On the Asset Allocation of a Default Pension Fund

Magnus Dahlquist   Ofer Setty   Roine Vestman[*]

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Abstract

We characterize the optimal default fund in a defined contribution (DC) pension plan. Using detailed data on individuals and their holdings inside and outside the pension system, we find substantial heterogeneity among default investors in terms of labor income, financial wealth, and stock market participation. We build a life-cycle consumption–savings model incorporating a DC pension account and realistic investor heterogeneity. We examine the optimal asset allocation for different realized equity returns and investors and compare it with age-based investing. The optimal asset allocation leads to less inequality in pensions while it moderates the risks through active rebalancing.

JEL classification: D91, E21, G11, H55.
Keywords: Age-based investing, default fund, life-cycle model, pension plan design.

1 Introduction

The worldwide shift from defined benefit (DB) to defined contribution (DC) pension plans challenges pension investors, who have been given greater responsibility to choose their contribution rates and manage their savings. Many investors seem uninterested, display inertia (Madrian and Shea, 2001), or lack financial literacy (Lusardi and Mitchell, 2014), and end up in the default option. Consequently, the design of the default option in a pension plan is a powerful tool for improving investment outcomes.[1]

This paper studies one important aspect of the design of the default option: the optimal asset allocation. Our contribution is both empirical and theoretical. We begin by constructing a dataset of Swedish investors' detailed asset holdings inside and outside the pension system.[2] We find that remaining in the default fund is a strong predictor of not having any equity exposure outside the pension system. Default investors have a 37% lower stock market participation rate outside the pension system than do active (non-default) investors, one third of the difference being unexplained by observable characteristics. Moreover, there is a great deal of heterogeneity among default investors. Default investors who participate in the stock market have financial wealth equal to 1.4 years of labor income, while default investors who do not participate have financial wealth equal to only five months of labor income. Similarly, participating default investors have 4.4 times as much financial wealth as do non-participating default investors. These basic facts make it reasonable to question the

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[1]Studies have examined the design of the enrollment features (Carrol et al., 2009), contribution rates (Madrian and Shea, 2001; Choi et al., 2003), choice menus (Cronqvist and Thaler, 2004), and equity exposures within pension plans (Benartzi and Thaler, 2001; Huberman and Jiang, 2006). Benartzi and Thaler (2007) have reviewed heuristics and biases in retirement savings behavior. More recently, Chetty et al. (2014) document inertia among 85% of Danish pension investors with respect to their contribution rates, Poterba (2014) discusses the savings rates required in order to obtain warranted replacement rates, and Sialm et al. (2015) argue that sponsors of DC plans adjust the options of the plan to overcome investor inertia.

[2]Calvet et al. (2007, 2009) have made use of the data on asset holdings outside the pension system. To the best of our knowledge, we are the first to combine these register-based data with information about savings inside the pension system. Bergstresser and Poterba (2004) and Christelis et al. (2011) use survey data when studying equity exposure and the location choice between taxable and tax-deferred accounts.

ability of a one-size-fits-all design of the default fund to meet all default investors' needs.

We then set up a model to examine the optimal asset allocation of the default fund in a DC pension plan. Our model belongs to the class of life-cycle portfolio choice models with risky labor income (see, e.g., Cocco et al., 2005; Gomes and Michaelides, 2005), which we extend to include a pension system as well. The pension system has a DC pension account so that liquid savings outside the pension system coexist with illiquid savings inside the pension system. Our rich model generates cross-sectional heterogeneity in wealth and labor income, as well as both stock market participants and non-participants outside the pension system. We can therefore study the optimal asset allocation for default investors with different individual characteristics and for investors who have realized different stock market returns. We find substantial heterogeneity in the optimal allocation to equity in the DC pension account. The year before retirement, the highest decile has an optimal equity share above 45%, while the lowest decile has an optimal equity share below 13%. We also find that the optimal equity share varies substantially with the stock market's past performance. The year before retirement, the optimal equity share for the average investor is above 44% with a 10% probability and below 21% with a 10% probability. The reason is that the optimal asset allocation involves active rebalancing.

We finally evaluate extensions to pure age-based investing rules. A common piece of investment advice, or rule of thumb, is to allocate a percentage share equal to 100 minus one's age in equity and the remainder in bonds. We demonstrate that accounting for past stock market returns and investor characteristics other than age has positive implications. By tailoring the asset allocation to investors' individual characteristics, it is possible to reduce inequality in pensions. We find that a shift from the "100-minus-age" rule to the optimal individually tailored allocation reduces pension inequality by 21%, primarily by raising the replacement rates of low-income investors. That is, the optimal asset allocation reduces the inequality and at the same time moderates the risks. The optimal asset allocation implies less

inequality in pensions even when compared with other age-based rules (with either higher or lower equity exposures) and holds qualitatively even if the equity risk premium is low.

Our work relates to that of Gomes et al. (2009) and Campanale et al. (2014). Gomes et al. (2009) study the effects of tax-deferred retirement accounts and find the largest effects on savings rates relative to a non-tax environment for investors with high savings rates. In contrast, our proposed allocation rule affects low-income investors the most. Campanale et al. (2014) investigate how stock market illiquidity affects a portfolio choice model's ability to replicate the distribution of stock holdings over the life cycle and the wealth distribution.

Our work also relates to that of Lucas and Zeldes (2009), who deal with the investment decisions of pension plans in the aggregate. However, our model is more quantitative, and we can consider individual outcomes beyond aggregate ones at the pension plan level. In this sense, Shiller's (2006) evaluation of the life-cycle personal accounts for Social Security is closer to our study. Our focus on investor heterogeneity is complementary to the work of Poterba et al. (2007), who simulate individuals' pension benefits in DB and DC plans and report distributions across individuals.

A designer of the default option in a DC pension plan (e.g., a plan sponsor such as an employer or the government) may find our results interesting. Paying attention to asset allocation will arguably increase in importance over time as DC plans become a common component of pension systems. As plan enrollment becomes widespread, the pool of investors in such plans will undergo a compositional shift in terms of education, labor income, and, not least, financial wealth outside the pension system. For default investors with little financial wealth outside the DC plan, the asset allocation of the default option is particularly important as it determines their overall investment portfolio. To illustrate how a default option designer could implement our insights, we propose a simple investment rule. At a conceptual level, the rule diverges in two ways from standard age-based investing or inter-temporal hedging (Merton, 1971). First, the DC account balance in itself is a useful

instrument guiding the asset allocation decision. If the account balance is low due to poor past equity returns, more equity risk can be assumed, while the reverse is the case if the account balance is high due to good past equity returns. We find this result particularly useful because the account balance itself is readily available information, making the rule cost effective to implement (see Bodie et al., 2009, for a discussion of the costs of individualized allocations). Second, we find that the stock market participation status outside the pension system provides considerable information about the investor. On average, non-participants should have a 20-percentage-point higher equity share relative to that of participants.

The paper proceeds as follows. Section 2 provides an overview of the Swedish pension system. Section 3 describes our data. Section 4 empirically analyzes individuals' portfolio choices inside and outside the pension system and how they are related. Section 5 presents our life-cycle model and its calibration. Section 6 analyzes the optimal design of a default pension fund. Section 7 presents the implications of alternative designs of the default fund. Finally, Section 8 concludes.

2   The Swedish pension system

The Swedish pension system rests on three pillars: public pensions, occupational pensions, and private savings. Below, we describe the public and occupational pensions.

The public pension system was reformed in 2000.[3] It has two major components referred to as the income-based pension and the premium pension. A means-tested benefit provides a minimum guaranteed pension.

The contribution to the income-based pension is 16% of an individual's income, though the income is capped (in 2014 the cap was SEK 426,750, or approximately USD 62,200).[4]

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[3]Individuals born between 1938 and 1954 are enrolled in a mix of the old and new pension systems, while individuals born after 1954 are enrolled entirely in the new system.

[4]In 2014 the SEK/USD exchange was around seven. During our sample period, the exchange rate has fluctuated between six and ten SEK per USD. We henceforth report numbers in SEK.

The return on the contribution equals the growth rate of aggregate labor income measured by an official "income index." Effectively, the return on the income-based pension is similar to that of a real bond. The income-based pension is notional in that it is not reserved for the individual but is instead used to fund current pension payments as in a traditional pay-as-you-go system. It is worth mentioning that the notional income-based pension is also DC, but to avoid confusion we simply refer to it as the notional pension.

The contribution to the premium pension is 2.5% of an individual's income (capped as above). Unlike the income-based pension, the premium pension is a fully funded DC account used to finance the individual's future pension. Individuals can choose to allocate their contributions to up to five mutual funds from a menu of several hundred. The premium pension makes it possible for individuals to gain equity exposure. Indeed, most of the investments in the system have been in equity funds (see, e.g., Dahlquist et al., 2015). A government agency manages a default fund for individuals who do not make an investment choice. Up to 2010, the default fund invested mainly in stocks but also in bonds and alternatives. In 2010, the default fund became a life-cycle fund. At the time of retirement, the savings in the income-based pension and the premium pension are transformed into actuarially fair life-long annuities.

In addition to public pensions, approximately 90% of the Swedish workforce is entitled to occupational pensions. Agreements between labor unions and employer organizations are broad and inclusive and have gradually been harmonized across educational and occupational groups. For individuals born after 1980, the rules are fairly homogenous, regardless of education and occupation. The contribution is 4.5% of an individual's income (capped as above) and it goes into a designated individual DC account. For the part of the income that exceeds the cap, the contribution rate is greater in order to achieve a high replacement rate even for high-income individuals. While the occupational pension is somewhat more complex and tailored to specific needs, it shares many features with the premium pension.

Specifically, it is an individual DC account.

Next, we discuss our data on individuals' savings inside and outside the pension system.

3   Data

We tailor a registry-based dataset to our specific needs. This dataset's foundation is a representative panel dataset for Sweden, the LINDA (Longitudinal Individual Data) dataset. LINDA covers more than 300,000 households and is compiled by Statistics Sweden. We use eight waves between 2000 and 2007 and consider socioeconomic information such as age, education, and labor income. Our sample period is determined by the launch of the new pension system in 2000 and by the availability of detailed financial wealth data (described below) up to 2007. The Online Appendix contains further information on LINDA. We match LINDA with data from two additional sources.

