market price of intangible capital. This is the *contemporaneous* channel of procyclicality.

An *intertemporal* channel further increases the price of intangible capital. Due to the persistence of the shock’s impact, firms expect the banking sector to be better capitalized for an extended period of time, and thereby, they expect to hold more deposits and intangible capital to grow faster going forward. This lifts up the expectation of the future prices of intangible capital, which feeds back into an even higher current price through the expected price appreciation $\mu_I$. Figure 3 illustrates the contemporaneous and intertemporal channels of the procyclicality of $q_I$.

**Dynamic inefficiency.** The endogenous variation in the price of intangible capital leads to a new form of investment inefficiency that only arises in this dynamic setting. Taking as given the price of intangible capital $q_I$, Corollary 2’ reveals a form of *static inefficiency*, measured by the wedge between firms’ deposits $m_I$ and the contemporaneous investment target $i^*_I$, defined by $q_I F'(i^*_I) = 1$. Static inefficiency arises because bankers’ current risk capacity is limited, and the current value of collateral may not be able to back enough loans. This echoes Corollary 2 of the static model.

In a dynamic setting, the investment target $i^*_I$ varies with the price of intangible capital. Due to the necessity and cost of carrying deposits, $q_I$ is smaller than $q_{FB}$, the price of intangible capital
Figure 4: Money Market.

in an unconstrained economy in which firms have perfect access to external funds for investment:

\[ q_{FB}^I = \frac{\alpha + \lambda [q_{FB}^F (i_{FB}^I) - i_{FB}^I]}{\rho + \delta}, \quad (14) \]

where the first-best investment is given by \( q_{FB}^I F'(i_{FB}^I) = 1 \). Because \( q_t^I < q_{FB}^I \), the investment target \( i_t^* \) is below the first-best investment rate \( i_{FB}^I \). Since \( q_t^I \) reflects the expectation of the money market conditions, the wedge, \( i_{FB}^I - i_t^* \), measures a form of dynamic inefficiency.

**Procyclical bank leverage.** Following good shocks, banks’ equity increases and they charge a lower price of risk \( \gamma_t^B \). As illustrated by Figure 4, the equilibrium point moves from “1” to “2.” The equilibrium quantity of deposits increases, and whether it increases faster or slower than banks’ equity determines whether bank leverage is procyclical and countercyclical.

Because the price of intangible capital is procyclical, firms’ money demand will also shift outward, so the equilibrium point moves further from “2” to “3,” which increases the equilibrium quantity of deposits even further. This endogenous expansion of firms’ money demand allows banks’ debt to grow faster than their equity, contributing to the procyclicality of bank leverage.  

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This paper shares with Kiyotaki and Moore (1997) the idea that certain types of intertemporal complementarity amplify economic fluctuations. Here, the procyclicality of \( q_t^I \) is amplified by the intertemporal feedback through the expected price appreciation. The procyclicality of \( q_t^I \) leads to procyclical money demand, which in turn, contributes...
What distinguishes this paper from the existing macro-finance models is precisely this endogenous expansion of the demand for intermediaries’ debt. A static demand usually leads to counter-cyclical leverage (e.g. He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)).

Even though the model features a specific type of money demand motivated by the corporate cash holdings, the insight that leverage procyclicality results from the procyclicality of money demand is general. We would naturally expect that a booming economy has a stronger transaction demand for money-like securities issued by the financial intermediaries. The firms’ money demand in this paper is only one particular characterization of procyclical money demand.

**Stagnation and instability.** The procyclicality of bank leverage has two critical implications: first, recession lasts a long period of time; second, the risk of recession accumulates in booms.

Recession states are defined as the states where the economy’s expected growth rate $\mu^K_t$ is negative. The growth is driven by intangible investment, which is tied to firms’ deposits. Recession is triggered by negative shocks. As banks’ equity is depleted, banks require a higher risk compensation $\gamma^B_t \sigma$, which pushes up the money premium, and the price of intangible capital decreases. As illustrated by Figure 2, we see both an upward shifting of bankers’ indifference curve and an inward shifting of firms’ money demand. In equilibrium, firms’ deposit holdings and $\mu^K_t$ decrease. A banking crisis affects the real economy through the collapse of inside money creation, which echoes the classic account of the Great Depression by Friedman and Schwartz (1963).

As the economy moves into a recession, firms’ money demand contracts, and banks deleverage. When the economy is close to the issuance boundary $\eta$, the impact of negative shocks is bounded: bank equity never decreases beyond the issuance boundary. Thus, low leverage offers a very small benefit by making the economy robust to negative shocks. In contrast, low leverage limits the impact of positive shocks on bank equity, so it takes a sequence of sufficiently large good shocks to speed up the accumulation of banks’ equity. Therefore, the recovery is slow.

Now, let us consider a boom triggered by positive shocks. As banks’ equity increases, they issue more deposits that facilitate trade and investment, so the economy grows faster. However,
due the procyclicality of firms’ money demand, banks’ leverage grows. Defined in Equation (8),
\[ \sigma^n_t = (x^B_t - 1) \sigma, \]
is the state variable \( \eta_t \)’s instantaneous shock elasticity. After a long period of
good shocks, a high bank leverage \( x^B_t \) makes the economy very sensitive to shocks. Even small
negative shocks can significantly deplete bank equity and drag the economy into recession states.

Proposition 3 solves the stationary probability density of \( \eta_t \) (likelihood of different states),
and the expected time to reach \( \eta \in [\eta, \bar{\eta}] \) from the bottom of recession \( \eta \) (recovery time).

**Proposition 3** The stationary probability density of state variable \( \eta_t \), \( p(\eta) \) can be solved by:
\[
\mu^n(\eta) p(\eta) - \frac{1}{2} \frac{d}{d\eta} \left( \sigma^n(\eta)^2 p(\eta) \right) = 0,
\]
where \( \mu^n(\eta) \) and \( \sigma^n(\eta) \) are defined in Equation (8). The expected time to reach \( \eta \) from \( \eta \), \( g(\eta) \)
can be solved by:
\[
1 - g'(\eta) \mu^n(\eta) - \frac{\sigma^n(\eta)^2}{2} g''(\eta) = 0,
\]
with the boundary conditions \( g(\eta) = 0 \) and \( g'(\eta) = 0 \).

**Collateral asset pricing.** The model offers a new channel that links intermediaries’ risk capacity to asset prices. As illustrated in Figure 2, the borrowing constraint of households tends to bind when \( \gamma^B_t \) is low. Tangible capital relaxes the borrowing constraint, so the collateral shadow value \( \kappa^T_t \)
reduces households’ required return and increases the tangible capital price. Proposition 4 shows households’ indifference condition on tangible capital holdings. I assume the spillover growth is not internalized, so the variation in \( q^T_t \) is entirely driven by the collateral premium \( \kappa^T_t \).

**Proposition 4 (Tangible Capital Valuation)** The equilibrium price of tangible capital \( q^T_t \) satisfies
\[
q^T_t = \left( \frac{p - \kappa^T_t}{\alpha} \right) - \left( \frac{\mu^T_t}{\rho} - \delta \right) + \frac{\sigma^T_t \sigma}{\rho \rho}.
\]
where \( \mu^T_t \) and \( \sigma^T_t \) are defined in the equilibrium capital price dynamics \( dq^T_t = \mu^T_t q^T_t dt + \sigma^T_t q^T_t dZ_t \).

\( \kappa^T_t \) is an intermediated collateral premium that ultimately comes from the convenience yield
that firms assign to bank deposits. To back deposits, bankers compete with each other to extend

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35The spillover growth from intangible capital to tangible capital can be endowments distributed among households.
loans to households, pushing down the loan rate, and thereby, allowing households to bid up the tangible capital price. In an unconstrained economy where firms have perfect access to external financing, the convenience yield on deposits is zero, and so is the collateral premium $\kappa_t$. The price of tangible capital is $\alpha_{\rho+\delta}$. When firms face the liquidity problem, the convenience yield on deposits is transformed partly into the collateral premium via bankers’ efforts to create deposits, so $q_t^T > \alpha_{\rho+\delta}$. The extra collateral value increases with bankers’ intermediation capacity.

**Solving the equilibrium.** The solution of the model is a set of functions defined on $[\eta, \bar{\eta}]$. Each function maps the value of state variable $\eta_t$ to the value of an endogenous variable of interest. We can separate the functions into two categories. The first set includes the forward-looking variables $q^B (\eta_t)$, $q^I (\eta_t)$, and $q^T (\eta_t)$. The second set are the variables, such as banks’ leverage $x_t^B$, firms’ deposits-to-capital ratio $m_t^I$, deposit rate $r_t$, and loan rate $R_t$, that can be solved directly once we know the first set of functions. Once these variables are known, using Itô’s lemma, we can convert Equation (9), (13), and (15) into a system differential equations of $(q^B (\eta_t), q^I (\eta_t), q^T (\eta_t))$. The details of the solution algorithm is provided in Appendix I, which also shows the existence and uniqueness of the Markov equilibrium (stated in Proposition 5) in a manner of constructive proof.

**Proposition 5 (Markov Equilibrium)** There exists a unique Markov equilibrium with state variable $\eta_t$ that follows an autonomous law of motion in $[\underline{\eta}, \bar{\eta}]$. Given functions $q^B (\eta_t)$, $q^I (\eta_t)$, and $q^T (\eta_t)$, agents’ optimality conditions and market clearing conditions solve firms’ deposit holdings, banks’ leverage, loan rate, and deposit rate as functions of $\eta_t$. Substituting these variables into bankers’ HJB equation, and the intangible and tangible capital pricing formula (i.e. Equations (9), (13), and (15)), we have a system of three second-order ordinary differential equations that solve the functions $q^B (\eta_t)$, $q^I (\eta_t)$, and $q^T (\eta_t)$, under the following boundary conditions:

At $\underline{\eta}$: (1) $\frac{dq^I (\eta_t)}{d\eta_t} = 0$; (2) $\frac{dq^T (\eta_t)}{d\eta_t} = 0$; (3) $q^B (\bar{\eta}) = 1 + \chi$; (4) $\frac{d(q^I (\eta_t))}{d\eta_t} = 0$;

At $\bar{\eta}$: (5) $\frac{dq^I (\eta_t)}{d\eta_t} = 0$; (6) $\frac{dq^T (\eta_t)}{d\eta_t} = 0$; (7) $q^B (\eta) = 1$; (8) $\frac{d(q^I (\eta_t))}{d\eta_t} = 1$.

We need a total of eight boundary conditions for three second-order ordinary differential equations and two endogenous boundaries to pin down a unique solution. The first two conditions at both boundaries prevent capital prices from jumping upon reflection. These conditions rule out arbitrage in the markets of intangible and tangible capital. Condition (3) and (7) are the value-matching conditions for banks’ issuance and payout respectively.
(4) and (8) are the “smooth-pasting” conditions. Note that \( q_t^B E_t = q_t^B \eta_t (K_t^I + K_t^T) \) is the market value of bank equity. At \( \eta_t \), condition (4) guarantees the market value of existing shares does not jump when news shares are issued. At \( \eta_t \), condition (8) guarantees the market value of bank equity declines by the exact amount of dividends paid out. Both conditions make sure that existing shareholders’ value does not jump at the reflecting boundaries. If either (4) or (8) is violated, taking as given the aggregate issuance and payout, individual banks will have incentive to deviate.

### 3.3 Solution

**Calibration.** To numerically solve the differential equations, we need to fix the parameter values. First, one unit of time is defined as one year. \( \delta = 4\% \) and \( \sigma = 2\% \), in line with mean and standard deviation of loan delinquency rates (source: FRED). \( \phi = 0.45 \) form Peters and Taylor (2016).36

The other parameters are chosen so that model moments match those of the data. All moments are evaluated at the stationary distribution. \( \rho = 0.04 \), so the mean of \( r_t \) matches the average yield of MZM (money of zero maturity) and three-month Treasury bills (source: FRED). \( \alpha = 0.1 \), so the mean of price-to-earnings ratio of firms’ equity matches that of S&P 500 (1990-2015). \( \iota = 0.02 \), so the mean of the ratio of cost of operations to banks’ net income matches the mean wages-to-net income ratio in Call reports. \( \lambda = 1/7 \), so the mean growth rate matches the U.S. economic growth from intangible investment calculated by Corrado and Hulten (2010).

The investment technology is \( F(i) = \omega_0 i^{\omega_1} \). \( \omega_0 = 0.8035 \), so the mean money premium matches the average GC repo/T-bill spread in Nagel (2016). I set \( \omega_1 \) equal to 0.99, so the mean of firms’ cash-to-asset ratio matches that of Compustat sample from 1971 to 2015. The dilution cost \( \chi \) governs the tail behavior, and thus, the standard deviation of the money premium. Acknowledging that in reality, the money premium varies due to forces beyond the model mechanism, I set \( \chi = 1 \) to produce half of the standard deviation of GC repo/T-bill spread in Nagel (2016).

**Bank equity cycle.** The state variable \( \eta_t \) measures the aggregate bank equity relative to the size of the real economy. Bank equity affects the real economy through its impact on deposit creation. When \( \eta_t \) increases, banks expand balance sheets and issue more deposits, so firms can hold more cash for investment, and the economy grows faster. Figure 5 shows the statistical properties of \( \eta_t \).

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36 Using Compustat database, they accumulate firms’ knowledge capital from R&D (research and development) expenses and organizational capital from SG&A (selling, general and administrative) expenses. Intangible capital is the sum of the two. Intangibility intensity is the ratio of intangible capital to total capital (i.e. intangible and physical capital, such as property, plant and equipment). The median intensity of firm-year observations is 45%.
Panel A plots several sample paths of $\eta_t$ by simulating the law of motion (Equation (8)).\textsuperscript{37} The paths are bounded by the issuance and payout boundaries.

Panel B of Figure 5 plots the impulse response function of $\eta_t$. It shows that shocks have persistent effects. Because the law of motion of $\eta_t$ is non-linear and state-dependent, we cannot define impulse response functions as in linear time-series analysis. Thus, to illustrate the persistent impact of shocks, I fix the initial value of $\eta_t$ to the median value under the stationary distribution, and consider an increase of $\eta_t$ to the 60th percentile. The figure plots the percentage change of the term structure of expectation, the expected value of $\eta_{t+T}$ with $T$ ranging from one month to ten years.\textsuperscript{38} In expectation, the impact of the shock dissipates gradually. The initial increase of less than 11% raises the expected value in ten years by more than 3%.

\textsuperscript{37}The discretization step is one day so the shock $\Delta Z_t$ is drawn from normal distribution $N(0, 1/365)$. Asmussen, Glynn, and Pitman (1995) discuss the discretization error. Simulating regulated diffusion has a weak order of convergence of 1/2, which is slower than the order 1 for simulations of diffusion processes without reflecting boundaries.

\textsuperscript{38}The expectation is calculated by the Kolmogorov backward equation (i.e. the Feynman-Kac formula), for reflected diffusion processes. The partial differential equations are solved by the Method of Lines (Schiesser and Griffiths (2009)). Borovička, Hansen, and Scheinkman (2014) provide an alternative, systematic framework to define and calculate impulse responses and the term structure of shock elasticity for non-linear diffusion processes.
The persistence is caused by banks’ precautionary behavior, which is in turn due to the dilution cost of issuance. If bankers could issue equity freely (i.e. $\chi = 0$), they no longer need to retain equity. Whenever $q_t^B$ is above one, signaling an improvement of the investment opportunity set, bankers raise equity from households; whenever $q_t^B$ is below one, bankers distribute dividends. The dilution cost implies that equity is only raised infrequently when $q_t^B$ reaches $1 + \chi$, signaling severe capital shortage. $\chi$ opens up a wedge between $q_t^B$, the value of one dollar inside banks as retained equity, and 1, the value of one dollar paid out as dividends. As a result, banks preserve a financial slackness. Unless the economy hits the payout boundary, banks accumulate equity.

Panel C shows the stationary cumulative probability function (c.d.f.) from Proposition 3. The curve starts from zero at the issuance boundary, and ends at one at the payout boundary. Around 50% of the time, the economy is in a region with negative growth. The economy spends more time in recessions than the U.S. economy, because its mean growth rate is calibrated to 0.74%, which is the growth rate of output per hour attributed to intangible investment from 1995 to 2007 (Corrado and Hulten (2010)). If other sources of growth work independently from the model mechanism, adding those back can make the numbers more realistic.