The first data source is the Swedish Tax Agency (through Statistics Sweden), whose data cover each individual's non-pension financial wealth. It is a registry-based source of financial holdings outside the public pension system. Specifically, the tax reporting allows us to compute the value of the holdings of all bonds, stocks, and mutual funds that an individual holds at the end of each year. There are two exceptions to these detailed tax reports. The first exception is the holdings of financial assets within private pension accounts, for which we observe only additions and withdrawals; the second exception is the so-called capital insurance accounts, for which we observe the account balances but not the detailed holdings.[5] There is also a tax on real estate, which allows us to accurately measure the value of owner-occupied single-family houses and second homes. Apartment values are also available, though they are less accurately measured.

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[5]Capital insurance accounts are savings vehicles that are not subject to the regular capital gains and dividend income taxes, but are instead taxed at a flat rate on the account balance. According to Calvet et al. (2007), these accounts accounted for 16% of aggregate financial wealth in 2002.

The second data source is the Swedish Pensions Agency, whose data cover pension savings. We have information on individuals' entry into the pension system and on their mutual fund holdings in their premium pension accounts at the end of each year. Unfortunately, it is impossible to match these data with occupational pension accounts because these accounts are administered by private entities. Moreover, individuals' holdings in occupational pension plans are not covered by the tax-based dataset described above. However, we know the typical contribution rates in occupational pension plans and the typical allocation of these plans to equities and bonds. We will assume that the typical contribution rate and allocation in occupational pension plans apply to all enrolled individuals.

In previous studies, the tax-based holdings information and records from the Swedish Pensions Agency have been used separately. Calvet et al. (2007, 2009), Vestman (2015), and Koijen et al. (2015) use non-pension financial wealth to answer questions related to investors' diversification, portfolio rebalancing, housing and stock market participation, and consumption expenses. Dahlquist et al. (2015) use information from the Swedish Pensions Agency to analyze the activity and performance of pension investors. To the best of our knowledge, we are the first to combine comprehensive and high-quality panel data on individuals' investments inside and outside the pension system.

4 Empirical analysis

4.1 Sample restrictions

We begin with all individuals in the 2007 wave of LINDA and match them with the Swedish Pensions Agency's records of DC account holdings at the end of every year between 2000 and 2007. There are 430,216 individuals covered in both datasets. We then impose three sample restrictions. We exclude individuals for whom we lack portfolio information at the end of each year since they entered the premium pension system. To better match the model to data, we

also exclude the richest percentile in terms of net worth. Finally, we exclude individuals for whom we lack educational information; this applies mainly to recent immigrants, the very old, and the very young. Our final sample consists of 318,345 individuals.

4.2 Three types of pension investors

We classify all individual investors into three categories to document how default investors differ from active investors. Inspired by Dahlquist et al. (2015), we base the classification on the activity in the DC account between 2000 and 2007:

1. *Default.* A default investor has had her premium pension in the default fund since entering the pension system.

2. *Initially active.* An initially active investor opted out of the default fund when entering the pension system but since then has never changed her fund allocation.

3. *Active.* An active investor, after entering the pension system, has made at least one change to her fund allocation.

Note that our classification based on activity relies on the panel dimension of the data. Previous analysis of the choice between taxable and tax-deferred accounts has relied on cross-sectional data (see, e.g., Christelis et al., 2011).

4.3 Summary statistics

Table 1 reports the averages of key variables in 2007. The first column reports the values for all investors and the remaining columns report the values for each of the three specific investor types. Default investors account for 34.1% of all investors while initially active and active investors account for 28.0% and 37.9%, respectively. Below, we mainly compare

the observable characteristics of default and active investors. Initially active investors have characteristics that lie between those of the default and active investors.

The average default investor is 42 years old, which is 4–5 years younger than the active investors. The average labor income of a default investor is SEK 192,096, or only 68% of the average labor income of active investors. In untabulated results, we find that this ratio is fairly stable over the life cycle. Hence, the difference in labor income between active and default investors is not attributable to age differences, but is likely an artifact of other differences (e.g., educational differences, as discussed below). Similarly, there is also a substantial difference in financial wealth. The wealth of the average default investor relative to that of the average active investor is only 56%. Taken together, this means that the pension savings become relatively more important to default investors. In untabulated results, we find that this is particularly so for older default investors who have built up their savings in the DC account.

The table also reports the stock market exposure outside the pension system. We define stock market participation as direct investments in stocks or investments in equity mutual funds. The stock market participation of default investors is 38.8%, which is 23 percentage points lower than that of active investors. That is, default investors have a 37% lower stock market participation rate than do active investors. The average equity shares differ across the investor types, ranging from 11.9% for the default investors to 18.4% for the active investors. Interestingly, these differences are driven entirely by differences in participation rates. Conditioning on stock market participation, the three investor types all have an average equity share of approximately 30%.[6]

There are also large differences in real estate ownership. The main reason for this difference is that the real estate ownership rate among default investors is 50%, much lower

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[6]Our definitions of stock market participation and equity share assume that there are no equity holdings in investors' capital insurance accounts and private pension accounts.

than the 79% among active investors. The differences in financial and real estate wealth are captured in net worth, which is the total wealth minus total liabilities. The differences in total wealth result from differences in both financial wealth and real estate wealth.

Finally, the investor types also differ in education. Though the fraction of high school graduates is fairly stable at approximately 55% across all investor types, the fraction of investors with a college degree is seven percentage points lower among default investors than among active investors (i.e., 25.1% versus 31.8%). Instead, default investors are much more likely than active investors to have finished only elementary school (i.e., 19.1% versus 11.5%).

4.4   Stock market participation

We next turn to a more formal comparison of investment behavior outside the pension system for the different investor types. Specifically, we study how activity in the pension system relates to stock market participation outside the pension system. We run the following regression:

$$D(\text{Participation}_i = 1) = \alpha + \beta_1 D(\text{Default}_i = 1) + \beta_2 D(\text{Initially active}_i = 1) + \gamma' X_i + \varepsilon_i, \quad (1)$$

where $D(\text{Participation}_i = 1)$ is a dummy variable that takes a value of one if the individual holds stocks directly or equity funds outside the pension system, $D(\text{Default}_i = 1)$ is a dummy variable that takes a value of one if the individual is a default investor in the pension system, $D(\text{Initially active}_i = 1)$ is a dummy variable that takes a value of one if the individual is an initially active investor, X_i is a vector of individual characteristics, and ε_i is an error term. As our classification of investors refers to the 2000–2007 period, we restrict ourselves to stock market participation at the end of 2007. In our initial specifications, the stock market participation is linear in the individual characteristics. However, we also consider piecewise linear splines for the continuous characteristics. The characteristics are largely chosen to

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be consistent with a structural life-cycle model of portfolio choice, similar to the model we set up in the next section. Hence, we include age, labor income, and financial wealth as individual characteristics; we also consider a real estate dummy, educational dummies, and geographical dummies. We argue that a linear regression model is sufficient to establish basic facts about investment behavior inside and outside the pension system.

Table 2 reports the results of these regressions. (Note that in the regressions age is scaled down by 100, and labor income and financial wealth are scaled down by 1,000,000.) Specification I serves as a benchmark and focuses on a linear specification without the default and initially active investor type dummies. Specification II includes the investor type dummies. The estimates indicate that when controlling for individual characteristics, there is a strong negative relationship between default investing and stock market participation. Being a default investor in the pension system reduces the likelihood of having equity exposure outside the pension system by 13.3 percentage points. This effect can be related to the effect of other variables. For example, the estimated effect of being ten years older is a 0.2 percentage-point-higher participation rate. Moreover, the effects of SEK 100,000 more in labor income and in financial wealth correspond to participation rates that are 1.2 and 2.9 percentage points higher. These effects can also be compared with the 23 percentage-point difference in the unconditional participation rate between default and active investors. That is, including a rich set of controls reduces the participation rate gap from 23.0 to 13.3 percentage points, though it is still substantial.

Specification III lets age, labor income, and financial wealth enter as piecewise linear splines. Specification IV also includes industry and occupational dummies. Even in these richer specifications, there is still a strong negative relationship between default investing and stock market participation. A default investor in the pension system is 8.7-percentage-points less likely to participate in the stock market outside the pension system. Our results suggest that 38% of the gap is driven by differences in unobservable variables. One such

unobservable variable could be the experience of making investment decisions.

The bottom-line finding of our regressions is that default investing in the pension system is strongly associated with lack of equity exposure outside the pension system. Even when controlling for individual characteristics that correspond to the state variables of a standard life-cycle portfolio choice model, the gap in stock market participation between active and default investors is substantial. These findings have implications for the design of an optimal default fund. In addition, the findings underscore the importance of modeling limited stock market participation outside the pension system. We will calibrate our model to the pool of default investors rather than to the average pension investor.

4.5   Heterogeneity among default investors

In this section we demonstrate that there is considerable heterogeneity among default investors. Understanding how these investors differ from one another is important for the design of a default fund. Table 3 presents the distributions of variables for default investors. Panel A presents the distributions for all default investors. It shows that default investors exist in all age categories and differ greatly in labor income, financial wealth, and equity exposure. Regarding the inequality in labor income, 25% of default investors earn more than SEK 277,860 whereas 25% earn less than SEK 52,446. The inequality in financial wealth is even greater: 25% have SEK 149,388 or more in financial wealth whereas 25% have essentially no savings, i.e., less than SEK 9283. This inequality applies to equity exposure as well, most default investors having no equity exposure outside the pension system, whereas 10% have at least 48.1% of their financial wealth allocated to equities.

In Panels B and C, default investors are split into stock market participants and non-participants. While participants and non-participants differ little in age, they differ somewhat in labor income and a great deal in financial wealth. The median non-participant earns 20% less than does the median participant. Furthermore, the median non-participant has

just 12% of the financial wealth of the median participant. Only 10% of participants have less financial wealth than does the median non-participant. Finally, financial wealth can be contrasted to labor income. Stock market participants have financial wealth worth 1.4 years of labor income, while non-participants have financial wealth worth just five months of labor income. As participants have higher labor income, the average participating default investor has 4.4 times as much financial wealth as does the average non-participating default investor. These basic facts make it reasonable to question the ability of a one-size-fits-all design of the default fund to meet all default investors' needs.