Panel D of Figure 5 plots the expected time to reach different values of $\eta_t$ from the issuance boundary $\eta$ (Proposition 3). The right bound marks the lowest value of $\eta_t$ that delivers a non-negative growth rate. In expectation, it takes ten years to recover from the bottom of a recession. During this period, banks are undercapitalized, and thus, cannot supply enough money that can be held by firms for intangible investment. Next, I will provide a deeper diagnosis of the cycle, and link the procyclicality of bank leverage to these statistical properties of the economy.

**Leverage Procyclicality.** As illustrated by Figure 2, the money market equilibrium varies with $(\gamma_t^B, q_t^I, q_t^T)$. $\gamma_t^B$ is the price of risk that bankers charge for issuing risk-free deposits backed by risky loans. $q_t^I$, the price of intangible capital, shifts firms’ money demand. $q_t^T$, the price of tangible capital, determines the households’ borrowing capacity, or the maximum value of bank loans. These variables, and the money market, evolve with $\eta_t$. For now, let us focus on $\gamma_t^B$ and $q_t^I$.

Panel A of Figure 6 shows $\gamma_t^B$ as a function of $\eta_t$. Because one unit of time is set to one year, $\gamma_t^B$ is the annual Sharpe ratio of risky lending financed by risk-free deposits. $\gamma_t^B$ decreases in $\eta_t$. When the economy is close to the bank issuance boundary, banks charge a price of risk higher than 0.2; near the bank payout boundary, $\gamma_t^B$ is close to zero. Good shocks increase $\eta_t$ and decrease $\gamma_t^B$, shifting downward the bankers’ indifference curve in Figure 4.
As well documented in the empirical literature, asset price variation is dominated by the variation in discount rate (e.g. Cochrane (2011)). In the model, discount rate is fixed at $\rho$, so the variation in $q^I_t$ is purely driven by firms’ cost of liquidity management (i.e. the money premium), and their choice of liquidity holdings that determines the capital growth. Thus, quantitatively, we would not look for large variation in the price of intangible capital over the cycle.
Bankers’ leverage is procyclical around 70% of the time. When their equity increases, they issue even more deposits, so their leverage increases. As firms’ deposits increase, the marginal value of deposits declines, so the equilibrium money premium and banks’ profit from money creation decreases. Eventually, bankers’ leverage declines, expedited by the collateral shortage.

The procyclicality of bank leverage helps explain the statistical properties of the economy that we see in Figure 5. In Panel C, a relatively small probability mass is in the states where banks’ equity is high. In good times, banks’ leverage is high, so the economy is very sensitive to shocks. When the economy is close to the payout boundary, the impact of good shocks is limited, because a sufficiently large shock triggers dividend distribution, which means the banking sector cannot grow beyond \( \bar{\eta} \). Thus, high leverage only serves to amplify the impact of negative shocks on bank equity, so even small shocks can significantly deplete bank equity. Because of this fragility, the economy spends less time in boom. Panel D of Figure 5 shows the slow recovery. As previously mentioned, when the economy is close to the issuance boundary, the impact of negative shocks is bounded. The decline of bank leverage only serves to reduce the impact of good shocks on bank equity, so banks accumulate equity slowly and the economy tends to get stuck in recessions.

Countercyclical leverage would have led to the exactly opposite pattern. Booms are stable, because banks’ low leverage reduces the shock elasticity of \( \eta_t \), which is \( \sigma_t^\eta = \left( x_t^B - 1 \right) \sigma \) (Equation (8)), which reduces the impact of negative shocks. High leverage in recessions would reduce the duration of recession by amplifying the impact of positive shocks on bank equity.\(^{40}\)

Reading Panel C and D of Figure 6 from the right to the left, we see how a crisis unfolds. Depending on where the economy is, a negative shock may increase or decrease bank leverage. In reality, the balance-sheet adjustments in crisis differ across different types of financial intermediaries. Adrian and Shin (2010) and Adrian, Etula, and Muir (2014) document the procyclicality of broker-dealers’ leverage. He, Khang, and Krishnamurthy (2010) show that commercial banks’ leverage actually increased in the 2007-09 financial crisis, as the shadow banking sector shrank its balance sheet (Krishnamurthy, Nagel, and Orlov (2014)) and assets moved to the commercial banking sector (Acharya, Schnabl, and Suarez (2013)).

Commercial banks’ capacity to create deposits not only depends on their financial soundness but also on the deposit insurance and regulations. Thus, banks in the model are closer to the finan-

\(^{40}\)Models with countercyclical leverage can generate instability through other mechanisms, such as fire sale in He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), which need intermediaries to hold long-term assets, and thereby, are exposed to endogenous volatility of asset prices. In order to highlight the direct link between leverage and financial instability, I shut down this potentially reinforcing channel by restricting banks’ investment to short-term loan contracts. In Appendix II.3, banks hold long-term assets.
cial intermediaries in the shadow banking sector. Many have argued that the demand for money-like securities is one of the major drivers behind the shadow banking development (e.g., Gorton (2010); Gorton and Metrick (2012); Gennaioli, Shleifer, and Vishny (2013); Pozsar (2014)).

Collateral asset pricing. Panel A of Figure 7 plots the collateral shadow value of tangible capital $\kappa_t^T$ against the cumulative probability, and Panel B plots the price of tangible capital $q_t^T$. In 95% of the time, the borrowing constraint of households does not bind ($\kappa_t^T = 0$), so collateral shortage is not an active limit on deposit creation. But as $\eta_t$ increases, the increase in $q_t^T$ cannot catch up with the growth of bankers’ risk-taking capacity, so when banks are extremely well-capitalized, they exhaust households’ borrowing capacity, and the shadow value of tangible capital emerges.

The collateral premium ultimately comes from the convenience yield that firms assign to deposits, which is then split between $\kappa_t^T$ and bankers’ risk compensation $\gamma_t B$ (Corollary 2$'$). Thus, the collateral premium tends to be large, when bankers are well capitalized and their risk aversion is low. In other words, bankers’ intermediation capacity determines the extent to which the money premium translates into the collateral premium. As shown in Proposition 4, the price of tangible capital $q_t^T$ incorporates the expectation of the future collateral premium, so it increases in $\eta_t$.

This intermeditated money premium links banks’ risk appetite to asset prices. This channel contributes to the intermediary asset pricing literature. Existing studies usually assume intermediaries’ exclusive access to assets (e.g. He and Krishnamurthy (2013); Gromb and Vayanos (2015)) or their low risk aversion (e.g., Gârleanu and Pedersen (2011); Longstaff and Wang (2012)).

Panel B of Figure 7 also shows the gap between the equilibrium price of tangible capital
and a constant $\frac{\alpha}{\rho+\delta}$, which is the price of tangible capital in the unconstrained economy where firms have perfect access to external funds and the money premium is zero. The gap is quantitatively small under the current calibration, because the collateral shortage only limits money creation in less than 5% of time. The key parameter is $\phi = \frac{K_I}{K_I+K_T}$. When this intangible intensity increases, the economy faces a stronger imbalance between firms’ demand for deposits and households’ borrowing capacity. Therefore, the collateral premium, and the deviation of $q_T^F$ from the unconstrained benchmark value, tend to be larger. The asset pricing implications of *intermediated money premium* should be more relevant in a more intangible-intensive economy.\textsuperscript{41}

**Static and dynamic inefficiencies over the cycle.** Corollary 1’ decomposes the money premium into two components, bankers’ required risk compensation $\gamma^B_t \sigma$, and the collateral premium $\kappa^T_t$. A higher money premium directly translates into a larger gap between the cash-constrained investment rate and the current investment target rate $i^*_t$, defined by $q_t^IF'(i^*_t) = 1$.

Panel A of Figure 8 shows the static inefficiency, measured by the percentage deviation of the investment rate $m^f_t$ (i.e., firms’ deposits-to-intangible capital ratio), from the current investment target $i^*_t$. More than 95% of the time, bankers’ risk capacity is the only determinant of the money premium, so as it declines, shown in Panel A of Figure 6, the money premium decreases, and firms hold more deposits for investment. Moving from the left to right, the investment wedge declines from 95% at the depth of recession to 30%. It rebounds when the collateral shortage arises, and

\textsuperscript{41}Corrado and Hulten (2010) document the increasing reliance of the U.S. economy on intangibles, such as technologies, marketing, and design. In 2007, the investment in intangibles amounts to $1.6$ trillion (11.3% of GDP).
\( \kappa_t^T \) becomes positive, as shown in Panel A of Figure 7.

Panel B of Figure 8 shows the dynamic inefficiency, measured by the percentage deviation of the current investment target from the first-best investment rate defined by Equation (14). Dynamic efficiency varies with the price of intangible capital. As shown in Panel B of Figure 6, \( q_t^I \) increases as \( \eta_t \) increases. For 60% of the time, the current investment rate is 70% or more below the first-best. This wedge declines to less than 55% as the economy moves close to the bank payout boundary.

### 3.4 Extension

I will close this section by briefly mentioning several extensions in Appendix II.

**Financial innovation.** Appendix II.1 allows households to borrow up to \( \theta \) times of the value of their tangible capital holdings (\( \theta > 1 \)). We can think of the collateral multiplier as a result of rehypothecation (Gorton and Muir (2016)) or synthetic products. Allowing \( \theta > 1 \) alleviates the collateral shortage as a limit on deposit creation. In the current setting, this collateral multiplier benefits the economy by allowing more money to be created that supports intangible investment. Moreover, because household borrowing constraint becomes less binding, the collateral premium is smaller, so banks make more profits from money creation. As a result, in booms, banks are willing to retain more equity by delaying dividend payout. As the payout boundary increases, the economy becomes more stable.

**Regulatory constraint.** Appendix II.2 introduces a regulatory constraint, a cap on bank leverage (i.e. \( x_t^B < \bar{x} \)). Even if the value of \( \bar{x} \) is intentionally chosen to be slightly below the maximum bank leverage in the benchmark case, the leverage constraint still has a significant impact on banks’ balance-sheet adjustments over the cycle. Because the regulatory constraint limits banks’ ability to earn the money premium, the payout boundary and issuance boundary both decline dramatically. Banks’ leverage is still procyclical, and averaged over the cycle, it is much higher than the benchmark case because leverage increases dramatically when the regulatory constraint does not bind. In sum, by crowding out banks’ profits, regulatory constraint crowds out bank equity, and the resulting leverage can be even higher.

**Long-term bank assets.** Appendix II.3 allows banks to directly hold tangible capital instead of lending to households in the form of short-term loans. Due to the money premium on deposits, banks’ funding cost is lower than households, which makes bankers the natural buyer of tangible capital and creates a fire sale mechanism in the sense of Shleifer and Vishny (1992) and Brun-
nermeier and Sannikov (2014) when banks are undercapitalized. Because the calibrated money premium is not large enough to create a significant wedge of funding cost between bankers and households, the fire sale mechanism is not strong.

In a related paper (Li (2016)), I show that allowing banks to invest in firms’ equity or directly hold intangible capital can lead to a strong amplification mechanism. Following good shocks, $q_t^I$ increases not only because of a lower money premium, but also because bankers have a larger risk-taking capacity. Therefore, $q_t^I$ increases even faster, so firms’ money demand becomes more procyclical, which in turn amplifies the procyclicity of bank leverage. Following bad shocks, the mechanism flips into a deleveraging and fire sale spiral.

**The credit view and leverage cycle.** Appendix II.4 extends the model to capture the credit view that banks are special because of their expertise in supplying credit (e.g. Bernanke (1983)). Since the financial crisis of 2007-09, tremendous effort and progress have been made to incorporate financial intermediaries as credit suppliers into models of macroeconomy (e.g., Gertler and Kiyotaki (2010)). In the spirit of Bolton and Freixas (2000), I model bankers as restructuring experts: they can get some value from the collateral of non-performing loans. This extension has a huge impact on the leverage cycle. Earning money premium is no longer the only way to compensate bankers’ risk exposure. Bankers can make a profit from restructuring. With this new source of profit, bankers are willing to borrow at a deposit rate equal to $\rho$. Following good shocks, the economy can grow into a region where firms hold enough deposits to meet the investment target, so the convenience yield on deposits is zero and $r_t = \rho$. At this point, households will be willing to hold deposits, which opens up a new source of funds for bankers. Banks’ leverage increases by more than 60% at the peak of the cycle, making booms more fragile.

4 Government Debt

The monetary services of government debt has long been recognized. Patinkin (1965) discusses the role of bonds in facilitating transactions, and Friedman (1969) refers in general to a non-pecuniary return to bonds. The liquidity of U.S. Treasury securities is further enhanced by the development of repo markets since 1980s (Fleming and Garbade (2003)).

This section explores the financial stability implications of government debt as alternative money. On the one hand, government debt expands the aggregate money supply and alleviates the money shortage faced by firms. On the other hand, government money supply squeezes the money
premium, and thus reduces banks’ profit and return on equity. As a result, bankers will choose to retain less equity as a risk buffer, and through this equity crowding-out effect, government debt can crowd out banks’ money supply and destabilize the banking sector.

4.1 Setup

I assume that firms can hold government debt as cash. Government debt is issued at $t$ with the same interest rate as deposits, $r_t$, and repaid at $t + dt$. The proceeds from issuance are transferred as lump-sum payments to households, and debt is repaid with lump-sum tax on households, so fiscal policy does not directly affect the state variable $\eta_t$.

The government faces a debt limit $M^G \left( K^T_t + K^I_t \right)$. Consider a simple debt management strategy in line with Friedman’s rule of money supply: the government maximizes issuance. Therefore, the government debt-to-GDP ratio is $\frac{M^G}{\alpha}$. The total money supply is $M^S_t = M^B_t + M^G$, where $M^B_t = (x^B_t - 1) \eta_t$ are the deposits issued by banks, scaled by aggregate capital stock.

Government debt allows firms to hold more cash, so the marginal benefit of cash declines,

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42The government’s sources and uses of funds may take other forms, as long as they do not change households’ credit demand, firms’ money demand, and banks’ leverage, payout, and issuance.

43Maximum issuance is in line with Friedman’s rule: individuals’ opportunity cost to hold money should be equal to the social cost of creating money (Friedman (1969); Woodford (1990a)). Firms’ opportunity cost to hold money is $\rho - r_t$, the money premium. The private sector’s marginal cost of money creation is $\gamma^B \sigma + \kappa^T$, the sum of risk compensation demanded by bankers and the collateral premium due to the collateral shortage. In contrast, the government’s cost of money creation is zero, because tax payments do not need the backing of collateral, and the government does not face the recapitalization frictions as banks do. Therefore, Friedman’s rule suggests that the government should maximize its money supply. However, as will be discussed later, because banks also face financial constraints, Friedman’s rule is not necessarily the optimal government debt management strategy.
so does the equilibrium money premium. We would also expect the government debt to have a positive effect on investment and growth, which is similar to the investment crowding-in effect in Woodford (1990b) and Holmström and Tirole (1998). However, the overall effect on investment and growth depends on how banks react.\footnote{The traditional crowding-out effect describes how the government debt supply raises interest rate in general, and thereby crowds out private investment through a higher financing cost. My model does not entertain this effect, because by shutting down external financing completely, the model has cash being the only determinant of investment.} The key determinant of banks’ operating environment is firms’ demand for deposits. After introducing the government debt, firms’ cash holdings per unit of capital are

\[
\frac{(M^B_t + M^G_t)}{K^I_t} (K^I_t + K^T_t) = m^I_t + \frac{M^G_t}{\phi},
\]

where \(m^I_t\) is firms’ deposits per unit of capital. Substituting this into firms’ cash policy in Lemma 1′, we have a new deposit demand curve that banks face:

\[
\rho - r_t = \lambda \left[ q^I_F F^I \left( m^I_t + \frac{M^G_t}{\phi} \right) - 1 \right].
\]

Because \(F (\cdot)\) is a concave function, the curve is shifted inward as illustrated by Figure 9.

Note that the setup can have an alternative interpretation: government debt serves as banks’ reserves (as in Bansal, Coleman, and Lundblad (2011)), which in turn backs an equal amount of deposits. It is mathematically equivalent to firms holding government debt directly.\footnote{In other words, whether government debt is held by firms directly or through banks is indeterminate.} Define the money multiplier, \(M^G_t + M^B_t \frac{M^G}{M^G + M^B}\), as the ratio of total money to government debt. Under this alternative interpretation, this multiplier resembles the textbook money multiplier. Since in reality, a central bank’s assets are mainly government securities, if we treat the balance sheet of a central bank as pass-through, we can treat reserves as banks’ holdings of government debt.\footnote{The contemporaneous version of reserves requirement, the liquidity coverage ratio, counts government securities as banks’ liquidity holdings (Basel Committee on Bank Supervision (2013)).}

### 4.2 Crowding-out, Instability, and Stagnation

**Leverage cycle and instability.** Figure 10 compares the model’s performances under different values of \(M^G\). It shows how the economy, and the banking sector in particular, reacts to government debt supply. Panel A plots bank leverage for different levels of government debt-to-output ratio (\(\frac{M^G}{\alpha}\)). Bank leverage is plotted against the stationary cumulative probability function. As government debt-to-output ratio increases from 0% to 50%, bank leverage becomes more procyclical.
Panel C of Figure 10 shows the amplified procyclicality of bank leverage has a direct impact on the stationary distribution of the state variable. Stronger procyclicality of leverage makes the economy more sensitive to shocks in good times (high $\eta_t$) and less sensitive to shocks in bad times (low $\eta_t$). Because near the payout boundary, bad shocks have a larger impact than good shocks, probability mass is shifted towards the bad states.

Another key message from Panel C is that both the issuance and payout boundaries decline when the government debt-to-output ratio increases from 0% to 50%. This shows the bank equity crowding-out effect of government debt. As shown in Panel D, government debt squeezes the money premium at all levels of bank equity, and thereby, raises banks’ debt cost.\textsuperscript{47} In response to a lower return on equity, banks reset their payout and issuance policies.

In bad times, banks raise equity only after sufficiently deleveraging. They wait for a severe money shortage and sufficiently high money premium that boosts return on equity and justifies

\textsuperscript{47}The model’s prediction that an increase in government debt supply decreases the money premium is consistent with a surging set of evidence, such as Krishnamurthy and Vissing-Jorgensen (2012), Greenwood and Vayanos (2014), Greenwood, Hanson, and Stein (2015), and Sunderam (2015).
the recapitalization cost. In good times, banks are more willing to reduce their equity through dividend payouts, facing the competition from the government in the money market. As a result, the procyclicality of leverage can be amplified, even if overall, banks issue less debt when facing the competition from the government, as shown in Panel B of Figure 10.48

Greenwood, Hanson, and Stein (2015), Krishnamurthy and Vissing-Jorgensen (2015), and Woodford (2016) also explore the financial stability implications of government debt as a substitute for bank-issued money. Specifically, these models predict that by narrowing the money premium, government debt crowds out bank debt, and thereby, decreases banks’ leverage and stabilizes the economy. However, banks’ dynamic equity management problem is ignored in these settings.

This model also predicts the debt crowding-out effect (Panel B of Figure 10), but on top of this, it reveals a more subtle but even stronger equity crowding-out effect that works through the endogenous responses in banks’ issuance and payout policies. By amplifying the procyclicality of bank leverage, this equity crowding-out effect destabilizes the economy.

**Stagnation.** Government debt supply has a non-monotonic impact on the depth and duration of recession that is caused by the contraction of banks’ money supply. Figure 11 shows the expected time to recovery from the bottom of a recession (i.e., the bank issuance boundary), for different levels of government debt-to-output ratio.49 Before the government debt-to-output ratio reaches around 90%, increasing government debt increases the duration of a recession.

48The bank debt crowding-out effect of government debt supply has been documented by Bansal, Coleman, and Lundblad (2011) for financial commercial papers and banker’s acceptance, Greenwood, Hanson, and Stein (2015) also for financial commercial papers, Sunderam (2015) for asset-backed commercial papers, and Krishnamurthy and Vissing-Jorgensen (2015) for the financial sector’s debt in general using a long history of data.

49The time to recovery is calculated following Proposition 5 using the model solution under different values of $M^G$. 

![Figure 11: Non-monotonic Effects of Government Debt on Recovery Time.](image-url)
Raising government debt lengthens a recession in two ways. First, because of a lower money premium, the expected return on banks’ equity is lower, which in turn slows down the accumulation of bank equity. Second, by squeezing banks’ profit, government debt also discourages banks from recapitalizing. Near the issuance boundary, the impact of bad shocks is limited, while the impact of good shocks can be relatively large. However, as the issuance boundary shifts to a lower level of bank equity, this beneficial asymmetry fades away even before the economy is able to accumulate a sufficiently high level of bank equity that supports a positive growth rate. Therefore, through the impact on the bank issuance boundary, increasing government debt makes the recovery path sensitive to bad shocks at too early a stage.

However, as the government debt-to-output ratio increases, the marginal benefit of bank deposits on investment and growth decreases, so eventually, the economy will exhibit less cyclical driven by the variation in bank equity, and both the depth and duration of the recession is reduced. Let us consider the extreme case where the government has unlimited debt capacity. It can eliminate the money shortage and recession, and the economy will grow at a constant rate \( \mu^K (i^I_{FB}) \) given by the first-best intangible investment. After the government debt-to-GDP ratio passes 90%, further increase of government debt decreases the duration of the recession.

The increasing leg in Figure 11 is particularly relevant for understanding the pre- and post-crisis dynamics of the U.S. economy. From 2001 to 2008, the U.S. public debt-to-GDP ratio increases gradually from around 55% to 70%. This coincides with a period of the strongest procyclicality of leverage in the financial sector, to which many have attributed the risk accumulation that finally brings the Great Recession (e.g. FSB (2009)). The post-crisis period saw an even more dramatic increase in public debt, in response to both fiscal stimulation and the quantitative easing done by the central bank. By the end of 2012, the public debt-to-GDP ratio had reached its current level, around 100%. The years after the 2007-2009 financial crisis features slow recovery. This paper contributes to the contemporaneous debate on the causes of stagnation by emphasizing the financial sector’s responses to increases in government debt.\footnote{The current literature has largely focused on the aggregate demand, demography, or technological progress (e.g., Eggertsson and Mehrotra (2014); Summers (2015)).}

The comparative statics in Figure 11 show the model’s performances in response to unexpected and permanent changes in the government debt supply. While the current practice of U.S. Treasury debt management emphasizes predictability (Garbade (2007)), since the financial crisis, there has been a considerable amount of uncertainty on fiscal policies, especially on the debt level.
This paper only considers the simplest case of fixed government debt supply, but the model does reveal an interesting trade-off that calls for an optimal strategy of government debt management. When the government issues more debt, it benefits the firms by reducing the money premium, but at the same time, it hurts the bankers who rely on the money premium as a source of profit. Whether the government should increase or decrease its debt supply depends on which sector is more constrained. In recession, both sectors tend to be severely constrained: the marginal value of banks’ equity is high because the sector is undercapitalized; the marginal value of firms’ cash holdings is also high because they are holding less cash. The government’s decision to increase or decrease its debt supply should balance the impact on both sectors.

4.3 Discussion

Optimal timing of government debt supply. To what extent increasing government debt supply benefits the firms depends on the banks’ deposit creation capacity. When $\eta_t$ is high and banks are already supplying a large amount of money, the marginal benefit of increasing government debt is small. When banks are undercapitalized and not creating enough deposits, increasing government debt can significantly relax the liquidity constraint on firms’ investments. This suggests a contingent strategy of government debt supply, one which favors countercyclicality: issuing more debt in bad times (low $\eta_t$) and less in good times (high $\eta_t$).

In contrast, the crowding-out effects on banks favor an exactly opposite strategy (i.e., procyclical government debt supply). In good times, banks’ leverage tends to be high, making the economy unstable. This suggests that the government should increase its debt to crowd out banks’ leverage. Moreover, since banks are well capitalized, the government does not need to worry much about crowding out banks’ profit and equity.\(^{51}\) In bad times, the equity crowding-out effect becomes a major concern. The government may want to reduce its debt, allowing the banking sector to speed up balance-sheet repairment by earning a high money premium.

In Appendix II.5, I solve the model with contingent government debt supply, which follows a simple procyclical rule: $M^G_0 + M^G_1 \eta_t$, where $M^G_1 > 0$, and $M^G_0$ and $M^G_1$ are chosen so that averaged over the cycle, the government debt-to-output ratio is 50%. The solution illustrates the

\(^{51}\) Increasing government debt gives banks incentive to pay out dividends at a lower level of equity. By decreasing the payout boundary, it strengthens the asymmetric impact of shocks, making the impact of good shocks more limited. This negative effect needs to be weighed in against the positive effect of stability from reducing banks’ leverage.
trade-off that government faces. Under this procyclical rule, the expected time to recovery is reduced by half in comparison to the model with constant government debt-to-output ratio equal to 50%. However, the money premium is higher in the depth of the recession, which translates into less investments. Therefore, the recession becomes deeper, but the recovery becomes faster.

**Bank equity crowding in.** So far, we have only considered the competition between government and intermediaries as money suppliers. Under additional frictions, government debt may be held by banks for their own liquidity needs. For instance, consider shocks from payment imbalances that lead to deposit migration across individual banks (e.g., Bianchi and Bigio (2014); Drechsler, Savov, and Schnabl (2014)). Banks hit by bad shocks need to pay those hit by good shocks. Failing to do so incurs a cost. If loans must be held until maturity and are thus illiquid, banks may hold government debt as a means of payment between each other. Banks may also hold government securities for regulatory purpose, for example to meet the liquidity coverage ratio (Basel Committee on Bank Supervision (2013)). Therefore, increases in government debt could relax banks’ liquidity constraint, and thereby, may increase their profit and crowd in equity.

**Optimal bailout scale.** In bad states, the government may intervene to recapitalize the banking sector, like the Troubled Asset Relief Program (“TARP”) in the financial crisis of 2007-09. On the one hand, the banking sector benefits from equity injection. On the other hand, banks’ profit from money creation is squeezed by the government debt that finances the bailout program. Balancing the two effects, the government may find an optimal scale of bailout financed by government debt in the sense that the expected time to recover is minimized.

**Regulated and unregulated banking.** We can reinterpret $M^G \left( K^T_t + K^I_t \right)$ as money supplied by a separate banking sector that is fully regulated and backs deposits with 100% reserves in the form of government debt. Regulated banks’ balance sheets are just a pass-through. Their money supply grows proportionally with the economy. Such a banking sector is in line with the Chicago Plan. The competition between government and intermediaries in supplying money can thus be reinterpreted as the competition between fully regulated banks and unregulated banks.

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52The illiquidity of loans may arise from relationship lending or other frictions that either limits end borrowers’ multilateral commitment or dilutes original lenders’ enforcement efforts. Government debt may also serve as collateral to reduce agency cost (e.g. Saint-Paul (2005)) or as collateral in transactions between intermediaries to mitigate counterparty risks (e.g, Gorton (2010); Duffie, Scheicher, and Vuillemey (2015)). In both cases, increasing government debt benefits the financial sector.

53It is a banking reform initially proposed by Frank Knight and Henry Simons of the University of Chicago and supported by Irving Fisher of Yale University (Phillips (1996)). Benes and Kumhof (2012) revisited the plan in the interest of financial stability.
Besides many issues surrounding the Chicago Plan, such as politicized credit allocation, this paper points to a particular concern. If private money cannot be fully forbidden, the leverage cycle in the unregulated shadow banking sector can be amplified by the introduction of fully government-backed money.

**Asset purchase programs.** The government may compete with financial intermediaries at another front, the credit supply. Through the lens of the model, the central bank’s purchases of mortgage-related securities through the quantitative easing programs can be understood as investments in collateralized loans to households that are financed by debt. The increase of government debt reduces the money premium, and the increase in government’s credit supply tends to push up the collateral premium, because the borrowing constraint of households is more likely to bind. According to Corollary 1’, banks’ expected profit from money creation is the difference between the money premium and the collateral premium. With the government supplying more debt and credit simultaneously, the pie (money premium) is smaller, and by carving out a larger share as collateral premium, banks are forced to take a smaller slice.

From a liquidity provision perspective, governments should back their debt with assets that are usually outside of financial intermediaries’ domain. However, other mechanisms may favor government purchases of assets traded and held exactly by financial intermediaries. In settings such as He and Krishnamurthy (2012) and Brunnermeier and Sannikov (2014), when intermediaries are the first-best owners of certain assets and face recapitalization frictions, public asset purchases may stop a fire sale by sustaining asset prices.

**Market liquidity and financial instability.** Firms can hold other firms’ or households’ liabilities as a liquidity buffer if there exists liquid secondary markets. Suppose \( M^F (K^T_t + K^I_t) \) is the maximum amount of liquid securities that firms and households can issue in aggregate. Since both are willing to issue any securities that promise an expected return less or equal to \( \rho \), firms and households maximize issuance in the presence of the money premium.

To analyze the effects of market liquidity, we can simply follow the analysis of government money supply. The conclusions carry through. An increase in the securities that can be readily sold in secondary markets may amplify the bank leverage cycles and prolong recessions. From 1975 to 2014, the stock market capitalization in the United States increased from around 40% of GDP to 140%. While the competition between intermediated and market liquidity is not the focus of this

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54 In the first and third stages of quantitative easing (QE1 and QE3), the Fed acquired newly originated mortgage-backed securities (Di Maggio, Kermani, and Palmer (2016)).
paper, the framework here does point out a quantitatively relevant possibility that from a monetary perspective, deep financial markets may destabilize the financial sector.

5 Conclusion

In the context of intangible investment and corporate cash holdings, this paper revisits the money view of financial intermediation in Friedman and Schwartz (1963). The theory has two ingredients: first, a bank balance-sheet channel of inside money creation; second, procyclical money demand driven by investment needs. The calibrated model produces quantitative implications on the cyclicalality of financial intermediaries’ leverage, the cyclicalality of money premium and credit spread, the frequency and duration of recessions, and the impact of government debt on the cycle.

Firms’ intangible capital requires complementary cash holdings to finance periodic growth opportunities. Cash is banks’ risk-free debt backed by risky loans (“inside money”). When bad shocks trigger massive loan defaults, banks become undercapitalized, so the supply of inside money shrinks and the money premium spikes. As holding cash becomes more expensive, firms hold less cash, and as a result, their intangible investment declines and economic growth slows down. Recession is caused by the collapse of inside money creation.

The probability and duration of recession depend on the cyclicality of banks’ leverage. Following good shocks, banks collect more loan repayments and their equity grows, so the supply of inside money expands and the money premium falls. Firms assign a higher valuation on intangible capital because it is now cheaper to hold the complementary cash. As intangible capital becomes more valuable, firms want to hold more deposits in anticipation of growth opportunities. Therefore, good shocks lead to the expansion of both the money supply and demand, so in equilibrium, banks’ debt can grow faster than their equity, leading to procyclical leverage. In contrast to the existing literature, the procyclicality of leverage arises from the real sector’s procyclical money demand. Combined with the asymmetric impact of shocks near bank issuance and payout boundaries, procyclical leverage leads to frequent and stagnant recessions.

The model also provides a laboratory to explore the financial stability implications of government debt as alternative money. By squeezing banks’ profit from inside money creation, government debt can crowd out bank equity, and thereby, amplifies the procyclicality of bank leverage and destabilizes the economy. The calibrated model shows these effects are quantitatively significant. The key mechanism is the endogenous responses of banks’ issuance and payout policies,
which is ignored by the first attempts in the literature.

Whether public and private money are complements or substitutes is still an open question. This paper focuses on the later, but a richer environment can definitely entertain both possibilities. Banks may hold government debt as a buffer against their own liquidity shocks. In the presence of large aggregate shocks that trigger banks’ default, the proceeds from government debt issuance can be used to bail out the banking sector in order to avoid a sudden evaporation of inside money.

Another direction of future research is to combine the money view with the credit view (Bernanke (1983)), by allowing firms to partially finance investment through bank credit. Whether cash and credit are substitutes or complements has critical implications on the cyclicity of bank leverage and instability. If the investment technology has decreasing returns to scale, cash and credit are substitutes. When banks have a strong balance sheet and are willing to extend credit, firms demand less bank debt because the marginal benefit of holding cash has declined. Countercyclical money demand could lead to countercyclical bank leverage, stable booms, and fast recovery from recessions. In contrast, if the investment technology has increasing returns to scale, cash and credit are complements. Extra bank credit amplifies the marginal benefit of cash holdings, so in good states when banks are well capitalized and willing to lend, firms’ demand for bank debt as cash holdings increases, which amplifies the procyclicality of bank leverage. Increasing returns to scale is particularly relevant for the new-economy industries that feature network externalities (Katz and Shapiro (1985)) or superstar economy (Rosen (1981)).