The takeaway is that there is considerable heterogeneity even among default fund investors. Specifically, stock market participation serves the function of an indicator variable, most participants being richer in terms of both labor income and financial wealth. This suggests that it may be beneficial to carefully design the default fund to suit each investor's specific situation as much as possible rather than imposing one allocation on all.

5   The model

Following the empirical analysis, we set up a life-cycle model for the economic situation of a default investor. The model builds on the work of Viceira (2001), Cocco et al. (2005), and Gomes and Michaelides (2005) and includes risky labor income, a consumption–savings choice, and a portfolio choice. We augment the model with a pension system in which individuals save in illiquid pension accounts, from which their pension is received as annuities.[7] Next we describe the model's building blocks.

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[7]Our model relates to Gomes et al. (2009), who consider portfolio choice in the presence of tax-deferred retirement accounts, and to Campanale et al. (2014), who consider a model in which stocks are subject to transaction costs, making them less liquid.

5.1   Demographics

We follow individuals from age 25 until the end of their lives.[8] The end of life occurs at the latest at age 100, but could occur before as individuals face an age-specific survival rate, ϕ_t. The life cycle is split into a working, or accumulation, phase and a retirement phase. From the ages of 25 to 64 years, individuals work and receive labor income exogenously. They retire at age 65.

5.2   Preferences

The individuals have Epstein and Zin (1989) preferences over a single consumption good. At age t, each individual maximizes the following:

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U_t \;=\; \left(c_t^{1-\rho} + \beta \phi_t E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \tag{2}
$$

$$
U_T \;=\; c_T, \tag{3}
$$

where β is the discount factor, $\psi = 1/\rho$ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, and $t = 25, 26, ..., T$ with $T = 100$. For notational convenience, we define the operator $\mathcal{R}_t(U_{t+1}) \equiv E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$.

5.3   Labor income

Let Y_{it} denote the labor income of employed individual i at age t. During the working phase (up to age 64), the individual faces a labor income process with a life-cycle trend and

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[8]We choose age 25 as the start of the working phase, as Swedish workers do not fully qualify for occupational pension plans before that age.

persistent income shocks:

$$y_{it} = g_t + z_{it}, \tag{4}$$

$$z_{it} = z_{it-1} + \eta_{it} + \theta\varepsilon_t, \tag{5}$$

where $y_{it} = \ln(Y_{it})$. The first component, g_t, is a hump-shaped life-cycle trend. The second component, z_{it}, is the permanent labor income component. It has an idiosyncratic shock, η_{it}, which is distributed $N(-\sigma_\eta^2/2, \sigma_\eta^2)$, and an aggregate shock, ε_t, which is distributed $N(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$. The aggregate shock also affects the stock return, and θ determines the contemporaneous correlation between the labor income and the stock return. We allow for heterogeneity in income as early as age 25 by letting the initial persistent shock, z_{i25}, be distributed $N(-\sigma_z^2/2, \sigma_z^2)$.

During the retirement phase (from age 65 and onwards), the individual has no labor income.[9] Pension is often modeled as a deterministic replacement rate relative to the labor income just before retirement.[10] However, in our model, the replacement rate is endogenously determined. The individual relies entirely on annuity payments from savings accounts. Later we discuss these accounts in detail.

5.4   Stock market participation costs and investor types

To enter the stock market outside the pension system, the individual must pay a one-time participation cost, κ_i. (The financial wealth and the decision to invest in the stock market are described later.) A one-time entry cost is common in portfolio-choice models (see, e.g., Alan, 2006; Gomes and Michaelides, 2005, 2008).

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[9]Hence, the retirement decision is not endogenous as in French and Jones (2011). More generally, we do not consider endogenous labor supply decisions as in Bodie et al. (1992) and Gomes et al. (2008).

[10]One exception is that of Cocco and Lopes (2011), who model the preferred DB or DC pension plan for different investors.

The state variable, I_{it}, tracks whether stock market entry has occurred between age 25 and age t; its initial value is zero (i.e., $I_{i25} = 0$). The law of motion for I_{it} is given by:

$$I_{it} = \begin{cases} 1 \text{ if } I_{it-1} = 1 \text{ or } \alpha_{it} > 0 \\ 0 \text{ otherwise} \end{cases} \tag{6}$$

where α_{it} is the fraction of financial wealth invested in the stock market. The cost associated with stock market entry then becomes $\kappa_i(I_{it} - I_{it-1})$.

A new feature of our model is that we allow for different costs for different investors. We assume a uniform distribution of the cost:

$$\kappa_i \sim U(\underline{\kappa}, \bar{\kappa}), \tag{7}$$

where $\underline{\kappa}$ and $\bar{\kappa}$ denote the lowest and highest costs among all investors, respectively. We justify the dispersion in cost with reference to the documented heterogeneity in financial literacy and financial sophistication (see Lusardi and Mitchell, 2014, for an overview). By introducing a cost distribution, we can replicate the fairly flat life-cycle participation profile in the data.[11] On average, low-cost investors will enter early in life whereas high-cost investors will enter later or never at all. With a sufficiently low value of $\underline{\kappa}$, some low-cost investors will enter immediately. At the end of life, more high-cost than low-cost investors will remain non-participants. For simplicity, we assume that the cost is independent of other characteristics.

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[11]Fagereng et al. (2015) present an alternative set-up to account for the empirical life-cycle profiles on portfolio choice. Their set-up involves a per-period cost and a loss probability on equity investments.

5.5 Asset returns

The gross return on the stock market, R_{t+1}, develops according to the following log-normal process:

$$\ln(R_{t+1}) = \ln(R_f) + \mu + \varepsilon_{t+1}, \tag{8}$$

where R_f is the gross return on a risk-free bond and μ is the equity premium. Recall that the shock, ε_t, is distributed $N(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$, so $E_t(R_{t+1} - R_f) = \mu$. Also recall that ε_t affects labor income in (5), and that the correlation between stock returns and labor income is governed by the weight θ.

5.6 Three accounts for financial wealth

An individual has three financial savings accounts: (i) a liquid account outside the pension system (which we simply refer to as financial wealth), (ii) a fully-funded DC account in the pension system, and (iii) a notional account belonging to the pension system. The notional account, which provides the basis for the pension, is income based and evolves at the rate of the risk-free bond. The DC account is also income based but the investor can choose how to allocate between bonds and stocks; it corresponds to the default fund we wish to design.

The account outside the pension system is accessible at any time. Each individual chooses freely how much to save and withdraw from it. In contrast, the contributions to the pension accounts during the working phase are determined by the pension policy (rather than by the individual) and are accessible only in the form of annuities during the retirement phase. Importantly, the two pension accounts include insurance against longevity risk.

Financial wealth

The individual starts the first year of the working phase with financial wealth, A_{i25}, outside the pension system. The log of initial financial wealth is distributed $N(\mu_A - \sigma_A^2/2, \sigma_A^2)$. In

each subsequent year, the individual can freely access the financial wealth, make deposits, and choose the fraction to be invested in risk-free bonds and in the stock market. However, the individual cannot borrow:

$$A_{it} \geq 0, \tag{9}$$

and the equity share is restricted to be between zero and one:

$$\alpha_{it} \in [0, 1]. \tag{10}$$

Taken together, (9) and (10) imply that individuals cannot borrow at the risk-free rate and that they cannot short the stock market or take leveraged positions in it.

The individual's cash in hand (i.e., the sum of financial wealth and labor income) develops according to:

$$X_{it+1} = A_{it} \left(R_f + \alpha_{it}(R_{t+1} - R_f) \right) + Y_{it+1}. \tag{11}$$

Supported by the analysis in Fischer et al. (2013), we do not model taxes on capital gains.

The DC account

Inside the pension system, each individual has a DC account with a balance equal to A_{it}^{DC}. During the working phase, the contribution rate equals λ^{DC}.[12]

The investor cannot short the stock market or take leveraged positions in it:

$$\alpha_{it}^{\mathrm{DC}} \in [0, 1]. \tag{12}$$

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[12]In line with the Swedish pension system, we implement the contribution as an employer tax. This means that the contributions do not show up as withdrawals from gross labor income in the individual's budget constraint. This is consistent with our calibration of the labor income process to micro data (i.e., our measure of gross labor income is net of the employer tax).

Before retirement, the law of motion for the DC account balance is:

$$A_{it+1}^{\mathrm{DC}} = A_{it}^{\mathrm{DC}}(R_f + \alpha_{it}^{\mathrm{DC}}(R_{t+1} - R_f)) + \lambda^{\mathrm{DC}} \min\{Y_{it}, \overline{Y}\}, \tag{13}$$

where $\overline{Y}$ is a the ceiling at which the individual's earnings no longer qualify for additional contributions. That is, $\lambda^{\mathrm{DC}}\overline{Y}$ is the maximum contribution to the DC account.

Upon retirement at age 65, withdrawal starts. We assume that the investor is allowed to make a one-time decision on the equity exposure for the remainder of her life (i.e., $\alpha_{i65}^{\mathrm{DC}} = \alpha_{i66}^{\mathrm{DC}} = ... = \alpha_{i100}^{\mathrm{DC}}$). Note that this variable becomes a state variable. We have considered an extension in which the investor can choose the equity exposure for each year in the retirement phase. We found that this extension does not materially affect the optimal choice in the working phase.

The notional account

The law of motion for the notional account balance during the working phase is:

$$A_{it+1}^{\mathrm{N}} = A_{it}^{\mathrm{N}}R_f + \lambda^{\mathrm{N}} \min\{Y_{it}, \overline{Y}\}, \tag{14}$$

where λ^{N} is the contribution rate for the notional account. Note that the ceiling on the contributions to the DC and notional accounts captures the progressive feature of the pension system.

To economize on state variables, we use z_{i64} to approximate the notional account balance at the time of retirement. This approximation is based on simulations of equations (4), (5), (8), and (14) to obtain the best fit between z_{i64} and A_{i64}^{N} using regression analysis. This approximation works well. We provide further details in the Online Appendix.

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Annuitization of the pension accounts

Upon retirement at age 65, the DC account and the notional account are converted into two actuarially fair life-long annuities. They insure against longevity risk through within-cohort transfers from individuals who die to survivors. The notional account provides a fixed annuity with a guaranteed minimum. If the balance of the account is lower than is required to meet the guaranteed level at age 65, we let the individual receive the remainder at age 65 in the form of a one-time transfer from the government. The annuity from the DC account is variable and depends on the choice of the equity exposure as well as realized returns. In expectation, the individual will receive a constant payment each year.

5.7 The individual's problem

Next we describe the individual's problem. To simplify the notation, we suppress the subscript i. Let $V_t\left(X_t, A_t^{\mathrm{DC}}, z_t, \kappa, I_t\right)$ be the value of an individual of age t with cash in hand X_t, DC account balance A_t^{DC}, a persistent income component z_t, cost κ, and stock market participation experience I_t. If the individual is 65 years or older, $\alpha_{65}^{\mathrm{DC}}$ is yet another state variable. For simplicity, we omit this variable from the value function.

The following describes the individual's problem when the equity share in the DC account is chosen optimally (i.e., conditional on all state variables, including age, financial wealth inside and outside the pension system, stock market participation, and labor income). This is equivalent to the best possible design of the default fund, directly tailored to the individual.

The participant's problem

An individual who has already entered the stock market solves the following problem:

$$V_t\left(X_t, A_t^{\mathrm{DC}}, z_t, \kappa, 1\right) = \max_{A_t, \alpha_t, \alpha_t^{\mathrm{DC}}} \left\{ \left((X_t - A_t)^{1-\rho} + \beta\phi_t \mathcal{R}_t \left(V_{t+1}\left(X_{t+1}, A_{t+1}^{\mathrm{DC}}, z_{t+1}, \kappa, 1\right)\right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \right\}$$

subject to equations (4)–(13).

The entrant's problem

Let $V_t^+\left(X_t, A_t^{\mathrm{DC}}, z_t, \kappa, 0\right)$ be the value for an individual with no previous stock market participation experience who decides to participate at t. This value can be formulated as:

$$V_t^+\left(X_t, A_t^{\mathrm{DC}}, z_t, \kappa, 0\right) = \max_{A_t, \alpha_t, \alpha_t^{\mathrm{DC}}} \left\{ \left((X_t - A_t - \kappa)^{1-\rho} + \beta\phi_t \mathcal{R}_t \left(V_{t+1}\left(X_{t+1}, A_{t+1}^{\mathrm{DC}}, z_{t+1}, \kappa, 1\right)\right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \right\}$$

subject to equations (4)–(13).

The non-participant's problem

Let $V_t^-\left(X_t, A_t^{\mathrm{DC}}, z_t, \kappa, 0\right)$ be the value for an individual with no previous stock market participation experience who decides not to participate at t. This value can be formulated as:

$$V_t^-\left(X_t, A_t^{\mathrm{DC}}, z_t, \kappa, 0\right) = \max_{A_t, \alpha_t^{\mathrm{DC}}} \left\{ \left((X_t - A_t)^{1-\rho} + \beta\phi_t \mathcal{R}_t \left(V_{t+1}\left(X_{t+1}, A_{t+1}^{\mathrm{DC}}, z_{t+1}, \kappa, 0\right)\right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \right\}$$

subject to equations (4)–(13).

Note that as $\alpha_t = 0$, the return on financial wealth is simply R_f.

Optimal stock market entry

Given the entrant's and non-participant's problems, the optimal stock market entry is given by:

$$V_t\left(X_t, A_t^{\text{DC}}, z_t, \kappa, 0\right) = \max\left\{V_t^+\left(X_t, A_t^{\text{DC}}, z_t, \kappa, 0\right), V_t^-\left(X_t, A_t^{\text{DC}}, z_t, \kappa, 0\right)\right\}.$$

5.8 Calibration

In this section we describe our calibration strategy. Table 4 reports the values of key parameters. Most parameters are set either according to the existing literature or to match Swedish institutional details; those parameters can be said to be set exogenously. Four parameters are set to match the data as well as possible; those parameters can be said to be determined endogenously.

Exogenous parameters

There are six sets of exogenous parameters.

First, we set the elasticity of intertemporal substitution to 0.5, which is a common value in life-cycle models of portfolio choice (see, e.g., Gomes and Michaelides, 2005).

Second, we set the equity premium to 4% and the standard deviation of the stock market return to 18%. These choices are in the range of commonly used parameter values in the literature. We set the simple risk-free rate to zero, which in other calibrations is often set to 1–2%. We argue that this is correct in our model as labor income does not include economic growth. Thus, we deflate the account returns by the expected growth to obtain coherent replacement rates. As replacement rates in our model are a function of returns, rather than a function of final labor income, this choice is more important to the present model than to previous models. Simulations of the labor income process and contributions to the pension accounts validate our strategy. These simulations indicate that replacement rates at age 65

relative to labor income at age 64 are coherent with Swedish Pensions Agency forecasts.

Third, we set labor income according to Swedish data. Following Carroll and Samwick (1997), we estimate the riskiness of labor income. We find that the standard deviation of permanent labor income equals 0.072, which is somewhat lower than in most representative samples. However, we are focusing on default investors who earn less than the average worker; for them the parental leave and sick leave benefits (included in our definition of labor income) are important relative to the wage earnings, resulting in a low estimate.[13] We set the one-year correlation between permanent income growth and stock market returns to 10%. This corresponds to a θ of 0.040. We approximate the distribution of initial labor income and financial wealth using log-normal distributions. The mean financial wealth for 25-year-old default investors is set to SEK 111,300. The cross-sectional standard deviations are set to 0.391 (σ_z) and 1.365 (σ_A) to match the data for 25-year-old individuals.

Fourth, we consider the contribution rates. We set the contribution rate for the notional account to 16%. The maximum contribution to this account is capped (corresponding to a labor income ceiling of SEK 344,250). We set the contribution rate for the DC account to 7% (also capped). This mirrors the premium pension account with a contribution rate of 2.5% and the occupational pension account with a typical contribution rate of 4.5%. The cap on the contributions means that the pension system has a progressive feature. The fraction of individuals with labor income above the ceiling depends on age. For example, 4% of 30-year-olds are above the ceiling and 21% of 50-year-olds are above. Overall, only 6% of aggregate income is not counted towards contributions.

Fifth, we determine the annuity divisor for the notional account in retirement. We use the unisex mortality table of Statistics Sweden to determine ϕ_t. We assume that the notional

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[13]Because lack of data, our measure of labor income does not include unemployment benefits and other kinds of allowances and government transfers. We therefore exclude individual-year observations from the estimation when labor income is sufficiently low (i.e., the individual is not working full time or would be eligible for benefits not included in our measure).

account continues to be invested in the risk-free bond and allow for inheritances within a cohort from dying to surviving individuals, incorporating those into the returns of the survivors. We then use the standard annuity formula to reach an annuity factor of 5.6% out of the account balance at age 65. We use the same formulas for the DC account, though we adjust the expected return to the endogenous choice of the DC equity share in retirement.

Finally, we determine the DC equity share profile of the calibration. This is important because equity exposure in the pension system determines individuals' demand for equity outside the pension system, which we in turn match to be consistent with our data in 2007. Each cohort born before 1973 has a unique equity share profile. This must be accounted for when matching wealth and equity share profiles outside the pension system. Note that cohorts born in 1973 or later were only 24 years old or younger in 1997 when contributions to the new pension system started to accrue.[14] Consequently, cohorts born before 1973 were 25 years or older in 1997 and their contributions during early working life (i.e., before 1997) were not credited to their premium pension accounts. Exactly how much was foregone depends on the cohort. To obtain a single life-cycle profile that can be used in the calibration, we mix the cohorts' profiles. Younger cohorts are given a greater weight in the early stages of the life-cycle and older cohorts are given a greater weight in the later stages. For practical purposes, a good fit is a linear profile that starts at an equity share of 71% at age 25 and falls to 40% at age 64. It then continues at 40% through the retirement phase. This profile is depicted in the lower-right panel of Figure 1. See the Online Appendix for further details.

Endogenous parameters and model fit

Four parameters are treated as endogenous in the calibration. We consider data from the working phase.[15] The discount factor is calibrated to match the 0.80 ratio of financial wealth

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[14]The premium pension system was launched in 2000 but contributions had been accumulating since 1997.

[15]Note that we match the model to data from 2007. This does not allow us to extract cohort or time effects as in, e.g., Ameriks and Zeldes (2004). However, Vestman (2015) finds that cohort and time effects

to labor income. A β of 0.932 provides an exact fit to the data. The top-left panel of Figure 1 shows the full life-cycle profile of financial wealth. The model fits the financial wealth well up to age 60 and undershoots after that. A higher discount factor would better fit financial wealth late in life but make the fit worse before.

The support of the cross-sectional distribution of participation costs is set so that we match the average stock market participation rate between ages 25 and 64. As can be seen in the top-right panel of Figure 1, participation is almost flat over the life-cycle. Intuitively, the parameter $\underline{\kappa}$ affects the participation rate among the young, who are poor in terms of financial wealth and reluctant to enter the stock market if the cost is high. The relatively high participation rate of young individuals therefore leads us to set $\underline{\kappa}$ equal to zero. The parameter $\bar{\kappa}$ is then determined to match the average participation rate from age 25 to 64, which is 0.37 in the model and 0.38 in the data. We obtain this participation rate by setting $\bar{\kappa} = 30,000$. As the distribution is uniform, this corresponds to an average participation cost of SEK 15,000. We find our modeling approach appealing as it enables us to keep the cost low for the average investor (see Vissing-Jørgensen, 2002). The uniform distribution of the cost enables the model to replicate the flat participation profile in the data.[16]

Finally, the relative risk aversion coefficient, γ, determines the conditional equity share. We weigh the equity shares of each age group by its financial wealth. A relative risk aversion of 12 provides a reasonable fit. The value-weighted conditional equity share is 0.50 in the model and 0.41 in the data. The lower-left panel of Figure 1 depicts the life-cycle profile. The model overshoots the data when financial wealth is low and undershoots when liquid financial wealth is high. We are reluctant to increase the relative risk aversion above 12, as this would lead to a worse discrepancy close to retirement age. In the model there is a noticeable increase in the equity share after age 80; however, if value-weighted, this increase

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are not strongly present in the data.