The setup in this paper is motivated by two concurring phenomena. In the last few decades, the financial sector has grown exponentially, fueled by short-term debt that is deemed money-like. Meanwhile, nonfinancial corporations in the United States hold an enormous amount of money-like securities, largely driven by the entry of R&D-intensive firms. Behind these two trends is a structural transformation towards a new economy where growth heavily relies on intangible investment (Corrado and Hulten (2010)). Against this broad landscape, the money view of financial intermediation may help us understand many other phenomena going forward, far beyond this simple account of procyclical financial intermediation, instability, and stagnation.
Appendix I  Proofs

Proof of Lemma 1. Let $k^I_I (s)$ denote the endowed intangible capital holdings of the representative firm $s$, so the aggregate intangible capital stock is $K^I_I = \int_{s \in [0,1]} k^I_I (s) \, ds$. To save notations, I will suppress the firm index $s$ going forward. Given $q_0^I$, the market price of intangible capital, the representative firm’s wealth is $e^I_I = q_0^I k^I_I$. The firm chooses $k^I_I$, units of intangible capital, $m^I_I$, deposits held per unit of intangible capital, and consumption $c^I_I$ at $t = 0$, and investment rate $i^I_I$ at $t = 1$. If $c^I_I < 0$, the firm issues equity to households. The value function $V^I \left( e^I_I \right) =$

$$\max_{c^I_I \geq 0, k^I_I \geq 0, m^I_I \geq 0} c^I_I + \frac{1}{1 + \rho} \left[ \alpha k^I_I + (1 + r_0) m^I_I k^I_I + \max_{i^I_I \geq 0} \lambda \left( \alpha F \left( i^I_I \right) - i^I_I \right) k^I_I \right],$$

subject to the liquidity constraint at $t = 1$,

$$i^I_I \leq m^I_I,$$

and the budget constraint at $t = 0$,

$$c^I_I + q_0^I k^I_I + m^I_I k^I_I \leq e^I_I.$$

Without the liquidity constraint, the first-best investment rate is given by $\alpha F' \left( i^I_{FB} \right) = 1$.

The Lagrange is:

$$V^I \left( e^I_I \right) = \max_{c^I_I \geq 0, k^I_I \geq 0, m^I_I \geq 0, i^I_I \geq 0} c^I_I + \frac{1}{1 + \rho} \left[ \alpha k^I_I + (1 + r_0) m^I_I k^I_I + \lambda \left( \alpha F \left( i^I_I \right) - i^I_I \right) k^I_I \right]$$

$$\quad + \kappa^L_0 \left( m^I_I - i^I_I \right) + \kappa^E_0 \left( e^I_I - c^I_I - q_0^I k^I_I - m^I_I k^I_I \right),$$

where $\kappa^L_0$ and $\kappa^E_0$ are the present (i.e., time 0) shadow values of deposits and firm equity respectively. The first order condition (F.O.C.) for $k^I_I$ is:

$$k^I_I \left[ \alpha + (1 + r_0) m^I_I + \lambda \left( \alpha F \left( i^I_I \right) - i^I_I \right) \right] = 0, \text{ and } k^I_I \geq 0,$$

F.O.C. for $i^I_I$:

$$i^I_I \left[ \lambda \left( \alpha F' \left( i^I_I \right) - 1 \right) k^I_I + (1 + \rho) \kappa^L_0 \right] = 0, \text{ and } i^I_I \geq 0,$$

47
F.O.C. for $m_0^I$

\[ m_0^I \left[ (1 + r_0) k_0^I + (1 + \rho) \kappa^L - (1 + \rho) \kappa_0^E k_0^I \right] = 0, \text{ and } m_0^I \geq 0, \]

and the complementary slackness conditions

\[ \kappa_0^L (m_0^I - i_1^I) = 0, \text{ and } \kappa_0^L \geq 0, \]

and

\[ \kappa_0^E (e_0^I - c_0^I - q_0 k_0 - m_0^I l_0^I) = 0, \text{ and } \kappa_0^E \geq 0. \]

Substitute the F.O.C.s into the Lagrange, we have the firm’s value function linear in equity $e_0^I$:

\[ V^I (e_0^I) = c_0^I + \kappa_0^E (e_0^I - c_0^I). \]

The return on retained equity, $e_0^I - c_0^I$, is $\kappa_0^E (1 + \rho)$.

Note that because entrepreneurs can issue equity ($c_0^I < 0$) or consume ($c_0^I > 0$), we must have $\kappa_0^E = 1$, so $V^I (e_0^I) = e_0^I$, the firm’s budget constraint binds, and the return on retained equity is $(1 + \rho)$. Substituting $\kappa_0^E = 1$ into the F.O.C. for $m_0^I$, we can solve the shadow value of deposits per unit of capital holdings:

\[ \kappa_0^L = \frac{1}{(1 + \rho) k_0^I (\rho - r_0)}. \]

When $r_0 < \rho$, $\kappa_0^L > 0$ and the liquidity constraint binds, (i.e. $i_1^I = m_0^I$). Substituting this shadow value into the F.O.C. for $i_1^I$, we get the optimality condition for deposit holdings. Note that $i_1^I = m_0^I$:

\[ r_0 + \lambda (\alpha F' (i_1^I) - 1) = \rho. \]

Q.E.D.

**Proof of Lemma 2.** I follow similar notations as in the proof of Lemma 1. At $t = 0$, given wealth $c_0^T = q_0^T k_0^T + g_0$, where $g_0$ is households’ goods endowment, the representative household chooses $k_0^T$ units of tangible capital, $m_0^T$ deposits held per unit of tangible capital, and $l_0^T$ bank loan. Households also choose the units of firms’ equity, $h_t^T$. As shown in the proof of Lemma 2, the expected return on the investment in firm equity is $1 + \rho$. Let $q_0^E$ denote the unit price of firm’s
equity. The value function

$$V^T (e^T_0) = \max_{c^T_0 \geq 0, k^T_0 \geq 0, m^T_0 \geq 0, l^T_0 \geq 0} c^T_0 + \frac{1}{1 + \rho} [(1 - \mathbb{E}_0 [\pi (Z_1)]) \alpha k^T_0 + (1 + r_0) m^T_0 k^T_0$$

$$+ (1 + \rho) h^T_0 - (1 + R_0) (1 - \mathbb{E}_0 [\pi (Z_1)]) l^T_0],$$

subject to the budget constraint

$$c^T_0 + q^T_0 k^T_0 + m^T_0 k^T_0 + q^E_0 h^T_0 \leq e^T_0 + l^T_0,$$

and the collateral constraint

$$l^T_0 (1 + R_0) \leq \alpha k^T_0.$$

Note that $\mathbb{E}_0 [\pi (Z_1)] = \delta$. The Lagrange is

$$V^T (e^T_0) = \max_{c^T_0 \geq 0, k^T_0 \geq 0, m^T_0 \geq 0, l^T_0 \geq 0} c^T_0 + \frac{1}{1 + \rho} [(1 - \delta) \alpha k^T_0 + (1 + r_0) m^T_0 k^T_0$$

$$+ (1 + \rho) h^T_0 - (1 - \delta) (1 + R_0) l^T_0] + \kappa^E_0 (e^T_0 + l^T_0 - c^T_0$$

$$- q^T_0 k^T_0 - m^T_0 k^T_0 - q^E_0 h^T_0) + \kappa^T_0 (\alpha k^T_0 - l^T_0 (1 + R_0)),$$

where $\kappa^E_0$ and $\kappa^T_0$ are the present (i.e., time 0) shadow values of financing and collateral respectively. The first order condition (F.O.C.) for $k^T_0$ is

$$k^T_0 [(1 - \delta) \alpha + (1 + r_0) m^T_0 - (1 + \rho) \kappa^T_0 (q^T_0 + m^T_0) + (1 + \rho) \kappa^T_0 \alpha] = 0, \text{ and } k^T_0 \geq 0,$$

F.O.C. for $h^T_0$

$$h^T_0 (1 - \kappa^E_0 q^E_0) = 0, \text{ and } h^T_0 \geq 0,$$

F.O.C. for $m^T_0$

$$m^T_0 [(1 + r_0) k^T_0 - (1 + \rho) \kappa^T_0 k^T_0] = 0, \text{ and } m^T_0 \geq 0,$$

F.O.C. for $l^T_0$

$$l^T_0 [-(1 - \delta) (1 + R_0) + (1 + \rho) \kappa^E_0 - (1 + \rho) \kappa^T_0 (1 + R_0)] = 0, \text{ and } l^T_0 \geq 0,$$
and the complementary slackness conditions,

$$\kappa^E_0 (e^T_0 + l^T_0 - c^T_0 - q^T_0 k^T_0 - m^T_0 k^T_0) = 0, \text{ and } \kappa^E_0 \geq 0,$$

and,

$$\kappa^T_0 (\alpha k^T_0 - l^T_0 (1 + R_0)) = 0, \text{ and } \kappa^T_0 \geq 0.$$

Substitute the F.O.C.s into the Lagrange, we have the household’s value function linear in wealth $e^T_0$:

$$V^T (e^T_0) = c^T_0 + \kappa^E_0 (e^T_0 - c^T_0).$$

On the one hand, by assumption, households’ goods endowment is sufficiently large, so to clear the goods market, households must consume (i.e. $e^T_0 > 0$). Therefore, we have $\kappa^T_0 \leq 1$. On the other hand, the market for tangible capital must clear, so households must find it optimal to retain some wealth that is stored in the tangible capital. Therefore, $\kappa^T_0 \geq 1$. Combining these two conditions, we have $\kappa^E_0 = 1$. And from the F.O.C. for $h^T_0$, we have the unit price of firms’ equity must also be one (i.e. $q^E_0 = 1$), if firms ever issue outside equity.

Substituting $\kappa^E_0 = 1$ into the F.O.C. for $m^T_0$:

$$m^T_0 (r_0 - \rho) = 0.$$

Therefore, $m^T_0 = 0$ if $r_0 < \rho$. Substituting $\kappa^E_0 = 1$ into the F.O.C. for $l^T_0$, we have (ignoring the product of two percentages, $R_0 \delta$)

$$\kappa^T_0 = \frac{\rho + \delta - R_0}{1 + \rho}.$$

Apparently, when the collateral constraint does not bind, $\kappa^T_0 = 0$, we have $R = \delta + \rho$. Rearranging the equation, we have (ignoring the product of two percentages, $\rho \kappa^T_0$):

$$R_0 = \rho + \delta - \kappa^T_0.$$

Q.E.D.

**Proof of Proposition 1.** The expected return on bank equity is:

$$x^B_0 (1 + R_0 - E[\pi(Z_1)]) - (x^B_0 - 1) (1 + r_0) = 1 + r_0 + x^B_0 (R_0 - E[\pi(Z_1)] - r),$$

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and when $Z_1 = -1$, the realized return on bank equity is:

$$1 + r_0 + x_0^B (R_0 - \delta - \sigma - r_0).$$

Note that $E[\pi_D(Z_1)] = \delta$. The representative bank’s value function is:

$$v(e_0^B; R_0, r_0) = \max_{c_0^B \geq 0, x_0^B \geq 0} c_0^B + \left(\frac{c_0^B}{e_0^B} - c_0^B\right) \left\{1 + r_0 + x_0^B (R_0 - \delta - r_0) + \xi_0 \left[1 + r_0 + x_0^B (R_0 - \delta - \sigma - r_0)\right]\right\}.$$

The first order condition for $x_0^B$ is:

$$R_0 - r_0 = \delta + \gamma_0^B \sigma,$$

where $\gamma_0^B = \frac{\xi_0}{1 + \xi_0} \in [0, 1)$ because $\xi_0 \geq 0$. Rearranging the equation, we have $\gamma_0^B$ equal to the Sharpe ratio of loans:

$$\gamma_0^B = \frac{R_0 - \delta - r_0}{\sigma}.$$

When $\gamma_0^B > 0$, the capital adequacy constraint binds. Substituting the F.O.C. for $x_0^B$ into the value function, we have:

$$v(e_0^B; R_0, r_0) = c_0^B + q_0^B (e_0^B - c_0^B),$$

where $q_0^B = \frac{(1 + r_0)(1 + \xi_0)}{(1 + \rho)}$. The bank chooses $c_0^B > 0$ only if $q_0^B \leq 1$.

**Proof of Corollary 1.** Proof is provided in the main text.

**Proof of Corollary 2.** Proof is provided in the main text.

**Proof of Lemma 1’ and Proposition 2.** First, I need to prove firms’ marginal value of equity is equal to one. Conjecture that a scalar diffusion process $\zeta_t^I$ summarizes firms’ equilibrium investment opportunity set.

$$d\zeta_t^I = \zeta_t^I \mu_t^c dt + \zeta_t^I \sigma_t^c dZ_t,$$

where $\mu_t^c$ and $\sigma_t^c$ are scalar diffusion processes of the instantaneous expectation and standard deviation of $\frac{d\zeta_t^I}{\zeta_t^I}$ respectively. Let $V_I(e_t^I; \zeta_t^I)$ denote the value function. Following previous nota-
investments, \( e_t^I \) is the representative firm’s equity. The Hamilton-Jacobi-Bellman (HJB) equation:

\[
\rho V^I (e_t^I; \zeta_t^I) = \max_{dc_t^I, k_t^I \geq 0, m_t^I \geq 0} \frac{dc_t^I}{dt} - \frac{\partial V^I}{\partial e_t^I} \frac{dc_t^I}{dt} + \frac{\partial V^I}{\partial \zeta_t^I} \mu_t^I \zeta_t^I + \frac{\partial V^I}{\partial e_t^I} \mu_t^I e_t^I + \frac{1}{2} \frac{\partial^2 V}{\partial (\zeta_t^I)^2} \left( \sigma_t^I \right)^2 + \frac{1}{2} \frac{\partial^2 V}{\partial (e_t^I)^2} \left( \sigma_t^I \right)^2 + \frac{\partial^2 V}{\partial e_t^I \partial \zeta_t^I} \left( \sigma_t^I \right) \left( \sigma_t^I e_t^I \right) + \lambda \left[ V (\tilde{e}_t^I; \tilde{\zeta}_t^I) - V (e_t^I; \zeta_t^I) \right],
\]

where \( \mu_t^I, \sigma_t^I, \) and \( \tilde{\zeta}_t^I \) are defined by the following dynamics of firm equity

\[
dc_t^I = -dc_t^I + \mu_t^I e_t^I dt + \sigma_t^I e_t^I dZ_t + (\tilde{e}_t^I - e_t^I) dN_t,
\]

where \( dN_t \) is the increment of the idiosyncratic counting (Poisson) process \( (dN_t = 1 \) when the investment opportunity arrives), and after the Poisson shock, firm equity jumps to

\[
\tilde{e}_t^I = e_t^I + q_t^I F (m_t^I) k_t^I - m_t^I k_t^I.
\]

Note that firms’ marginal value of wealth, \( \zeta_t^I \), is a summary statistic of firms’ investment opportunity set, which depends on the overall industry dynamics, so it does not jump when an individual firm is hit by investment opportunities.

Conjecture the value function is linear in equity, \( V^I (e_t^I; \zeta_t^I) = \zeta_t^I e_t^I \). The HJB equation can be simplified as:

\[
\rho V^I (e_t^I; \zeta_t^I) = \max_{dc_t^I, k_t^I \geq 0, m_t^I \geq 0} \frac{dc_t^I}{dt} - \zeta_t^I \frac{dc_t^I}{dt} + e_t^I \zeta_t^I \mu_t^I + e_t^I \zeta_t^I \mu_t^I + \left( \sigma_t^I \right) \left( \sigma_t^I e_t^I \right) + \lambda \tilde{\zeta}_t^I \left[ \tilde{e}_t^I - e_t^I \right].
\]

Firms can choose any \( dc_t^I \in \mathbb{R} \) (raise equity if \( dc_t^I < 0 \), so \( \zeta_t^I \) must be equal to one, and thus, I have also confirmed the value function conjecture.