[16]Technically, we approximate the uniform distribution using seven equally weighted discrete types (the seven costs are equally spaced between zero and SEK 30,000).

is negligible as the financial wealth is small then.

Figure 2 shows that the distribution of entry costs produces an endogenous sorting of individuals into stock market participants and non-participants that matches the data well. The left panel shows that the average labor income of non-participants is similar in the model and the data. The average labor income of participants is somewhat lower than in the data. The right panel shows the financial wealth in the model and in the data. The sorting by financial wealth to participants and non-participants is consistent with the data but weaker.[17] Financial wealth in the model peaks just before retirement, earlier than in the data. In the years after retirement, financial wealth decumulates in the model and the data, but much less so in the latter. In particular, the gap widens for participants. There could be several reasons for this, one being the lack of a bequest motive in the model.

6 The optimal design

In this section we discuss the optimal design of the default fund. The asset allocation is the solution to the individual's optimization problem conditioning on all state variables. It is thus identical to that of a rational default investor who is aware of her characteristics and how they affect the optimal DC equity share.

We first describe our simulation method. We then report the optimal asset allocation averaged over economies and individuals. Finally, we report how the DC equity share is distributed across economies with different stock market realizations (aggregate equity risk) and across individuals (inequality).

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[17]It is well known that it is difficult to generate wealth inequality in life-cycle models with incomplete markets. This has been addressed by incorporating heterogeneity in discount factors (Krusell and Smith, 1998) or a right-skewed income process (Castaneda et al., 2003). In our model the progressive feature of the pension system helps us match the data.

6.1 Simulation method

There are two main sources of risk in our model: (i) aggregate equity returns and (ii) idiosyncratic labor income shocks. Our simulation method lets us separately study the two risk sources. We consider 500 individuals with different idiosyncratic labor income shocks. For each of the 500 individuals, we assume seven different entry costs, so in total there are 3500 individuals in an economy. Strictly speaking, an economy is a single birth cohort, which we follow over its life. (Note that this differs from the calibration in which we matched parameters to a mix of cohorts.) The economy faces one equity return realization of 75 annual returns, common to all individuals in the economy. We simulate a total of 50 economies.

When we average for each individual over the 50 economies, we obtain ex ante life-cycle profiles of 3500 individuals; this distribution represents the inequality across individuals. When we instead average over the 3500 individuals within an economy, we are able to analyze the role of aggregate equity risk.[18]

6.2 Average optimal asset allocation

Figure 3 shows simple averages over both economies and individuals. The top-left panel reports labor income during the working phase and pension (i.e., annuities from the DC and notional accounts) during the retirement phase. The average labor income at age 64 equals SEK 191,400 and the average pension equals SEK 149,200, yielding a replacement rate of 78%. The top-right panel shows consumption, which is hump shaped due to individuals' inability to fully smooth their consumption. At retirement, there is a small jump in consumption partly due to the resolution of uncertainty, as there is then no labor income risk

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[18]For every economy, the same idiosyncratic income shocks are used. The cross-sectional average of these shocks is zero for each year. Furthermore, we reuse the idiosyncratic income shocks and stock market returns for all cost types and all designs of the default fund. We also reuse initial draws of z_{i25} and A_{i25}. This simulation method is similar to that of Campbell and Cocco (2015), who also distinguish between aggregate and idiosyncratic shocks.

and no risk in the annuity based on the notional account.

The three following panels show the notional account, the DC account, and financial wealth, all of which are distinctly hump shaped. The high contribution rates for the two pension accounts make their balances large relative to financial wealth even at a young age. At age 30, the DC account is as large as the financial wealth. The importance of the DC account then increases and at retirement it is three times as large as the financial wealth. This illustrates how potent the optimal asset allocation of the default fund is: The total equity exposure will essentially be determined by the equity share in the DC account. Financial wealth mainly serves as a buffer for precautionary motives, peaking just before retirement and then quickly depleting. While the notional account is the largest account, the DC account catches up over time due to its equity exposure. Just before retirement, there is a jump in the notional account. This discontinuity arises in the model as individuals who do not reach the implied pension floor receive an additional government transfer to their notional accounts. This is another reason why consumption jumps at retirement.

The third panel to the right shows average stock market participation reaching 50% late in life. The bottom panel to the left shows the equity share in financial wealth conditional on participation. This is the equity share outside the pension system. It first increases slightly and then falls until retirement. Note that its increase after retirement is economically not important as the financial wealth is low, so a high equity share can be tolerated. Finally, the bottom-right panel shows the DC equity share. At age 25 it is 100%. It remains high until age 34 when it starts to decrease almost linearly, by approximately 2.5 percentage points per year, until retirement. The decrease during the working phase is due to the gradual shift in the composition of investors' total wealth. As the present value of labor income diminishes and the account balances increase, a high equity share cannot be tolerated (Merton, 1971; Cocco et al., 2005). After retirement, individuals tolerate, on average, a somewhat higher equity exposure. We next turn to the economic mechanisms underlying the development of

the DC equity share.

6.3   Aggregate equity risk and inequality

Figure 4 shows the aggregate equity risk and inequality implied by the optimal asset alloca-
tion. The panels to the left refer to averages over individuals, highlighting the equity risk;
the panels to the right refer to averages over economies, highlighting the inequality across
individuals. We sort the variables by the DC equity share in the top panels, maintaining
that sorting for the remaining panels.

The top-left panel shows how the DC equity share varies over the economies, i.e., how
much it varies with the realized equity returns. The second decile (i.e., average of economies
6–10, denoted by a red dotted line) indicates that, with a probability of 10%, the DC
equity share exceeds at least 45% throughout the working phase, jumping to approximately
60% at retirement. The ninth decile (i.e., average of economies 41–45, denoted by a green
dashed line) indicates that, with a probability of 10%, the DC equity share decreases to less
than 20% just before retirement, jumping approximately to 37% at retirement. The panel
below shows the corresponding values for the DC account. It indicates a strong negative
correlation between the DC equity share and the DC account balance, a high equity share
corresponding to a low account balance and vice versa. The remaining three panels to the
left show the corresponding values for labor income and pension, consumption, and stock
market participation. None of these variables seems to correlate as strongly with the DC
equity share as does the DC account balance.

As the DC equity share correlates negatively with the DC account balance while labor
income (and hence contributions to the DC account) does not, the analysis suggests that
realized equity returns affect the optimal asset allocation. A small difference in returns over
many years result in large differences in DC account balances. For example, assuming that
contributions are constant, the average annual return is 0.8 percentage points above expec-

tations for economies in the ninth decile and 1.1 percentage points below expectations for economies in the second decile. This seemingly small difference in realized returns and large difference in DC equity share implies that the optimal allocation involves active rebalancing. As returns exceed expectations, it is optimal to invest less in equity and vice versa. The mechanism behind this property of the optimal allocation is that a key determinant of the DC equity share is the value of the DC account relative to other accounts and relative to the present value of labor income. The high sensitivity to realized returns means that the optimal equity share can differ markedly between cohorts that have experienced different return histories.[19]

The top-right panel shows the inequality in the DC equity share. The second decile (i.e., average of individuals 351–750) has a high DC equity share that stays above 45% throughout the working phase. At retirement, the DC equity share jumps to 60%. The ninth decile (i.e., average of individuals 2801–3150) has a DC equity share that declines below 30% at age 50 and is just 13% at age 64; at retirement, the DC equity share jumps to 33%. Notably, the gap between the second and ninth deciles is already large at age 40 and even greater among 50-year-olds. The four panels below reveal the explanation for this gap. Individuals with a high optimal DC equity share have low DC account balances, are income poor, and are unlikely to participate in the stock market; individuals with a low optimal DC equity share have high DC account balances, high labor income, and a high stock market participation rate.

To sum up, variation across economies suggests that equity return realizations matter for the DC equity share. This means that different birth cohorts can have different optimal

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[19]In untabulated results, we compute the effect of an annual return of zero for ten consecutive years (i.e., 4% below expectation for ten years in a row). Such realized returns imply that the DC equity share of a 55-year-old individual should be nine percentage points higher than the DC equity share of a 55-year-old born ten years earlier and who experienced realized returns of 4%. Conversely, a 55-year-old who for ten years has been exposed to realized returns of 8% has an optimal DC equity share eight percentage points below that of a 55-year-old who was born ten years earlier and who experienced realized returns of 4%.

allocations at the same age. Moreover, the large dispersion in optimal equity shares empha-
sizes the potential of an asset allocation conditioned on investor-specific characteristics. In
other words, different default investors have different needs.

7   Accounting for heterogeneity beyond age

It is common to formulate investment rules that depend on age. One such rule is to invest
the percentage 100 minus one's age in equity and the remainder in bonds. According to
this rule, a 30-year-old would invest 70% in equities and a 70-year-old would invest 30% in
equities. We refer to this as the "100-minus-age" rule. This rule can certainly be modified to
have different equity exposures at the beginning of the working phase and in the retirement
phase. Nevertheless, it would still be an age-based investment rule.

In this section, we relate our optimal design to age-based investing. We begin by re-
gressing the optimal equity share on age and other state variables to evaluate their hedging
efficiency. We then report the changes in pensions associated with a shift from the "100-
minus-age" rule to the optimal allocation, and how these changes are distributed in the
cross-section of investors.

7.1   Regression-based approximation of the optimal allocation

The optimal asset allocation depends on cross-sectional differences and realized returns.
Our model moves beyond age-based investment rules as it conditions on both individual
characteristics (e.g., labor income and stock market participation) and the DC account
balance (which, conditional on age and labor income, proxies for realized returns). To
illustrate the determinants of the optimal asset allocation in the default fund, we estimate

the following regression on model-generated data:

$$\alpha_{it}^{\mathrm{DC}} = \beta_0 + \beta_1 t + \beta_2 A_{it} + \beta_3 A_{it}^{\mathrm{DC}} + \beta_4 Y_{it} + \beta_5 I_{it} + \varepsilon_{it}, \tag{15}$$

where the dependent variable is the optimal DC equity share of individual i of age t, and where all covariates are state variables of the model. We do not include the participation cost, κ_i, as it would be unobservable in actual data.[20] We run the regression on individuals during their working phase. Note that the R-squared in the regression captures the efficiency of the investment rule.