Next, I solve the optimality condition for deposits holdings in Lemma 1’, and the capital pricing formula in Lemma 2. Since \( \zeta_t^I \) is a constant equal to one, a constant, \( \mu_t^I \) and \( \sigma_t^I \) are both zero. Therefore, the HJB equation can be further simplified:

\[
\rho V^I (e_t^I; \zeta_t^I) = \max_{k_t^I \geq 0, m_t^I \geq 0} \mu_t^I e_t^I + \lambda \left[ q_t^I F (m_t^I) - m_t^I \right] k_t^I.
\]

Because \( V^I (e_t^I; \zeta_t^I) = e_t^I \), the maximized expected growth rate of wealth is equal to \( \rho e_t^I \). The ex-
expected changed of wealth after consumption contains the production flow from capital, the change of capital value (from both price change and potential destruction), the return on bank deposits, and the net gain from potential investment opportunities:

\[
\alpha k_t^I dt + \mathbb{E}_t \left( q_{t+dt}^I k_{t+dt}^I - q_t^I k_t^I \right) + r_t m_t^I k_t^I dt + \lambda dt \left[ q_t^I F \left( m_t^I \right) - m_t^I \right] k_t^I
\]

The firm sets its balance sheet to maximize the expected growth of wealth under the budget constraint:

\[
\max_{k_t^I \geq 0, m_t^I \geq 0} \quad \alpha k_t^I dt + \mathbb{E}_t \left( q_{t+dt}^I k_{t+dt}^I - q_t^I k_t^I \right) + r_t m_t^I k_t^I dt + \lambda dt \left[ q_t^I F \left( m_t^I \right) - m_t^I \right] k_t^I
\]

\[+ \left( e_t^I - q_t^I k_t^I - m_t^I k_t^I \right) \mu_t^I dt, \]

where \( \mu_t^I \) is the Lagrange multiplier of the budget constraint, \( q_t^I k_t^I + m_t^I k_t^I \leq e_t^I \).

The optimal deposit holdings per unit of capital are given by the first-order condition (F.O.C.):

\[
m_t^I \left\{ r_t dt + \lambda dt \left[ q_t^I F' \left( m_t^I \right) - 1 \right] - \mu_t^I dt - \sigma_t^I dt + \sigma_t^I dt \right\} = 0, \quad \text{and} \quad m_t^I \geq 0.
\]

A fraction \((\delta dt - \sigma dZ_t)\) of capital is expected to be destroyed, so the capital evolves as

\[
k_{t+dt}^I = k_t^I - (\delta dt - \sigma dZ_t) k_t^I.
\]

Combining with the equilibrium price dynamics, we have, under Itô calculus,

\[
q_{t+dt}^I k_{t+dt}^I - q_t^I k_t^I = q_t^I k_t^I \left[ - (\delta dt - \sigma dZ_t) + \mu_t^I dt + \sigma_t^I dZ_t + \sigma_t^I dt \right].
\]

The optimal capital holdings is given by the first-order condition (F.O.C.): \( k_t^I \geq 0 \), and

\[
k_t^I \left\{ \alpha dt + q_t^I \left( -\delta + \mu_t^I + \sigma_t^I \right) dt + r_t m_t^I dt + \lambda dt \left[ q_t^I F \left( m_t^I \right) - m_t^I \right] - \left( q_t^I + m_t^I \right) \right\} = 0.
\]

Finally, we have the complementary slackness condition: \( \mu_t^I \geq 0 \), and

\[
\left( e_t^I - q_t^I k_t^I - m_t^I k_t^I \right) \mu_t^I = 0.
\]
Substituting these optimality conditions into the objective function, we have

$$\max_{k_t^I \geq 0, m_t^I \geq 0, l_t^I \geq 0, h_t^I \geq 0} \mu_t^I e_t^I dt + \lambda dt \left[ q_t^I F (m_t^I) - m_t^I \right] k_t^I = e_t^I d\kappa_t^E.$$  

From Equation (16), the left hand side is equal to \( \rho e_t^I dt \), so the Lagrange multiplier \( d\kappa_t^E = \rho dt \).

Substituting \( d\kappa_t^E = \rho dt \) into the F.O.C. for \( m_t^I \), we have

$$r_t + \lambda \left[ q_t^I F' (m_t^I) - 1 \right] = \rho.$$

Substituting \( d\kappa_t^E = \rho dt \) into the F.O.C. for \( k_t^I \) and rearranging the equation, we have

$$q_t^I = \frac{\alpha - (\rho - r_t) m_t^I + \lambda \left[ q_t^I F (m_t^I) - m_t^I \right]}{\rho - (\mu_t^I - \delta + \sigma \sigma_t^I)}.$$

Q.E.D.

**Proof of Lemma 2'**. Following the same steps in the proof of Lemma 1' and Proposition 2, we know the marginal value of wealth for households is equal to one, and the representative household’s HJB equation is:

$$\rho V^T (e_t^T, \zeta_t^T) = \rho e_t^T = \max_{k_t^I \geq 0, m_t^I \geq 0, l_t^I \geq 0, h_t^I \geq 0} \mu_t^I e_t^I,$$  

where, following the same notations, \( e_t^T \) is the household’s wealth, \( k_t^T \) tangible capital holdings, \( m_t^T \) deposits held per unit of tangible capital, \( l_t^T \) borrowing from banks, and \( h_t^T \) is the household’s investment in firms’ equity. Let \( q_t^E \) denote the price of one unit of firms’ equity. I assume that households do not internalize the spillover growth of tangible capital.

The household sets its balance sheet to maximize \( \mu_t^E e_t^T dt \), which contains the production flow from capital, the change of capital value (from both price change and potential destruction), the expected return on bank deposits, the expected return on firm equity, and the expected loan payment:

$$\alpha k_t^T dt + \mathbb{E}_t \left( q_t^T k_t^T dt - q_t^T k_t^T \right) + r_t m_t^T k_t^T dt + \frac{\rho}{q_t^E} h_t^T dt - l_t^T \mathbb{E}_t \left[ R_t dt - (\delta dt - \sigma dZ_t) \right].$$

Since \( (\delta dt - \sigma dZ_t) \) fraction of capital will be destroyed, loans backed by such capital do not need
to be repaid, so the total repayment per dollar of borrowing is \((1 + R_t dt) [1 - (\delta dt - \sigma dZ_t)]\), which can be simplified to \(1 + R_t dt - (\delta dt - \sigma dZ_t)\) under Itô calculus, ignoring \(R_t dt (\delta dt - \sigma dZ_t)\), which is infinitesimal of higher order.

Similar to firms, the representative household solves the following problem:

\[
\max_{k_t^T \geq 0, m_t^T \geq 0, l_t^T \geq 0, h_t^T \geq 0} \alpha k_t^T dt + \mathbb{E}_t \left( q_t^T k_{t+dt}^T - q_t^T k_t^T \right) + r_t m_t^T k_t^T dt + \rho h_t^T dt
\]

\[
- l_t^T \mathbb{E}_t [R_t dt - (\delta dt - \sigma dZ_t)]
\]

\[
+ \left( e_t^T + l_t^T - q_t^T k_t^T - m_t^T k_t^T - q_t^E h_t^T \right) d\kappa_t^E + \left( q_t^T k_t^T - l_t^T \right) d\kappa_t^L,
\]

where \(d\kappa_t^E\) is the Lagrange multiplier of the budget constraint, \(q_t^T k_t^T + m_t^T k_t^T + q_t^E h_t^T \leq e_t^T + l_t^T\), and \(d\kappa_t^L\) is the Lagrange multiplier of the collateral constraint, \(l_t^T \leq q_t^T k_t^T\).

The optimal deposits holdings per unit of capital is given by the first-order condition (F.O.C.):

\[
\left( r_t dt - d\kappa_t^E \right) m_t^T = 0.
\]

A fraction \((\delta dt - \sigma dZ_t)\) of capital is expected to be destroyed, so the capital evolves as

\[
k_{t+dt}^T = k_t^T - (\delta dt - \sigma dZ_t) k_t^T.
\]

Combining with the equilibrium price dynamics, we have, under Itô calculus,

\[
q_t^T k_{t+dt}^T - q_t^T k_t^T = q_t^T k_t^T \left[ - (\delta dt - \sigma dZ_t) + \mu_t^T dt + \sigma_t^T dZ_t + \sigma_t^T dt \right].
\]

The first-order condition (F.O.C.) for tangible capital holdings: \(k_t^T \geq 0\), and

\[
k_t^T \left[ \alpha dt + q_t^T \left( -\delta + \mu_t^T + \sigma_t^T \right) dt + r_t m_t^T dt + q_t^E d\kappa_t^L - (q_t^T + m_t^T) d\kappa_t^E \right] = 0.
\]

The F.O.C. for investment in firms’ equity: \(h_t^T \geq 0\), and

\[
h_t^T \left( \rho dt - q_t^E d\kappa_t^F \right) = 0.
\]

The F.O.C. for loan: \(l_t^T \geq 0\), and

\[
l_t^T \left( R_t dt - \delta dt + d\kappa_t^L - d\kappa_t^E \right) = 0.
\]
The complementary slackness conditions: $d\kappa_t^E \geq 0$, and

$$d\kappa_t^E (e_t^T + l_t^T - q_t^T k_t - m_t^T k_t^T) = 0,$$

and $d\kappa_t^L \geq 0$, and

$$d\kappa_t^L (q_t^T k_t - l_t^T) = 0.$$

Substituting these optimality conditions into the maximized $\mu_t^T e_t^T dt$, we have

$$\max_{k_t^T \geq 0, m_t^T \geq 0, l_t^T \geq 0, h_t^T \geq 0} \mu_t^T e_t^T dt = e_t^T d\kappa_t^E.$$

Note that in Equation (17), $\max_{k_t^T \geq 0, m_t^T \geq 0, l_t^T \geq 0, h_t^T \geq 0} \mu_t^T e_t^I = \rho e_t^I$, so the Lagrange multiplier $d\kappa_t^E = \rho dt$. From the F.O.C. for $h_t^T$, we know the price of one unit of firms’ equity is $q_t^E = 1$. Substituting $d\kappa_t^E = \rho dt$ into the F.O.C. for $m_t^T$, we have $m_t^T = 0$ if $r_t < \rho$. Therefore, in equilibrium, households do not hold bank deposits. Substituting $d\kappa_t^E = \rho dt$ and $m_t^T = 0$ into the F.O.C. for $k_t^T$ and rearranging the equation, we have

$$q_t^T = \frac{\alpha}{\left(\rho - \frac{d\kappa_t^L}{dt}\right) - (\mu_t^T - \delta + \sigma \sigma_t^T)}.$$

Since I have already shown that $\kappa_t^L$ is a diffusion process with smooth sample path, in the main text, I define $\kappa_t^T = \frac{d\kappa_t^L}{dt}$ for the collateral premium of tangible capital. Q.E.D.

**Proof of Proposition 1′.** Conjecture that a scalar diffusion process $q_t^B$ summarizes banks’ equilibrium investment opportunity set:

$$dq_t^B = q_t^B \mu_t^B dt + q_t^B \sigma_t^B dZ_t.$$

Following the notations in the main text, the representative bank’s value function is $v(e_t^B; q_t^B)$, and the HJB equation is

$$\rho v(e_t^B; q_t^B) = \max_{dy_t^B \geq 0} \left(1 - \frac{\partial v}{\partial e_t^B} \right) I_{\{dy_t^B > 0\}} e_t^B dy_t^B + \left(\frac{\partial v}{\partial e_t^B} - 1 - \chi\right) I_{\{dy_t^B < 0\}} e_t^B (-dy_t^B) +$$

$$\frac{\partial V}{\partial q_t^B} q_t^B \mu_t^B + \frac{\partial^2 V}{\partial q_t^B \partial q_t^B} (q_t^B \sigma_t^B)^2 + \frac{\partial V}{\partial e_t^B} e_t^B \mu_t^B + \frac{1}{2} \frac{\partial^2 V}{\partial (e_t^B)^2} (\sigma_t^B e_t^B)^2 + \frac{\partial^2 V}{\partial e_t^B \partial q_t^B} (q_t^B \sigma_t^B) (\sigma_t^B e_t^B).$$
From the bank’s budget constraint, Equation (6), we have:

\[ \mu^e_t = r_t + x^B_t (R_t - \delta - r_t), \quad \text{and} \quad \sigma^e_t = x^B_t \sigma. \]

Conjecture that the bank’s value function takes the linear form: \( v(e^B_t; q_t^B) = q_t^B e^B_t \). Substituting this conjecture into the bank’s HJB equation and dividing both sides by \( q_t^B e^B_t \), we have

\[
\rho = \max_{dy^B_t} \left\{ \left(1 - q_t^B\right) \mathbb{I}_{\{dy^B_t > 0\}} dy^B_t + \frac{(q_t^B - 1 - \chi)}{q_t^B} \mathbb{I}_{\{dy^B_t < 0\}} (-dy^B_t) \right\} + \mu^B_t + \max_{x^B_t \geq 0} \left\{ r_t + x^B_t (R_t - \delta - r_t) - x^B_t \gamma_t^B \sigma \right\} - \ell,
\]

where \( \gamma_t = -\sigma^B_t \).

\( q_t^B \) is the marginal value of equity. Paying out one dollar of dividend, the bank’s shareholders receive 1, but lose \( q_t^B \). Only when \( q_t^B \leq 1, \; dy^B_t > 0 \). When the bank issues equity, it incurs a dilution cost. From the existing shareholders’ perspective, one dollar equity is sold to outside investors at price \( q_t^B + \frac{\chi}{1 + \chi} \). To raise \( -(dy^B_t) e^B_t \) that is worth \( q_t^B (-dy^B_t) e^B_t \), the bank must issue \( \frac{(1 + \chi)(-dy^B_t) e^B_t}{q_t^B} \) shares, and thus, the existing shareholders give up total value of \( q_t^B \frac{(1 + \chi)(-dy^B_t) e^B_t}{q_t^B} = (1 + \chi) (-dy^B_t) e^B_t \). Therefore, the bank raises equity only if \( q_t^B \geq 1 + \chi \).

Finally, the indifference condition for \( x^B_t \) is \( R_t - \delta - r_t = \gamma_t \sigma \). If \( R_t - \delta - r_t < \gamma_t \sigma \), the bank sets \( x^B_t = 0 \). If \( R_t - \delta - r_t > \gamma_t \sigma \), the bank sets \( x^B_t \) to infinity. Q.E.D.

**Proof of Corollary 1′.** Proof is provided in the main text.

**Proof of Corollary 2′.** Proof is provided in the main text.

**Proof of Corollary 3.** First, I derive equation (8). Because individual banks share the same \( \mu^e_t, \sigma^e_t \), and payout/issuance rate \( dy^B_t \), aggregating over banks, the law of motion of \( E^B_t \) is

\[
dE^B_t = \mu^e_t E^B_t \, dt + \sigma^e_t E^B_t \, dZ_t - dy^B_t E^B_t.
\]

Given the expected growth rate, \( \mu^K_t = \lambda F \left( m_t^e \right) - \delta \), which is driven by intangible investment, the
aggregate capital stock, $K_t = K_t^I + K_t^T$, evolves as:

$$dK_t = \mu^K_t K_t dt + \sigma K_t dZ_t,$$

where the diffusion term comes from the stochastic fraction of capital destroyed.

By Itô’s lemma, the ratio, $\eta_t = \frac{E_t^B}{K_t}$, has the following law of motion:

$$d\eta_t = \frac{1}{K_t} dE_t^B - \frac{E_t^B}{K_t^2} dK_t + \frac{1}{K_t^3} (dK_t, dK_t) - \frac{1}{K_t^2} \langle dE_t^B, dK_t \rangle,$$

where $\langle dX_t, dY_t \rangle$ denotes the quadratic covariation of diffusion processes $X_t$ and $Y_t$, so we have $\langle dK_t, dK_t \rangle = \sigma^2 K_t^2 dt$, and $\langle dE_t^B, dK_t \rangle = \sigma_t^e E_t^B K_t dt$. Dividing both sides by $\eta_t$, we have

$$\frac{d\eta_t}{\eta_t} = \frac{dE_t^B}{E_t^B} - \frac{dK_t}{K_t} + \sigma^2 dt - \sigma_t^e dt.$$

Substituting the law of motions of $E_t^B$ and $K_t$, we have Equation (8).

The boundaries are given by banks’ optimal payout and issuance policies in Proposition 1′.

Q.E.D.

**Proof of Proposition 2.** Please refer to the proof of Lemma 1′.

**Proof of Proposition 3.** The proof follows Brunnermeier and Sannikov (2014). First, I derive the stationary probability density function. The Kolmogorov forward equation that characterizes $p(\eta, t)$, the probability density of $\eta_t$ at time $t$, is:

$$\frac{\partial}{\partial t} p(\eta, t) = - \frac{\partial}{\partial \eta} \left( \eta \mu^\eta_t(\eta) p(\eta, t) \right) + \frac{1}{2} \frac{\partial^2}{\partial \eta^2} \left( \eta^2 \sigma^\eta_t(\eta)^2 p(\eta, t) \right).$$

Note that in a Markov equilibrium, $\mu^\eta_t$ and $\sigma^\eta_t$ are functions of $\eta_t$.