This analysis relates to Merton (1971), who derive the inter-temporal hedging motive that arises from the present discounted value of labor income. Cocco et al. (2005) discuss the role of financial wealth relative to total wealth (including the present value of labor income) when labor income is uninsurable. In our model the value of the three accounts (A_{it}, A_{it}^{DC}, and A_{it}^{N}) and the present value of labor income guide optimal equity shares inside and outside the pension system. If the aim was to contrast our model to theirs, we could report the total equity share as a function of account balances and labor income. Since some of our accounts remain illiquid until retirement, the result would not be entirely identical. Note, for instance, that if equity returns are high and the balance on the DC account increases it is not possible to consume out of the account during the working phase. However, the purpose of our analysis is to obtain an asset allocation rule which is implementable for a designer of a default fund. We therefore focus on different subsets of the state variables and do not include wealth ratios in the analysis. Related to this, Dammon et al. (2004) focus on the optimal equity share in a tax-deferrable (retirement) account as a function of age and account balance.

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[20]Note that as we abstract from transitory income risk, it is possible to use cash in hand, X_{it}, and financial wealth, A_{it}, interchangeably as long as labor income, Y_{it}, is included. We choose to use financial wealth in the regression because it has a distinct meaning. Moreover, the omission of transitory risk also means that the permanent component of labor income, z_{it}, and labor income can be used interchangeably.

Table 5 reports the regression results using different specifications. Specification I mimics the simple age-based investment rule. The estimate suggests that individuals should decrease their DC equity exposure by 2.1 percentage points every year. This linear specification is admittedly a crude regression specification, because it results in many young individuals being forced into a DC equity share of 100%. The estimated intercept indeed implies that the predicted DC equity share for a 25-year-old is 112.2%. Nevertheless, the interpretation is that a better rule for the DC equity share would be to have it at 100% until about age 30 and thereafter let it fall by two percentage points per year. Note that this is a steeper reduction in equity exposure over time than that of the "100-minus-age" rule. Interestingly, the R-squared for our rule is as high as 54.9%. In untabulated results, we find that non-linear specifications in age only improves the R-squared marginally.

To better understand the role of incremental information in the form of additional state variables, Specifications II–V add one additional state variable at a time to the age variable. All additions significantly improve the regression fit. In Specifications II and III, labor income and financial wealth add 7–8 percentage points to the R-squared; however, the stock market participation status in Specification IV and the DC account balance in Specification V add more. The R-squared associated with conditioning on stock market participation is as high as 66.8%. The DC equity share should be differentiated by participation status. Stock market participants should have 23.2 percentage points less exposure to equity than do non-participants, a substantial difference. However, the single most influential state variable is the DC account balance. Specification V shows that the DC equity share should be reduced by 0.5 percentage points per year as a direct effect of age. The remaining reduction is contingent on the development of the account balance. In addition to the direct effect, the DC equity share should be reduced by 6.9 percentage points for every increase of SEK 100,000 in the account balance. This increase is in turn a function of the contribution to the account (i.e., labor income) and the realized equity return. If the return is negative, it

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may well be that the DC equity share should increase rather than decrease. The R-squared associated with this simple asset allocation rule increases by 22 percentage points relative to Specification I, implying that the rule can account for an impressive 76.9% of the variation in the optimal allocation. In untabulated results, we find that cumulative returns matter as much for the improvement in the R-squared as labor income does. It is particularly encouraging that the account balance is the single best piece of incremental information, as it is directly observable.

Based on the results for Specifications II–V, Specification VI shows the effects of the rule based on the two individual characteristics (in addition to age) that matter the most: the DC account balance and stock market participation. The optimal rule can be stated as follows: Reduce the DC equity share by half a percentage point every year. In addition, reduce the DC equity share by 6.3 percentage points for every SEK 100,000 invested in the account. Finally, reduce the life-cycle path by 19.5 percentage points if the individual is a stock market participant. This rule summarizes the model implications well and accounts for 85.2% of the model's optimal asset allocation rule. The R-squared for Specification VII reveals that labor income and financial wealth add little on the margin.

7.2   Inequality in pensions

We examine the distribution of pension at age 65 across individuals when using the "100-minus-age" rule. The left panel of Figure 5 shows the distribution of pensions under this rule. All individuals receive pensions of between SEK 100,000 and SEK 250,000, but the mode is relatively low, followed by a thin right tail.

The right panel of the figure shows the changes in pensions associated with a shift from the "100-minus-age" rule to the optimal allocation at each interval in the distribution. The increases are concentrated in the thick, left-hand part of the distribution. For individuals with a pension below SEK 137,000, the increase associated with a shift to the optimal

allocation exceeds 5%. The average changes are positive up to pensions of SEK 162,000, which implies that 75% of all individuals benefit in terms of realized pension from a shift in asset allocation. For the remainder, a shift from the simple life-cycle rule to the optimal rule is associated with an average decrease in pension.

The large range in changes (from –6% to 8%) is a consequence of heterogeneity in optimal allocations. As noted earlier, poor investors have a higher optimal equity share than do rich investors. Therefore, relative to the "100-minus-age" rule that sets the equity share regardless of the DC account balance, the pension of the poor rises while that of the rich declines.

Quantitatively, the average increase in pension is 3%. As the increases are concentrated among low-pension individuals and the decreases among high-pension individuals, a shift to the optimal allocation implies a compression of the distribution of pensions. Under the "100-minus-age" rule, the standard deviation of the logarithm of pension is 0.19, while under the optimal allocation, the standard deviation is 0.15, a 21% reduction.[21]

7.3   Other benchmarks and a lower equity premium

We have considered other benchmark comparisons that highlight the role of the optimal ex ante risk-reward trade-off and ex post realized pensions. First, we considered comparisons with a more aggressive age-based rule, namely, "125-minus-age." In this case, pensions increase relative to the optimal asset allocation ex post as a result of additional risk-taking that, from an ex ante perspective, is undesired. In other words, the optimal allocations moderate the risks better. Nevertheless, our finding that pension inequality is reduced under the optimal asset allocation holds also in comparison with this investment rule.

Second, we considered the distribution of pensions and the distribution over economies instead of over individuals. We find that in those economies in which pensions are the

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[21]We have also examined the effect of optimal allocation on consumption after retirement and found compression in it as well, though less than in pensions.

highest, the optimal allocation leads to lower pension compared with the "100-minus-age" rule. This is because the optimal allocation is associated with an active rebalancing to a smaller equity share in the event of high realized returns. However, by construction, each investor prefers the optimal equity share ex ante, as it better trades off risks and returns.

Finally, we considered an equity risk premium of just 2%, and all of our findings proved to be qualitatively robust to this change. We elaborate further on this in the Online Appendix.

8 Concluding remarks

We examine the effects of different equity exposures in the default fund in a defined contribution plan whose participants are heterogeneous. Using detailed data on individuals and their holdings inside and outside the pension system, we compare different investor types. We find that default investors are less educated, have less labor income, have less wealth, and are more reliant on their pension savings than active investors. Taken together, default investors can be viewed as less sophisticated investors. The observable variables capture a large part, but not all, of the unconditional differences in stock market participation. Potentially important unobservable variables could be related to financial literacy (e.g., experience of making investment decisions and various costs associated with investing). We also find that there is great heterogeneity among default investors. Whether or not a default investor participates in the stock market serves as an indicator variable. Most stock market participants are richer in terms of both labor income and financial wealth than are non-participants.

Following these findings, we set up a life-cycle model that captures the economic situation of a default investor. The model has standard building blocks such as risky labor income, a consumption–savings decision, and a portfolio choice. We augment the standard model with a pension system in which individuals save in illiquid pension accounts from which their pension is received as annuities. Prompted by documented heterogeneity among default

investors, we introduce different stock market participation costs for different investors. This helps us match stock market participation in the data and gives us heterogeneity over and above the observable variables. We use the model to discuss the optimal asset allocation of the default fund and relate it to common age-based investment rules.

With heterogeneity among investors, it becomes important to derive the optimal asset allocation for each default investors' individual characteristics. We explore how the optimal design relates to other variables in addition to the age of the investor. We find that age is indeed important but that the optimal rule also relies heavily on the pension account balance and stock market participation outside the pension system. We encourage pension agencies and plan sponsors to study the practical and legal aspects of designing a default fund based not only on age but also on other observable characteristics.

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Table 1: Averages of variables

	All	Default	Initially active	Active
Investors				
Number of investors	318,345	108,465	89,218	120,662
Fraction of investors	1.000	0.341	0.280	0.379
State variables				
Age	45.9	42.1	47.9	46.7
Labor income	242,377	192,096	247,834	283,541
Financial wealth	238,888	164,079	258,312	291,772
Stock market exposure				
Participation dummy	0.517	0.388	0.537	0.618
Equity share (unconditional)	0.155	0.119	0.158	0.184
Equity share (conditional)	0.299	0.307	0.294	0.298
Real estate ownership and net worth				
Real estate dummy	0.678	0.504	0.745	0.785
Real estate wealth	852,599	632,416	922,374	998,932
Net worth	703,827	484,629	787,829	838,758
Educational dummies				
Elementary school	0.154	0.191	0.163	0.115
High school	0.555	0.548	0.567	0.554
College	0.279	0.251	0.261	0.318
PhD	0.011	0.010	0.009	0.013

The table presents averages of variables for all investors and investor categories in 2007. "Default" refers to investors who are invested in the default fund. "Initially active" refers to investors who opted out of the default fund when entering the pension system but have since never changed the fund allocation. "Active" refers to investors who, after entering the pension system, made at least one change to their fund allocation. The number of investors refers to the number of investors in each category. The fraction of investors refers to the number of investors in each category relative to the total number of investors. Labor income refers to gross annual labor income. Financial wealth includes financial wealth outside the pension system, i.e., bank accounts, direct bond and stock holdings, mutual funds, as well as the balances in private pension accounts and capital insurance. The participation dummy is assigned a value of one if the investor holds either stocks or equity funds outside the pension system. The unconditional equity share is the value of direct stock holdings and equity funds divided by financial wealth (excluding private pension accounts and capital insurance). The conditional equity share is the equity share for investors who participate in the stock market. The real estate dummy is assigned a value of one if the investor owns either a house or an apartment. Real estate wealth is the value of houses and apartments (not conditioning on owning real estate). Net worth is the sum of financial wealth and real estate wealth minus total debt (e.g., mortgages, credit card debt, and student loans). The educational dummies are assigned a value of one for the investor's highest obtained education.

Table 2: Stock market participation outside the pension system

	I	II	III	IV
Default investor dummy		–0.133***	–0.087***	–0.087***
		(0.002)	(0.002)	(0.003)
Initially active investor dummy		–0.055***	–0.037***	–0.038***
		(0.002)	(0.002)	(0.002)
Age	0.080***	0.022***	—	—
	(0.007)	(0.007)		
Labor income	0.153***	0.119***	—	—
	(0.004)	(0.004)		
Financial wealth	0.293***	0.289***	—	—
	(0.002)	(0.002)		
Real estate dummy	0.149***	0.127***	0.063***	0.054***
	(0.002)	(0.002)	(0.002)	(0.002)
Educational dummies	Yes	Yes	Yes	Yes
Geographical dummies	Yes	Yes	Yes	Yes
Industry & occupational dummies	No	No	No	Yes
Age/income/wealth splines	No	No	Yes	Yes
R-squared	0.141	0.153	0.295	0.283
Number of observations	318,345	318,345	318,345	186,651

The table presents the results of regressions of a participation dummy (one if the investor participates in the stock market directly or indirectly outside the pension system, zero otherwise) on a default investor dummy (one if the investor holds the default fund, zero otherwise) and on an initially active investor dummy (one if the investor was initially active but then passive, zero otherwise). The other covariates are the state variables of a life-cycle portfolio choice model (i.e., age, labor income, and financial wealth) and a dummy for real estate ownership. Age is scaled down by 100, and labor income and financial wealth are scaled down by 1,000,000. All specifications include educational and geographical dummy variables. Specifications III and IV replace the linear specifications of age, labor income, and financial wealth with piecewise linear splines. For brevity, the coefficients of these variables are not presented in the table. Specification IV includes industry and occupational dummy variables. The number of observations in this specification is lower due to missing industry or occupational information for some investors. The sample consists of investors in 2007. Standard errors, robust to conditional heteroscedasticity, are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table 3: Distribution of variables for default investors

	10%	25%	50%	75%	90%	Mean
A. All default investors						
Age	24	29	42	53	63	42.1
Labor income	0	52,446	190,961	277,860	364,184	192,096
Financial wealth	7,135	9,283	40,920	149,388	415,102	164,079
Equity share	0.000	0.000	0.000	0.129	0.481	0.119
B. Participants						
Age	23	29	43	56	64	43.2
Labor income	0	74,593	220,137	311,839	423,176	224,700
Financial wealth	17,141	47,429	132,028	349,771	793,259	311,524
Equity share	0.018	0.072	0.231	0.491	0.736	0.307
C. Non-participants						
Age	24	30	41	51	61	43.2
Labor income	0	40,577	174,824	257,725	328,518	171,387
Financial wealth	7,135	7,135	16,485	60,764	167,724	70,429
Equity share	0.000	0.000	0.000	0.000	0.000	0.000

The table presents the averages of variables for default investors by percentiles in 2007. Panel A refers to all default investors. Panel B refers to default investors who participate in the stock market. Panel C refers to default investors who do not participate in the stock market. Labor income refers to gross annual labor income. A total of 108,465 investors are represented in Panel A, 42,132 in Panel B, and 66,333 in Panel C. Financial wealth includes financial wealth outside the pension system, i.e., bank accounts, direct bond and stock holdings, mutual funds, as well as the balances in private pension accounts and capital insurance. The equity share is the value of direct stock holdings and equity funds divided by financial wealth (excluding private pension accounts and capital insurance): the equity share in Panel B is the equity share of investors who participate in the stock market; the equity share in Panel C is that of investors who do not participate in the stock market and by definition equals zero.

Table 4: Model parameters

|	Notation	Value
Preferences and stock market entry cost		
Discount factor*	β	0.932
Elasticity of intertemporal substitution	$1/\rho$	0.500
Relative risk aversion*	γ	12
Ceiling for stock market entry cost*	$\overline{\kappa}$	30,000
Floor for stock market entry cost*	$\underline{\kappa}$	0
Returns		
Gross risk-free rate	R_f	1.00
Equity premium	μ	0.04
Standard deviation of stock market return	σ_ε	0.18
Labor income and financial wealth		
Standard deviation of idiosyncratic labor income shock	σ_η	0.072
Weight of stock market shock in labor income	θ	0.040
Standard deviation of initial labor income	σ_z	0.391
Standard deviation of initial financial wealth	σ_A	1.365
Mean of initial financial wealth	**	111,300
Ceiling for contributions to DC and notional accounts	$\overline{Y}$	344,250
Floor for notional pension	$\underline{Y}$	85,829
Contribution rates in pension accounts		
DC account	λ^{DC}	7%
Notional account	λ^{N}	16%
Life-cycle profiles		
Labor-income profile	g_t	—
Survival rates	ϕ_t	—

The table presents the parameter values of the model. * The parameter value has been determined endogenously by simulation of the model. ** The mean initial financial wealth for 25-year-old default investors, $\exp(\mu_A - \sigma_A^2/2)$, is set to SEK 111,300. The labor-income profiles are discussed in detail in the main text. The survival rates are computed from unisex statistics provided by Statistics Sweden.

Table 5: DC account equity share in model simulated data

|	I	II	III	IV	V	VI	VII
Constant	1.647***	1.760***	1.580***	1.657***	1.213***	1.255***	1.174***
	(0.022)	(0.022)	(0.024)	(0.022)	(0.010)	(0.008)	(0.013)
Age	−0.021***	−0.020***	−0.018***	−0.019***	−0.005***	−0.005***	−0.004***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Labor income		−0.704***					0.287***
		(0.053)					(0.038)
Financial wealth			−0.323***				−0.116***
			(0.069)				(0.024)
Participation dummy				−0.232***		−0.195***	−0.197***
				(0.007)		(0.005)	(0.005)
DC account balance					−0.686***	−0.633***	−0.649***
					(0.023)	(0.016)	(0.017)
R-squared	0.549	0.619	0.629	0.668	0.769	0.852	0.858
Number of observations	7,000,000	7,000,000	7,000,000	7,000,000	7,000,000	7,000,000	7,000,000

The table presents the results of regressions of the model's optimal DC equity share on some of the model's state variables. The simulated data are based on 50 economies, each of which has 3500 investors (500 individuals with seven different stock market participation costs) who each work for 40 years. This gives a total of 7,000,000 simulated observations. Labor income, financial wealth, and DC account balance are scaled down by 1,000,000. Standard errors, robust to conditional heteroscedasticity and clustered over economy and individual, are reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

# Figure 1: Calibration and model fit
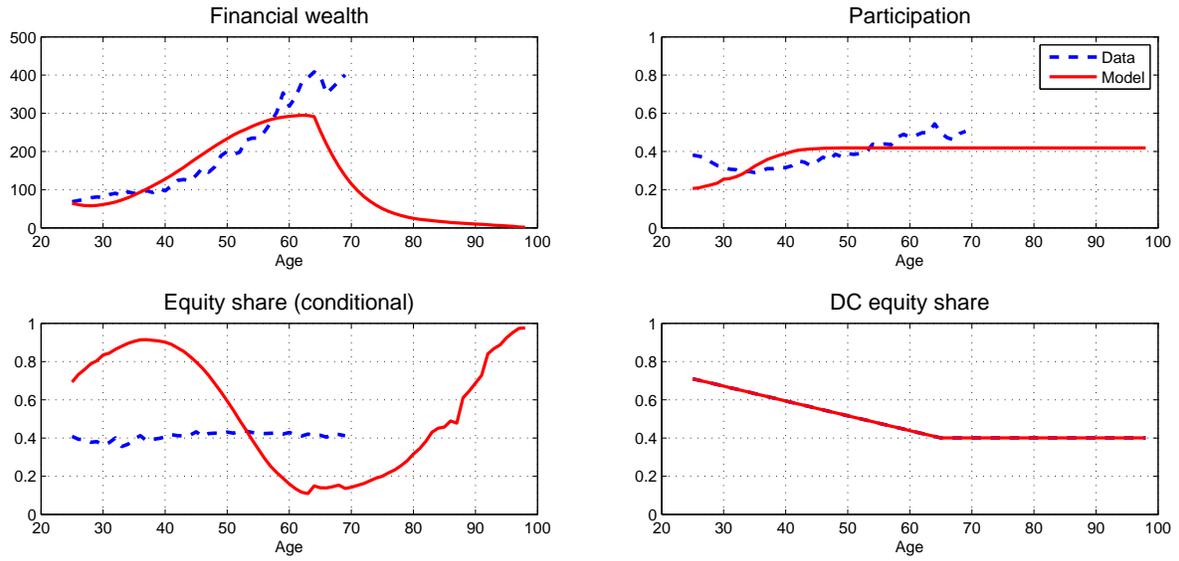
The figure shows the fit of the model relative to data and the DC equity share. The model simulation is based on 50 economies and 3500 individuals. Financial wealth is expressed in SEK 1000s.

## Figure 2: Model fit of participants and non-participants
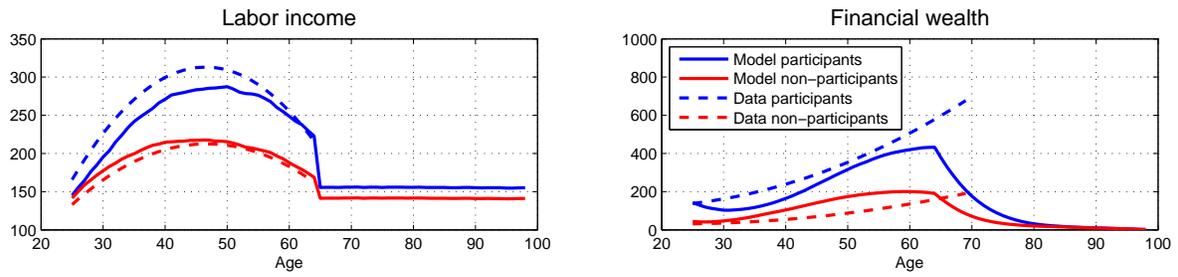
The figure shows the average labor income and financial wealth (both in SEK 1000s) for participants and non-participants due to endogenous sorting.