A stationary density is a function that solves the forward equation and does not change with time (i.e. $\frac{\partial}{\partial t} p(\eta, t) = 0$). So I suppress the time variable, and denote stationary density as $p(\eta)$. Integrating the forward equation over $\eta$, $p(\eta)$ solves the following first-order ordinary differential equation within the two reflecting boundaries:

$$0 = C - \eta \mu^\eta(\eta) p(\eta) + \frac{1}{2} \frac{d}{d\eta} \left( \eta^2 \sigma^\eta(\eta)^2 p(\eta) \right), \quad \eta \in [\underline{\eta}, \bar{\eta}].$$
Note that the integration constant $C$ is zero because of the reflecting boundaries. The boundary condition is the requirement that the probability density is integrated to one (i.e. $\int_\frac{1}{2}^\frac{3}{2} p(\eta) d\eta = 1$).

Next, I solve the expected time to reach from $\eta$. Define $f_{n_0}(\eta)$ the expected amount of time it takes to reach a point $n_0$ starting from $\eta \leq n_0$. Define $g(\eta_0) = f_{n_0}(\eta)$ the expected time to reach $n_0$ from $\eta$. One has to reach $\eta \in \{\frac{1}{2}, n_0\}$ first and then reach $n_0$ from $\eta$. Therefore, $g(\eta) + f_{n_0}(\eta) = g(\eta_0)$. Since $g(\eta_0)$ is a constant, we can differentiate both sides to have $g'(\eta) = -f_{n_0}'(\eta)$ and $g''(\eta) = -f_{n_0}''(\eta)$.

For any $\eta_t$, $f_{n_0}(\eta_t)$, the expected time to reach $\eta_0$ can be decomposed into $s - t$, and $E_t [f_{n_0}(\eta_s)]$, the expected time to reach $\eta_0$ from $\eta_s$ ($s \geq t$) after $s - t$ amount of time has passed. We get:

$$f_{n_0}(\eta_t) = E_t [f_{n_0}(\eta_s)] + s - t.$$ 

Therefore, $t + f_{n_0}(\eta_t)$ is a martingale, so $f_{n_0}$ satisfies the ordinary differential equation (i.e. zero drift):

$$1 + f_{n_0}'(\eta) \mu^n(\eta) + \frac{\sigma^2(\eta)}{2} f_{n_0}''(\eta) = 0.$$ 

Therefore, $g(\eta)$ must satisfy

$$1 - g'(\eta) \mu^n(\eta) - \frac{\sigma^2(\eta)}{2} g''(\eta) = 0.$$ 

It takes no time to reach $\frac{1}{2}$, so $g\left(\frac{1}{2}\right) = 0$. Moreover, since $\frac{1}{2}$ is a reflecting boundary, $g'(\frac{1}{2}) = 0$. $Q.E.D.$

**Proof of Proposition 4.** Please refer to the proof of Lemma 2$'$.

**Proof of Proposition 5.** Figure 12 shows an alternative graphic illustration of the deposit market. Cash firms’ aggregate deposit holdings (scaled by capital stock) is $M^D_t = \frac{m_t K^t}{K^t + K^\phi} = m^\phi_t$. Under concave investment technology ($F''(\cdot) < 0$), Lemma 1$'$ gives us a downward sloping money demand curve in the space of quantity and premium:

$$\lambda \left[ q^I_t F'(\frac{1}{\phi} M^D_t) - 1 \right] = \rho - r_t.$$ 

To derive the supply curve, we use Equation (11). Define the aggregate deposits issued by
Money premium
\[ \rho - r_t = \lambda \left[ q_I^t F' \left( \frac{1}{\phi} M_t^B \right) \right] - 1 \]

Money supply curve:
\[ \rho - r_t = \frac{\rho}{\phi} M_t^B + \kappa_t^T \]

Panels A and B: intermediary margin and collateral margin, respectively.

Figure 12: Money Market Equilibrium

For banks, \( M_t^S = \frac{(x_t^B - 1) E_t}{K_t^I + K_t^T} = \left( x_t^B - 1 \right) \eta_t \), so \( \sigma_t^\eta = \sigma \left( x_t^B - 1 \right) = \frac{\sigma}{\eta_t} M_t^S \). Substituting \( \sigma_t^\eta \) in the decomposition of \( \gamma_t^B \), we have \( \gamma_t^B = -\frac{\epsilon_t^B \sigma}{\eta_t} M_t^S \) and an upward sloping supply curve:

\[ -\frac{\epsilon_t^B \sigma^2}{\eta_t} M_t^S + \kappa_t^T = \rho - r_t. \]

Equation (12) can be rewritten as the deposit market clearing condition, where the money demand meets supply at equilibrium money premium:

\[ \lambda \left[ q_I^t F' \left( \frac{1}{\phi} M_t^B \right) \right] - 1 = -\frac{\epsilon_t^B \sigma^2}{\eta_t} M_t^S + \kappa_t^T. \]

Panel A of Figure 12 takes a snapshot of the dynamic equilibrium. The deposit market equilibrium moves with the quadruple \( \left( \eta_t, \epsilon_t^B, q_t^T, q_t^I \right) \) in response to shocks. When the borrowing constraint of households binds, \( M_t^S = \frac{q_t^T K_t^T - E_t}{K_t^I + K_t^T} = q_t^T (1 - \phi) - \eta_t \), money supply becomes inelastic (Panel B of Figure 12). A share of money premium \( \kappa_t^T \) is carved out and passed through bank balance sheets to households. The demand curve shifts with \( q_t^I \), which determines the profitability of investment.

Therefore, we can solve the endogenous variables, \( r_t, \kappa_t^T \) (and \( R_t \) by Lemma 2'), \( m_t^I \), and \( x_t^B \), as functions of \( \left( \eta_t, \epsilon_t^B, q_t^T, q_t^I \right) \). Once we know these variables, we can solve the dynamics of \( \eta_t \), and in particular, \( \mu_t^\eta \) and \( \sigma_t^\eta \). Then using Itô’s lemma, banks’ HJB equation, and the capital pricing equations (Equation (13) and (15)), we can form a system of differential equations for \( \left( q_t^B (\eta), q_t^I (\eta), q_t^T (\eta) \right) \). Next, I will describe the details of the solution algorithm. To simplify the
notations, I suppress the time subscripts.

To construct differential equations, all we need is a mapping from $\left( \eta, q^B, q^T, q^I, \frac{dq^B}{dq} \frac{dq^I}{dq} \frac{dq^T}{dq} \right)$ to $\left( \frac{d^2q^B}{d\eta^2} \frac{d^2q^I}{d\eta^2} \frac{d^2q^T}{d\eta^2} \right)$. We can replace the first derivatives with elasticities of $(q^B, q^I, q^T)$, $\epsilon^X = \frac{dq^X}{dq^Y}$, $X \in \{B, I, T\}$. Working with elasticities simplify the expressions. In the following, I take as given $(\eta, q^B, q^T, q^I)\epsilon^B, \epsilon^I, \epsilon^T)$ to solve $\left( \frac{d^2q^B}{d\eta^2} \frac{d^2q^I}{d\eta^2} \frac{d^2q^T}{d\eta^2} \right)$. I will use the calibrated investment function: $F(i) = \omega_0 i^{\omega_1}$. First, I solve $m^I, x^B, r, R$, and $\kappa_T$.

Consider two cases. First, $\kappa_T = 0$, so from Equation (12), we have

$$\gamma^B \sigma = \lambda \left[ q^I \omega_0 (m^I)^{\omega_1 - 1} \right].$$

Substituting $\gamma^B = -\epsilon^B \sigma \eta = -\epsilon^B \sigma^2 (x^B - 1)$ and deposit market clearing $(x^B - 1) \eta = \phi m^I$ into the equation, we have a polynomial equation for $m^I$. After solving $m^I$, we can use the deposit market clearing condition to solve $x^B$, and check whether the borrowing constraint of households is violated (i.e. if $x^B \eta > q^T (1 - \phi)$). If not, we have $\kappa_T = 0$, and proceed to solve the convenience yield $\lambda \left[ q^I F^I (m^I) - 1 \right]$, the deposit rate $r = \rho - \lambda \left[ q^I F^I (m^I) - 1 \right]$, and the loan rate $R = \rho + \delta$.

If the constraint is violated, we move to the second case where $\kappa_T > 0$. When $\kappa_T > 0$, we know $x^B$ from $x^B \eta = q^T (1 - \phi)$. We can use the deposit market clearing condition to solve $m^I$, and then, we can proceed to solve the convenience yield $\lambda \left[ q^I F^I (m^I) - 1 \right]$ and the deposit rate $r = \rho - \lambda \left[ q^I F^I (m^I) - 1 \right]$. To solve the loan rate $R$, we need $\kappa_T$. By Equation (12),

$$\kappa_T = \lambda \left[ q^I F^I (m^I) - 1 \right] - \gamma^B \sigma,$$

where $\gamma^B = -\epsilon^B \sigma^2 (x^B - 1)$, is already known. The loan rate $R_t = \rho + \delta - \kappa_T$.

Next, given $x^B, R$, and $r$, we can calculate $\mu^e = r + x^B (R - \delta - r)$ and $\sigma^e = x^B \sigma$, and given $m^I, \mu^K = \lambda \omega_0 (m^I)^{\omega_1 - 1} \delta$. $\mu^\eta$ and $\sigma^\eta$ are defined in Equation (8):

$$\mu^\eta = \mu^e - \mu^K - \sigma^e \sigma + \sigma^2, \quad \text{and} \quad \sigma^\eta = (x^B - 1) \sigma.$$

The final step is to calculate $\left( \frac{d^2q^B}{d\eta^2} \frac{d^2q^I}{d\eta^2} \frac{d^2q^T}{d\eta^2} \right)$. $\mu^B, \mu^I$ and $\mu^T$ are calculated from banks’ HJB equation (Proposition 1'), Proposition 2, and Proposition 4:

$$\mu^B = \rho + \iota - r,$$
\[ \mu^I = \rho + \delta - \sigma^I \sigma - \frac{\alpha}{q^I} + (\rho - r) m^I - \lambda \left[ F (m^I) - \frac{m^I}{q^I} \right], \]

and

\[ \mu^T = \rho - \theta \kappa^T + \delta - \sigma^T \sigma - \frac{\alpha}{q^T}, \]

where the diffusion terms are \( \sigma^T = \epsilon^T \sigma^\eta \) and \( \sigma^I = \epsilon^I \sigma^\eta \). Using Itô’s lemma, we know

\[ \frac{d^2 q^X}{d\eta^2} = 2q^X \frac{(\mu^X - \epsilon^X \mu^\eta)}{(\sigma^\eta \eta)^2}, \quad X \in \{B,I,T\}. \]

Therefore, we have constructed a mapping from \((\eta, q^B, q^T, \frac{dq^B}{d\eta}, \frac{dq^T}{d\eta})\) to \((\frac{d^2 q^B}{d\eta^2}, \frac{d^2 q^T}{d\eta^2})\).

Note that given the boundary conditions explained in the main text, the system of differential equations uniquely pins down a solution \((q^B(\eta), q^I(\eta), q^T(\eta))\). Therefore, as long as the polynomial equation given by the deposit market clearing condition (i.e. Equation (12)) has a unique solution of \(m^I\) in \([0, \infty)\), the mapping from \((\eta, q^B, q^T, \frac{dq^B}{d\eta}, \frac{dq^T}{d\eta})\) to \((\frac{d^2 q^B}{d\eta^2}, \frac{d^2 q^T}{d\eta^2})\) is unique, so we have a unique Markov equilibrium with state variable \(\eta\). Q.E.D.

**Proof of Proposition 6.** Proof is provided in Appendix II.1.

**Proof of Proposition 7.** Proof is provided in Appendix II.2.

**Proof of Proposition 8.** Proof is provided in Appendix II.3.

**Proof of Proposition 9.** Proof is provided in Appendix II.4.

**Appendix II  Extensions**

In this section, I illustrate how the model accommodates a variety of extensions that may improve the model’s quantitative performances and open up new areas for future research.
II.1 Financial Innovation

When the financial sector is well capitalized, the expansion of its balance sheet and risk-taking is constrained by the available collateral that supports credit demand. One aspect of financial innovation can be understood as ways to circumvent this constraint, for example, collateral rehypothecation as noted by Gorton and Muir (2016). A common practice in the financial system (Singh and Aitken (2010)), rehypothecation is mostly used on financial collaterals. We can interpret the tangible capital as including synthetic financial products, which can be more easily re-pledged than physical capital. As shown by Bottazzi, Luque, and Pascoa (2012), rehypothecation is a way to relax the leverage constraint of households in the current setting.

Therefore, one way to capture financial innovation is to introduce a parameter $\theta$, the “collateral velocity”, into the borrowing constraint of households $l^T_t \leq \theta q_T^T k^T_t$, where $\theta > 1$. If $\theta$ is sufficiently large, the collateral margin that limits money creation is gone.

**Proposition 6 (Collateral Velocity and Money Creation)** When $\theta$, the collateral velocity, is sufficiently high, the only margin that limits inside money creation is the intermediary margin. The equilibrium money premium is: $\rho - r_t = \gamma^B_t \sigma$.

The question is whether financial innovation improves efficiency and stability. By relaxing the restriction on balance-sheet expansion, intermediaries can create more inside money in good times, making it easier for firms to hold cash for investment. This improves efficiency. On the other hand, banks’ leverage will be higher in boom, which makes the economy more fragile. Next, I solve the full model with $\theta = 3$ and compare it with the benchmark case.

Because in the benchmark case, collateral shortage only happens in a limited amount of time. Relaxing the borrowing constraint of households does not significantly change the behavior of the economy. But comparing the new model with the baseline case, we see that banks’ payout boundary is higher, as shown in Panel A of Figure 13. The stationary cumulative probability function shifts more weight towards states with a higher level of bank equity, indicating that this particular type of financial innovation motivates banks to retain more equity. Banks no longer need to forgo the $\kappa^T_t$ part of money premium to borrowers, so now they capture the full money premium and make more profits.

The key takeaway in Panel B of Figure 13 is that lifting the collateral constraint in the economy, for example by allowing rehypothecation, does not necessarily lead to higher bank leverage, or more risk-taking. When banks are able to lend more and capture more profit per dollar of money
created, banks optimally choose to stretch their activities over a wider range of equity values, so that on average, the level of their leverage supports a sufficiently high return on equity that justifies occasional costly recapitalization. Panel C of Figure 13 shows that by removing the collateral margin that limits money creation, the economy will see a lower money premium, particularly in the region where money creation is constrained by collateral shortage.

II.2 Regulatory constraint

Regulation on bank leverage, such as value-at-risk constraint (Adrian and Shin (2014)) or capital adequacy requirement, can be easily incorporated. Macro-prudential regulation may require banks to decrease their leverage in good times. As discussed in the main text, high leverage in boom is an undesirable feature. It amplifies the shocks’ impact on the economy, and because of the reflecting payout boundary, bad shocks are amplified disproportionately, which increases the probability of crisis. As a result, small bad shocks can trigger big crisis, while good shocks only trigger banks’ payout.

Suppose $x_t^B \leq \tilde{x}_t$, where $\tilde{x}_t$ could be a state-contingent (macro-prudential) restriction. We need to augment banks’ HJB equation (Proposition 1’) with $\kappa^R_t \left( \tilde{x}_t - x_t^B \right)$, where $\kappa^R_t$ is the Lagrange multiplier of the regulatory constraint (“R” for regulation). The new equilibrium condition
for bank leverage $x^B_t$ is:

$$R_t - r_t = \delta + \gamma^B_t \sigma + \kappa^R_t.$$

The following proposition reveals that regulation adds another wedge in money premium, which transforms into more severe investment inefficiencies. Therefore, regulators face the trade-off between growth and stability.

**Proposition 7 (Money Premium under Regulatory Constraint)** The equilibrium money premium is the sum of intermediary premium, collateral value, and the shadow value of regulatory constraint, $\kappa^R_t$:

$$\rho - r_t = \gamma^B_t \sigma + \kappa^T_t + \kappa^R_t. \quad (18)$$

Following the same procedure in the solution algorithm (Proof of Proposition 5), we can derive a new money supply curve:

$$\frac{\gamma^B_t \sigma^2}{\eta_t} M^S_t + \kappa^T_t + \kappa^R_t = \rho - r_t.$$

When $\kappa^R_t > 0$, the regulatory constraint binds. As shown in Figure 14, which is constructed in the same way as Figure 12 in the proof of Proposition 5, money supply is limited by the regulatory constraint and the availability of tangible collateral that supports credit demand. The figure illus-
trates the case where the regulatory constraint binds but the household borrowing constraint does not.

Next, I solve the full model with a collateral multiplier ($\theta = 3$), so the economy does not face a collateral shortage that limits the loan demand and banks’ expansion, and the focus is on regulatory constraint and banks’ balance-sheet capacity.