## Figure 3: Averages implied by the optimal asset allocation
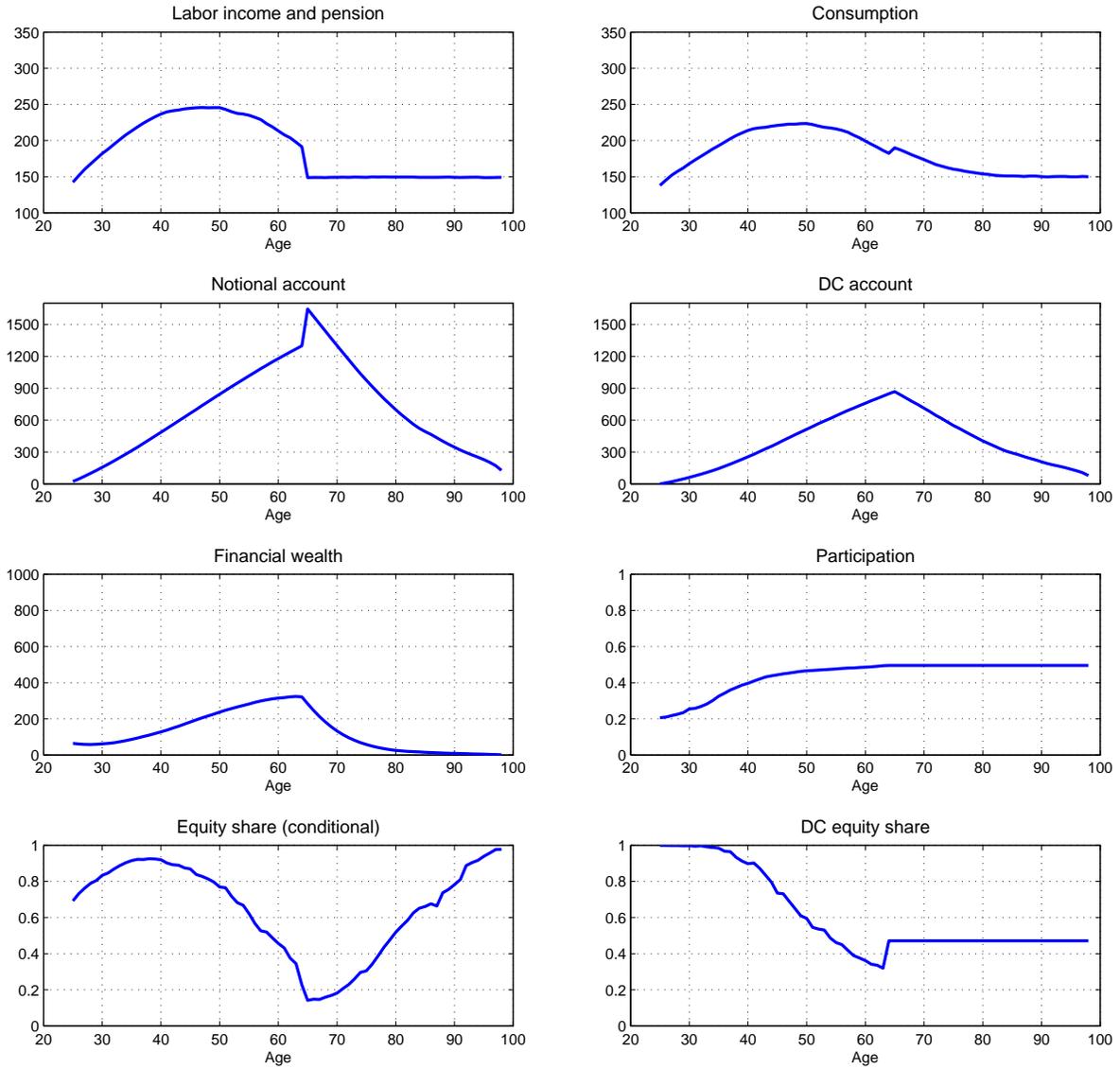
The figure shows averages over 50 economies and 3500 individuals for the optimal asset allocation. Labor income and pension, consumption, notional and DC accounts, and financial wealth are expressed in SEK 1000s.

Figure 4: Aggregate equity risk and inequality implied by the optimal asset allocation
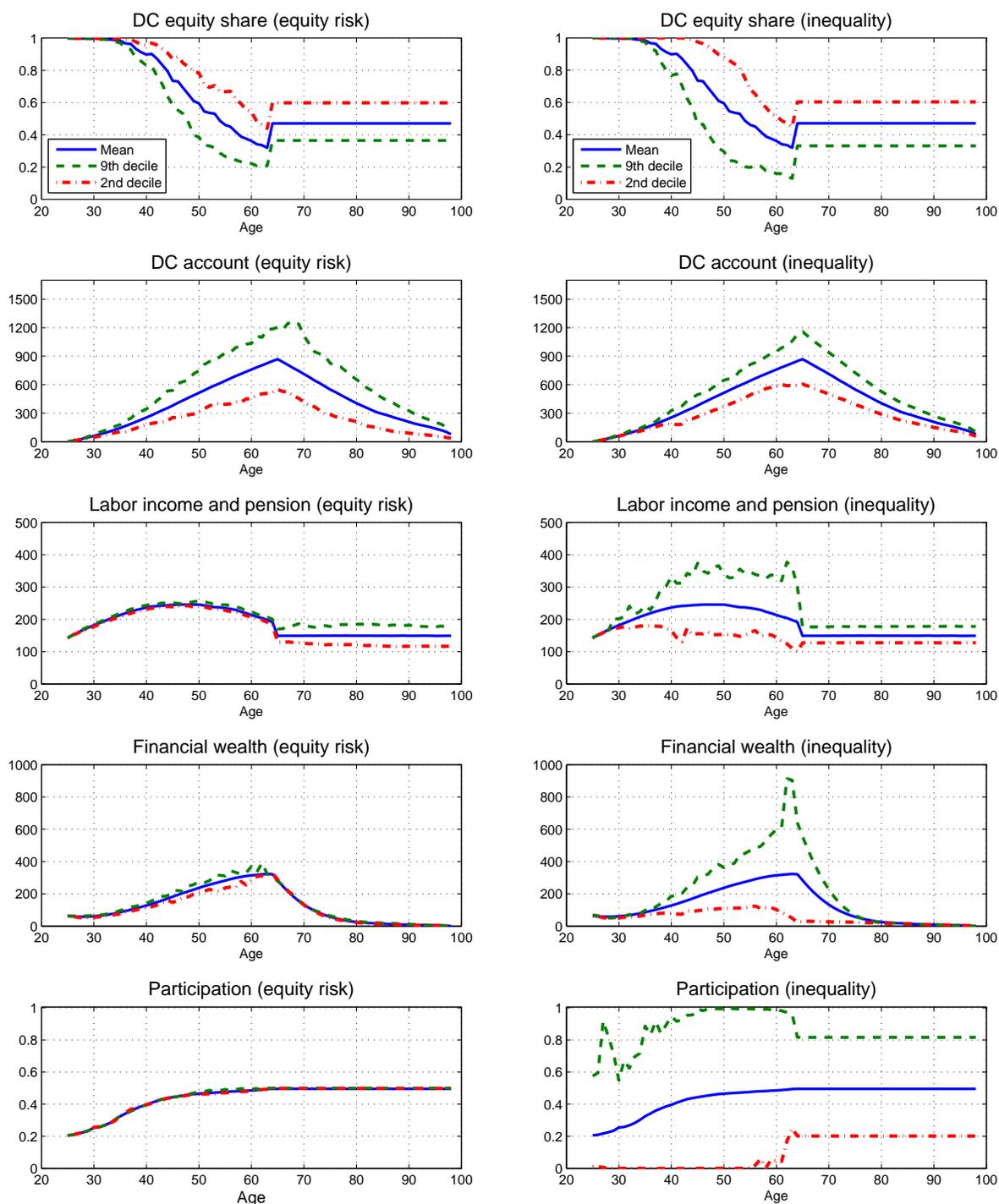
The figure shows the aggregate equity risk and inequality for the optimal asset allocation. The simulation is based on 50 economies and 3500 individuals. The left panels show how the averages vary over 50 economies. The second decile refers to the average of economies 6–10 (sorted). The ninth decile refers to the average of economies 41–45 (sorted). The right panels show how the averages vary over 3500 individuals. The second decile is based on the average of individuals 351–750. The ninth decile is based on the average of individuals 2801–3150. Note that the same economies and individuals are not tracked over time, i.e., the sorting at one age is independent of the sorting at another age. The DC account, labor income and pension, and financial wealth are expressed in SEK 1000s.

Figure 5: Pension inequality
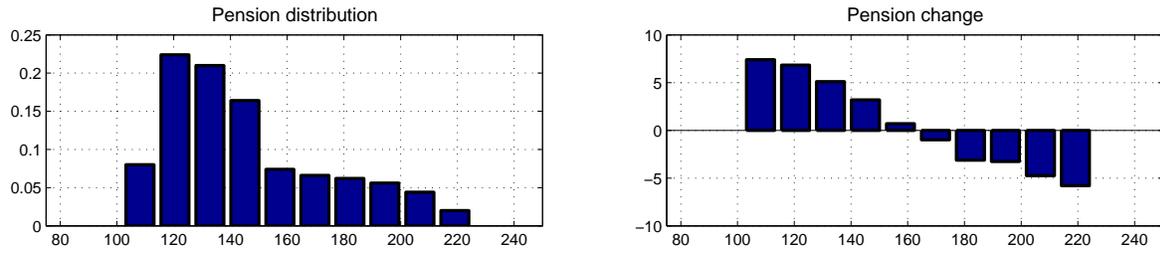
The figure on the left shows the distribution of pension (in SEK 1000s) at age 65 in cross-section under the "100-minus-age" rule. For each bar in the distribution, the figure to the right shows the pension changes (in %) associated with a shift from this asset allocation rule to the optimal allocation.