The regulatory constraint takes a simple form: banks asset-to-equity ratio $x_t^B \leq 22$. The number 22 is chosen to be slightly below the highest level of bank leverage in the case without any regulatory constraint. Therefore, we would expect little impact of the regulation on the model’s performance. Quite to the contrary, Figure 15 compares the model’s performance with and without the leverage constraint, and shows a striking difference.

The regulatory restriction on leverage does improve stability. Panel A of Figure 15 shows less curvature in the stationary C.D.F. curve when the regulation is introduced, which indicates a more evenly distributed probability mass. However, both banks’ payout and issuance boundaries decrease significantly. Regulation makes the banking sector less profitable. In response, banks now are only willing to hold a lower level of equity.

With a smaller banking sector, money supply retreats, which leads to higher level of money premium shown in Panel B. This indicates that the firms have to hold less cash, and more profitable investment opportunities are foregone as a result. Also shown is the regulatory shadow value $\kappa_t^R$ (dotted line). The restriction on bank leverage binds 80% of the time, and its shadow value accounts

Figure 15: Regulatory Constraint.
for a significant part of money premium.

A positive shadow value of regulatory constraint, $\kappa^R_t > 0$, indicates the marginal profit from money creation that is made possible by circumventing the regulation. As pointed out by Gorton (2010), Gorton and Metrick (2012), and Stein (2012), circumventing regulations on leverage to capture the money premium is one of driving forces behind the growth of shadow banking. Acharya, Schnabl, and Suarez (2013) document that commercial banks set up conduits to securitize assets while insuring the newly securitized assets using “liquidity” guarantees, which were structured to reduce bank capital requirements but provide recourse to bank balance sheets for investors. The shadow banking sector earns this regulatory shadow value. In a richer setting, the size of a shadow banking sector can be determined by equating this regulatory shadow value with the marginal cost of setting up shadow banking entity.

Panel C plots bank leverage against the stationary C.D.F., and Panel D plots leverage against the state variable. Introducing regulation does not necessarily lead to lower bank leverage. The regulation is set in a very conservative way, because the maximum leverage allowed is almost as high as the highest leverage in the Laissez-faire economy. But it still has huge impact on banks’ decision. Without the profit made in high leverage states, banks increase leverage over the cycle and retain less equity to sustain a sufficiently high return on equity that justifies the costs of occasional recapitalization.

II.3 Long-Term Bank Asset

In this section, banks can directly hold tangible capital, instead of extending short-term collateralized loans to households. Due to the money premium, banks have a lower marginal cost of financing than households (i.e. $r_t < \rho$), which gives banks a comparative advantage in holding tangible capital. When banks, the “natural buyer”, become richer, the price of tangible capital increases. Let $\epsilon_t^T$ denote the elasticity of $q_t^T$ with respect to $\eta_t$. We have $\epsilon_t^T = \frac{dq_t^T}{q_t^T} \cdot \frac{d\eta_t}{\eta_t} \geq 0$.

Investing in long-term assets expose banks to the endogenous variation of asset price. To create one dollar of deposits, banks add $(\sigma + \sigma_t^T)$ units of risk exposure. By Itô’s lemma, $\sigma_t^T = \epsilon_t^T \sigma_t^\eta \geq 0$. Note that in the baseline model, creating one dollar of deposits only adds $\sigma$ units of risk exposure. Therefore, when deposits are backed by long-term assets, the equilibrium money premium tends to increase in order to compensate banks for the extra endogenous risk.

The intermediated collateral premium $\kappa_T^T$ disappears, because banks’ deposit creation is no
longer dependent on households’ credit demand. This decreases the equilibrium money premium.

Proposition 8 (Money Premium and Long-Term Assets) The equilibrium money premium is:

$$\rho - r_t = \gamma_B^t \left( \sigma + \sigma_T^t \right).$$

(19)

Figure 16 shows that because \(q_T^t\) varies by little over the cycle, \(\sigma_T^t\) is negligible, and the model’s performance does not differ much from the baseline model, except that in the region where banks are extremely well capitalized, the money premium is lower (Panel C) because money creation is no longer constrained by insufficient credit demand. We still see banks’ leverage start to decrease, once they have accumulated a high level of equity. The decrease is more dramatic after banks own all of the tangible capital, because at this point, banks can only rely on the price appreciation of tangible capital to back more money creation (price effect) instead of acquiring more capital from households (reallocation effect).

To solve the model, note that the state variable’s instantaneous shock elasticity has changed:

$$\sigma_i^\eta = x_i^B \left( \sigma + \sigma_T^i \right) - \sigma,$$

because banks’ equity is exposed to the changes in tangible capital price. By Itô’s lemma, \(\sigma_i^T = \)
\( \epsilon_t^T \sigma_t^\eta \), so we can solve
\[
\sigma_t^\eta = \left( \frac{x_t^B - 1}{1 - x_t^B \epsilon_t^T} \right) \sigma,
\]
and the intermediary premium
\[
\gamma_t^B (\sigma + \sigma_t^T) = \epsilon_t^B \sigma_t^\eta (\sigma + \epsilon_t^T \sigma_t^\eta) = \epsilon_t^B \left( \frac{x_t^B - 1}{1 - x_t^B \epsilon_t^T} \right) \left[ 1 + \epsilon_t^T \left( \frac{x_t^B - 1}{1 - x_t^B \epsilon_t^T} \right) \right] \sigma^2,
\]
which is equal to the money premium, \( \rho - r_t \), and the equilibrium convenience yield of deposits
\[
\lambda \left[ q_t^l F' \left( \frac{(x_t^B - 1) \eta_t}{\phi} \right) - 1 \right],
\]
where I have substituted in the deposit market clearing condition \( \phi m_t^l = (x_t^B - 1) \eta_t \). Therefore, we have an equation that solves the equilibrium bank leverage \( x_t^B \) as a function of \( (\eta_t, \epsilon_t^B, q_t^l, \epsilon_t^T) \).

We can construct the differential equations as in the baseline case, and solve the model.

### II.4 The Credit View

In this section, the money view of banking is combined with the credit view (e.g. Bernanke (1983)). I model a particular form of banks’ advantage in supplying credit – the ability to restructure non-performing loans (Bolton and Freixas (2000)). I assume that on top of the capital destruction shock, a constant fraction \( (\delta - \delta) \) of tangible capital becomes obsolete per unit of time. Households cannot extract any value from such capital, so they default on loans and relinquish such capital to banks. As a result, households now are willing to borrow as long as \( R_t \leq \delta + \rho \).

**Lemma 2” (New Credit Demand)** *The equilibrium loan rate is given by: \( R_t = \delta + \rho - \kappa^T_t \).*

The defaulting loans backed by obsolete (instead of destroyed) capital, banks can recover the full value by reviving an equal amount of obsolete capital, which I interpret as a reduced-form representation of the outcome of banks’ restructuring effort. Therefore, banks’ first-order condition for leverage is the same as the baseline model. We still have
\[
R_t - r_t = \delta + \gamma_t^B \sigma.
\]
Substituting the equilibrium loan rate into this equation, we can see that banks’ restructuring ability leads to an reduction of the equilibrium money premium by \( (\delta - \delta) \).
Proposition 9 (Money Premium and the Credit Channel) The equilibrium money premium is

\[ \rho - r_t = \gamma^B \sigma + \kappa^T_t - (\bar{\delta} - \delta) \]  

(20)

Banks now have a new source of profit, the restructuring spread, \( \bar{\delta} - \delta \), because they can deal with the obsolescence of tangible capital better than households. As a result, the money supply curve is shifted downward by \( \bar{\delta} - \delta \) as shown in Figure 17, which is constructed in the same way as Figure 12 in the proof of Proposition 5. Now banks are willing to issue deposits at zero money premium (i.e. \( r_t = \rho \)), because even if the money premium is zero, banks still earn the restructuring spread that compensates for their risk exposure. Once \( r_t \) reaches \( \rho \), households become willing to hold deposits, which opens up a new source of funds for banks.

I solve the full model with the collateral multiplier \( \theta \) equal to 3, so we can ignore the collateral shortage (\( \kappa^T_t = 0 \)). Panel A of Figure 18 shows that banks expands its balance sheet through leverage, even passing the point of zero money premium (the vertical dotted line). Once the deposit rate reaches \( \rho \), households start to hold deposits, so banks’ leverage increase dramatically, with the highest level of leverage 60% higher than the highest level in the baseline model.

Solving the model with the credit channel requires an additional step. Same as before, we need a mapping from \( \left( \eta, q^B, q^T, q^I, \frac{dq^B}{d\eta}, \frac{dq^T}{d\eta}, \frac{dq^I}{d\eta} \right) \) to \( \left( \frac{d^2q^B}{d\eta^2}, \frac{d^2q^T}{d\eta^2}, \frac{d^2q^I}{d\eta^2} \right) \). First, we need to check whether banks supply sufficient inside money that supports the first-best intangible investment,
or whether \( r \) is equal to \( \rho \). As shown in Figure 17, this happens when \( \frac{\epsilon_B \sigma^2}{\eta_t} M_t^* - (\bar{\delta} - \delta) \leq 0 \), where \( M_t^* = \phi i_t^* \). If this condition holds, \( r = \rho \), \( i = i_t^* \), and given \( \kappa_t^T = 0 \), we can obtain the equilibrium deposit quantity \( M_t = \frac{(\bar{\delta} - \delta) \eta_t}{\epsilon_B \sigma^2} \) by plugging \( r_t = \rho \) into the money supply curve. If the condition does not hold, we solve the equilibrium amount of deposits the same way as in the baseline model. Once we solve the equilibrium deposit quantity, the rest follows as in the baseline case. I set \( \bar{\delta} = 0.0402 \), which is 0.5% higher than the value of \( \delta \) (0.04). \(^{55}\)

### II.5 Contingent Government Debt Supply

In this section, the government debt supply is allowed to be contingent on the state of the world (i.e. \( \eta_t \)). In particular, it follows a linear rule: the government issues \( (M^G_0 + M^G_1 \eta_t) (K^I_t + K^T_t) \) amount of debt, so that the debt-to-output ratio equal to \( \frac{M^G_0 + M^G_1 \eta_t}{\alpha} \). Figure 19 illustrates how this contingent arrangement affects the economy. \( M^G_0 \) and \( M^G_1 \) set to 0 and 8 respectively so that the mean government debt-to-output ratio, evaluated at the stationary c.d.f., is 50%.

The government debt supply is procyclical \( (M^G_1 > 0) \) to avoid crowding out banks’ profits and equity in the states where banks are already undercapitalized. However, reducing government money supply in these states deprives firms with scarce cash instruments, because this is precisely

\(^{55}\)For simplicity, I assume the economy is endowed with a constant measure \( (\bar{\delta} - \delta) \) of tangible capital per unit of time, which is distributed among households, so that the growth rate of tangible and intangible capital stay the same.
when banks have already scaled down their deposit creation.

Panel A of Figure 19 shows that under procyclical government debt, banks’ leverage becomes less procyclical. In bad states where firms’ money demands are weak, banks deleverage. However, due to the retreat of government debt supply, firms’ demands for bank deposits decline less than in the baseline case with constant government debt supply. In good states where firms’ money demands are strong, banks want to increase leverage, but as the government expands its debt issuance, firms’ demands for bank deposits expand less than in the baseline case.

As shown by Panel C, the reduced procyclicality of leverage makes the stationary probability distribution less concentrated in the bad states with low \( \eta_t \). Moreover, the issuance boundary shifts up (gray to black) relative to the constant government debt case, reflecting a weaker equity crowding-out effect in bad states. The payout boundary decrease (gray to black), because the government distributes more of its debt capacity in good states, intensifying the competition that banks face in the money market.

Panel B plots the money premium against the stationary c.d.f. Both scenarios have very similar money premium except in the relatively bad states when banks are undercapitalized. Under procyclical government debt, when banks deleverage, the government also decreases its debt supply, so the money premium is higher in the relatively bad states. At the bank issuance boundary, the money premium is 10 basis points higher under procyclical government debt.

Panel D plots the expected time to reach the recovery point (i.e. the lowest value of \( \eta_t \) that
brings positive growth). The recovery points are similar under both scenarios. Under procyclical
government debt, because the equity crowding-out effect is weaker in the bad states, the bank
issuance boundary is higher, which limits the impact of negative shock, and because the money
premium is higher, which translates into a higher expected return on bank equity, the rebuilding of
bank equity is faster, which is reflected in a lower slope of the recovery path. The expected time of
recovery declines from 18 years to 9 years when the government moves from a constant debt level
to procyclical debt issuance.

Comparing Panel B and Panel D reveals the trade-off that the government faces. By setting a
procyclical debt issuance policy, the government shortens the duration of recession, but increases
the severity of it. In the states where banks are extremely undercapitalized, the government reduces
its debt, allowing banks to speed up recovery by earning a large money premium, but at the same
time, a large money premium leads to less cash holdings of the firms, and less investments.

Future research may consider an optimal timing problem of government debt supply. Given
that the its average debt-to-output ratio cannot exceed a certain level, the government can choose
how to optimally distribute its debt capacity over the cycle. In each state of the world, the govern-
ment trades off the benefit from relaxing firms’ liquidity constraint and the cost from crowding out
banks, and also, it takes into consideration how the overall dynamics of the cycle reacts through
the responses in banks’ leverage, issuance, and payout decisions.

Appendix III Preliminary Evidence

III.1 The Structure of Inside Money

A key message from the model is that to understand the dynamics of intermediary leverage, we
need to pay attention to money demand. Leverage cycle crucially depends on the extent to which
money demand is time-varying and sensitive to changes in the money premium. One novel feature
of the model is the intertemporal complementarity of firms’ money holdings, which arises because
firms hold liquidity for investment. But, in reality, is corporate money demand large enough to
influence banks’ choice of leverage?

Figure 20 shows that nonfinancial corporations not only depend on financial intermediaries’
supply of money-like assets, but are also among the largest owners of intermediaries debt. Money-
like assets include a variety of financial instruments. Accordingly, “deposits” in the model should
be interpreted broadly as including short-term debt issued by financial intermediaries that either serves as money, such as deposits at commercial banks, or can be easily converted to money, such as repurchase agreements signed with broker-dealers and high-quality asset-backed commercial papers issued by structured investment vehicles. Corporate treasuries hold a variety of such securities as “cash holdings”, particularly through money market funds.

The left panel of Figure 20 decomposes the liquidity holdings of U.S. non-financial corporations by the types of money-like securities, and for each security, by the types of its issuers. I use the data as of December 2015 in the March 10, 2016 release of the Financial Accounts of the United States (previously known as the “Flow of Funds”). From this graph, we can get a sense that who are supplying money to the U.S. nonfinancial corporations and in what forms.

Foreign deposits and time deposits are all issued by depository institutions, and Treasury securities by the government. 19% of commercial papers are issued by the nonfinancial corporations themselves, 34% by domestic financial intermediaries, and 47% by foreign entities, of which 90% are issued by foreign financial firms (defined in the Financial Accounts). 72% of repurchase agreements are issued by the financial sector, and 27% by the foreign entities. Checkable deposits and currency are reported together in the Financial Accounts, of which 42% issued by the government are “currency outside banks”, and the remaining 58% are the liabilities of depository institutions. Given that firms usually do not hold currency directly, 58% underestimates the contribution of financial intermediaries.
The right panel of Figure 20 decomposes the outstanding money-like securities issued by financial intermediaries by different types of owners. Nonfinancial corporations only hold commercial papers through money market funds and mutual funds, which in sum account for a small fraction of total outstanding commercial papers issued by financial intermediaries. Nonfinancial corporations hold a little less than a third of repurchase agreements, most of which are also held indirectly through money market funds and mutual funds. They also account for a significant share of other money-like liabilities issued by depository institutions, such as checkable deposits and large time deposits.

Here are some details on how to construct this graph from the data in Financial Accounts. Table L.103 records nonfinancial corporations’ assets and liabilities. The liquidity holdings include money market mutual fund shares and mutual fund shares, which I break down into financial instruments using Table L.121 and L.122 respectively under the assumption that the funds held by nonfinancial firms invest in the same portfolio as the aggregate sector does.\footnote{Corporate and foreign bonds, loans, and miscellaneous assets, all held indirectly through money market mutual funds and mutual funds, are excluded. Agency- and GSE-backed securities are excluded because of the potential spikes of repo haircuts and the secondary market illiquidity during crisis times. Municipal securities are excluded because of secondary market illiquidity.} Corporate and foreign bonds, loans, and miscellaneous assets, all held indirectly through money market mutual funds and mutual funds, are excluded. Agency- and GSE-backed securities are excluded because of the potential spikes of repo haircuts and the secondary market illiquidity during crisis times. Municipal securities are excluded because of secondary market illiquidity.

Next, for each financial instrument, I calculate the net supply by each type of issuers using the instrument-level tables. Financial intermediaries are defined as in Krishnamurthy and Vissing-Jorgensen (2015). I only include the domestic financial institutions: U.S.-chartered depository institutions, foreign banking offices in U.S., banks in U.S. affiliated areas, credit unions, issuers of asset-backed securities, finance companies, mortgage real estate investment trusts, security brokers and dealers, holding companies, and funding corporations. Insurance companies are not included as financial intermediaries because their liabilities are long-term and usually held until maturity by insurance policy holders instead of resold in secondary market. Their liabilities are not money-like.

### III.2 Negative Correlation Between Money Premium and Corporate Cash

With a few exceptions, theories of corporate liquidity holdings abstract away money supply by assuming a storage technology (e.g., Froot, Scharfstein, and Stein (1993); Bolton, Chen, and Wang (2011)). Holmström and Tirole (1998) explicitly point out the possibility that the private sector is implicitly assumed that funds are just pass-through and do not provide any liquidity services beyond the securities they hold. This ignores the potential sharing of idiosyncratic liquidity shocks through funds.\footnote{It is implicitly assumed that funds are just pass-through and do not provide any liquidity services beyond the securities they hold. This ignores the potential sharing of idiosyncratic liquidity shocks through funds.}
not able to supply enough assets to satisfy the saving needs of its own under limited pledgeability of future income. This paper adds an intermediary margin into the liquidity supply.

Banks affect firms’ money holdings through the equilibrium money premium. In the model, firms’ money demand function echoes Baumol (1952) and Tobin (1956), with the interest rate replaced by the money premium as the opportunity cost. As the money supply varies with intermediation capital, a negative relation emerges between the equilibrium money premium and firms’ money holdings. Figure 21 confirms this negative correlation in data.

Using Compustat data from 1971 to 2014, Panel A of Figure 21 plots the cross-section average of firms’ cash-to-asset ratio each year against the average money premium in that year, which is measured by the spread between three-month certificate of deposit (CD) rate and three-month Treasury bill (T-bill) rate following Nagel (2016). Panel B divides the firm-year observations by

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57 A dominant fraction of corporate liquidity holdings are now interest-bearing. Thus, the opportunity cost is no longer interest rate, but the money premium. Regulation Q prohibited interest-bearing demand deposits and imposed interest rate ceilings on other types of deposit accounts. These restrictions were gradually lifted by mid 1980s: NOW accounts were legalized, interest rate ceilings were lifted, and money market funds emerged. Azar, Kagy, and Schmalz (2016) document how the composition of corporate liquidity evolves towards interest-bearing assets. In a broader context, Lucas and Nicolini (2015) discuss how replacing interest rate with money premium and adding money market accounts into money aggregate restore the negative relationship between the cost of holding money and the quantity of money that has been claimed to be “broken” in the macroeconomics literature since 1980s.

58 In Nagel (2016), the preferred measure of money premium is the spread between general collateral repo rate (GC) and Treasury bill rate, both at three-month horizon. However, repo rates are available only back to 1991. Using the GC/T-bill spread, we will lose a lot of firm-year variation in the panel data. Nagel (2016) argues that outside the crisis periods (financial crisis 2007-2009; savings and loans crisis in the 1980s), this credit risk component is small, and shows that in the periods since 1990s, when both CD rates and GC repo rates are available, there is only a small
the ratio of collateral (plant, property, and equipment, i.e. “PP&E”) to research and development expenses (“R&D”), or the “collateral coverage” (“C.C.”).\(^{59}\) It is important to note that while Figure 21 confirms the negative equilibrium correlation, it does not identify the corporate money demand curve. The money premium is an endogenous variable, and in the model, the process of money premium is an equilibrium outcome from the dynamic interaction between money demand and supply.

Shown in Lemma 1\(^{'}\), the model has a money demand that is downward sloping in the money premium, only after controlling for \(q_t^I\), the endogenous valuation of intangible capital. Therefore, the negative relation between money premium and firms’ cash holdings is far from assumed. In the calibration, \(\omega_1 = 0.99\), which makes sure that the concavity of investment technology is small and thus, what matters is not the curvature of money demand curve at each point in time, but how the money demand curve moves with \(q_t^I\). The relation crucially depends on the dynamics of \(q_t^I\), which in turn is an equilibrium object and varies with the balance-sheet condition of the banking sector.

If \(q_t^I\) were countercyclical and decreasing in \(\eta_t\), the model could have predicted a positive correlation between firms’ money holdings and money premium. Figure 21 confirms the equilibrium the negative relationship between firms’ cash and the money premium that arises because \(q_t^I\) is procyclical. Panel B reveals that this negative relationship is most prominent for firms with high R&D investment and less collateral.\(^{60}\) This paper provides a general equilibrium perspective on the business-cycle variation of corporate cash holdings. It is able to make such predictions precisely by departing from the existing literature that assumes a storage technology and by incorporating the endogenous money supply by financial intermediaries.

The regression results in Table 1 confirm the negative correlation between corporate cash holdings and the money premium in Figure 21. Regression allows us to examine the correlation on a conditional basis, controlling for the known firm characteristics that affect firms’ cash holdings. Panel data regression also allows us to control for time fixed effect, which could potentially summarize the unobserved macroeconomic variables that confounds the relationship between corporate cash holdings and the money premium.\(^{61}\) Following Petersen (2009), I use two-way cluster

\(^{59}\)R&D is arguably the most cleanly measured among all categories of intangible investment. For example, the calculation of investment in organization capital relies on many assumptions. It is commonly measured by 30% of SG&A (selling, general, and administrative expenses) (e.g. Peters and Taylor (2016)).

\(^{60}\)These firms hold more cash in line with the findings that collateral is an important determinant of cash holdings (e.g. Almeida and Campello (2007)) and so is R&D (e.g. Falato and Sim (2014); Pinkowitz, Stulz, and Williamson (2016); Begenau and Palazzo (2015)).

\(^{61}\)There is still the concern that individual firms’ money demand may have heterogeneous loadings on the some
Here are some details of the regression. The sample is from 1971 to 2014. In 1971, “The Reserve Fund” was established as the first money market fund in the United States. Money market funds created a way of getting around Regulation Q, which then prohibited demand deposit accounts from paying interest and capped the rate of interest on other types of bank accounts. The money market funds essentially introduced interest-bearing money. The opportunity cost for holding money used to be inflation and interest rate, but since then, became the money premium (Lucas and Nicolini (2015)).

FRED and Compustat are the two main data sources. CD rates and T-bill rates are from FRED. I use Compustat/CRSP merged annual database to calculate firm-level cash-to-assets ratio, which is the left-hand-side variable of the panel data regression, collateral coverage ratio, which is a key right-hand-side variable, and firm-level control variables that are included in the analysis of cash holdings in the corporate finance literature. Cash-to-assets ratios is the ratio of cash and cash unobserved macro factors. One strategy is to directly model the error by a latent factor structure with heterogeneous factor loadings of individual firms. I did not pursue this strategy because its validity highly depends on the assumed factor structure of $\varepsilon_{i,t}$.

<table>
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<tr>
<th></th>
<th>(1) Cash</th>
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<th>(4) Cash</th>
<th>(5) Cash</th>
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<td>Money premium</td>
<td>-0.0979*** (0.0128)</td>
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<td>C.C. * M.P.</td>
<td>0.0970*** (0.0121)</td>
<td>0.0944*** (0.0120)</td>
<td>0.0204*** (0.00294)</td>
<td>0.0829*** (0.0106)</td>
<td>0.0413*** (0.00532)</td>
<td>0.0104*** (0.00295)</td>
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<td>Collateral Coverage (PPE / R&amp;D)</td>
<td>-0.152*** (0.0112)</td>
<td>-0.147*** (0.0113)</td>
<td>-0.0286*** (0.00208)</td>
<td>-0.128*** (0.0102)</td>
<td>-0.0689*** (0.00469)</td>
<td>-0.0157*** (0.00452)</td>
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<td>0.0390*** (0.00142)</td>
<td>0.00602*** (0.000835)</td>
<td>0.0335*** (0.00138)</td>
<td>0.0203*** (0.00123)</td>
<td>0.0155*** (0.000806)</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
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<td>Firm FE</td>
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Observations: 163738 163738 147920 163738 122145 163738
Adjusted $R^2$: 0.234 0.249 0.747 0.300 0.481 0.696

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 1: Money Premium and Corporate Cash Holdings
equivalents (Compustat-CHE) to total assets (Compustat-AT), winsorized at 1% level at two tails. Collateral coverage ratio is the ratio of PP&E (Compustat-PPENT) to R&D expenses (Compustat-XRD). Following Frank and Goyal (2009), I use PP&E as a measure of the tangibility of firms’ assets, and interpret it as the available collateral on balance sheet.

Following Bates, Kahle, and Stulz (2009), the firm-level control variables are defined as follows. Firm size is the natural logarithm of total assets. Market-to-book ratio of assets is the ratio of asset market value to total assets. Asset market value is the sum of equity market capitalization, which is the product of shares outstanding (Compustat-CSHO) and close price, and book value of debt, which is the difference between total assets and book value of equity (Compustat-CEQ). Cash flow is measured by operating income before depreciation (Compustat-OIBDP) minus interest expenses (Compustat-XINT), dividend payment (Compustat-DVC), and income taxes (Compustat-TXT), and then divided by total assets. Industry cash-flow volatility is calculated as the two-digit SIC industry average of firms’ cash-flow standard deviation in the past ten years. Net working capital is the difference between working capital (Compustat-WCAP) and cash and cash equivalents, divided by total assets. Capex is Compustat-CAPX divided by total assets. Leverage is the sum of long-term debt (Compustat-DLTT) and debt in current liabilities (Compustat-DLC), dividend by total assets. Acquisition activities are measured by Compustat-AQC divided by total assets. Dividend dummy is equal to one if the firm pays dividend (Compustat-DVC > 0), and zero otherwise. I follow the steps in Bates, Kahle, and Stulz (2009) to winsorize these variables.

### III.3 Procyclical Leverage and Government Debt

This section provides some preliminary evidence on how the changes in the government debt affect the cyclicity of financial intermediaries’ leverage. In particular, I am going to focus on one type of financial intermediaries, broker-dealers. Adrian and Shin (2010) document the procyclicality of broker-dealers’ leverage: when their assets grow, their debt grows faster, leading to higher leverage. Broker-dealers, or commonly known as the investment banks, issue money-like securities (mostly repurchase agreements) that are largely held by money market mutual funds, which are in turn held by firms and other entities as cash substitutes.

I focus on investment banks because relative to commercial banks, their choice of leverage is subject to less regulatory constraints, and thereby, they are closer to the laissez-faire banks in the model. Leverage is defined as the ratio of total book assets to book equity of the broker-dealer.
Table 2: Government Debt and Procyclical Leverage.

sector in the U.S. Financial Accounts (formerly known as the “Flow of Funds”) following Adrian and Shin (2010) and He, Kelly, and Manela (2016). To measure government debt, I use the ratio Treasury bills to GDP (source: U.S. Financial Accounts), because compared with long-maturity bonds, Treasury bills are more money-like in the sense that their value is relative more stable, the secondary market is more liquid, and the haircut in repo transactions is smaller. The sample is from 1968Q3 to 2015Q3. Adrian, Etula, and Muir (2014) discuss the data quality concerns of the Flow of Funds prior to 1968. Table 2 reports the results.

The first column of Table 2 replicates the main regression in Adrian and Shin (2010). The left-hand side variable is the log-difference of leverage and the right-hand side variable is the log-difference of assets. A positive and significant coefficient indicates the procyclicality of leverage. All the regressions are estimated by General Method of Moments with the Newey-West Heteroskedasticity and Autocorrelation Consistent (HAC) standard errors. The optimal number of lags is chosen by the method proposed by Newey and West (1994).

The fourth column of Table 2 shows the coefficients from regressing leverage change on both asset change and the interaction between asset change and change in the government debt. A positive coefficient on the interaction term suggests that when government debt increases, leverage tends to be more procyclical, i.e. intermediaries’ leverage increases more in response to trading gains or other sources of income that raises their asset value. This is consistent with the model’s prediction shown in Figure 10.

A clear identification of the impact of government debt on leverage cyclicality is very chal-

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<tbody>
<tr>
<td>IV for $\Delta \ln(Leverage)$</td>
<td>MP-FFR</td>
<td>MP-Comp</td>
<td>MP-FFR</td>
<td>MP-Comp</td>
<td></td>
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<tr>
<td>$\Delta \ln(Assets)$</td>
<td>0.910***</td>
<td>3.873***</td>
<td>2.527***</td>
<td>0.203</td>
<td>0.991**</td>
<td>0.867**</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>(0.113)</td>
<td>(1.451)</td>
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<tr>
<td>IV for $\Delta \ln(\frac{TBill}{GDP})$</td>
<td>Q2</td>
<td>Q2</td>
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<tr>
<td>$\Delta \ln(\frac{TBill}{GDP}) \cdot \Delta \ln(Assets)$</td>
<td>24.40</td>
<td>10.28***</td>
<td>8.999***</td>
<td></td>
<td></td>
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<td></td>
<td>(38.57)</td>
<td>(2.174)</td>
</tr>
<tr>
<td>Observations</td>
<td>189</td>
<td>77</td>
<td>77</td>
<td>188</td>
<td>77</td>
<td>77</td>
</tr>
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</table>
lenging, given that both the changes in the level of government debt and the changes of broker-dealers’ asset value are highly endogenous. Next, I take an instrument variable approach to address the endogeneity. The choice of instruments and the methodology here serve as merely a first attempt that calls for more careful investigation in the future.

The instrument variable for the changes in the Treasury bill is the dummy variable for the second quarter. Greenwood, Hanson, and Stein (2015) show that the Treasury expands the supply of bills ahead of statutory tax deadlines (e.g., April 15) to meet its ongoing needs, and repay following the deadlines. Greenwood, Hanson, and Stein (2015) and Nagel (2016) use week and month dummies respectively to instrument Treasury supply variation. The strongest seasonality of Treasury bill supply shows up in the second quarter of the year, so I use the second-quarter dummy as an instrument for the changes in the Treasury bill-to-GDP ratio.

The instrument variable for the changes in broker-dealers’ asset value is monetary shock. I consider two measures: the unanticipated changes in the Fed Funds Rate around the FOMC (Federal Open Market Committee) announcement windows (“MP-FFR”), and a composite measure of unanticipated changes in a variety of interest rates around the FOMC announcement windows proposed by Nakamura and Steinsson (2016) (“MP-Comp”). A monetary policy shock is calculated using a 30-minute window from 10 minutes before the FOMC announcement to 20 minutes after it. The calculation follows Nakamura and Steinsson (2016).\(^\text{62}\) Shocks are aggregated to the quarterly level. Calculated from high-frequency data, these monetary policy shocks arguably reflect the changes in the interest rates that only come from the unexpected content of Federal Reserve’s FOMC announcements, and thus, tend to be orthogonal to contemporaneous variations in other economic variables. Through its impact on the interest rates and various spreads, monetary policy shocks affect the value of broker-dealers’ assets.\(^\text{63}\)

Column 2 and 3 show the procyclicality of leverage with different measures of monetary policy shocks as instruments for the changes in broker-dealers’ assets. Both estimates are positive and significant, consistent with the baseline specification without using instrument variables.

Column 5 and 6 use the second-quarter dummy as an instrument variable for the Treasury bill supply, and use two measures of monetary policy shocks respectively to instrument the changes in broker-dealers’ assets. The coefficient on the interaction term is positive and highly significant.


\(^{63}\)In the recent literature, Gertler and Karadi (2015) show that these high-frequency monetary policy shocks affect term premia and credit spreads, and Hanson and Stein (2015) show the strong effect on forward real rates even in the distant future.
This confirms the finding in the baseline specification that the procyclicality of broker-dealers’ leverage is amplified by increases in the Treasury bill supply.

While the findings in Table 2 are consistent with the model’s prediction on the leverage cyclicality and the impact of government debt on it, the evidence is far from conclusive. The rich set of the model’s empirical implications, including the responses to government debt supply in financial intermediaries’ issuance and payout decisions, will be better evaluated in future research based on longer sample, international data, and alternative identification strategies.
References


